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Evaluation of LDPC codes efficiency in channels with fading

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Abstract. The article discusses the operation of communication systems using a low-density parity check codes in channels with multipath propagation. This class of codes has found wide application in modern communication and broadcasting systems and is one of the most effective in now. Some theoretical and practical issues of combating the effects arising from multipath propagation are presented. The description of the developed simulation model, channel codec, equalizer, and transmission channel is given. The channel encoder in the model is designed in accordance with the specifications for the 5G New Radio (5G NR) communication standard. The multipath parameters in the transmission channel correspond to the TDL models for 5G NR, with the exception of the Doppler shift, which is not taken into account in the simulation. In addition, there is additive interference in the channel. The decoding algorithm used was also described in detail. As a result of the study, the dependences of the bit error probability on the signal-to-noise ratio were obtained for the code rates 1/2 and 1/8, as well as for the system without coding. Based on the results obtained, it is possible to judge the correcting ability of this class of codes. Keywords: LDPC, low-density parity-check codes, multipath propagation, equalization, 5G NR.

1 Introduction

In the modern world, wireless communication has become widespread in many different applications. Due to the increase in traffic volume, the requirements for noise immunity and spectral efficiency are constantly growing.

To increase noise immunity, various methods and algorithms for signal generation and their processing are used. One of these methods is noise-immune coding, which directly affects the probability of a bit error (BER, Bit Error Rate), as well as the transmission rate and range.

The idea behind the noise-immune coding is to add redundancy (parity bits) to the information message, which makes it possible to detect and correct errors occurred during signal transmission over the communication channel. At the moment, there are many different algorithms for noise-immune coding. One of the most efficient is LDPC codes (Low-Density Parity-Check codes) [1].

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2 LDPC codes

Shannon's theorem establishes that at a transmission rate lower than a channel bandwidth, there is such an noise-immune codec, using which one can achieve an arbitrarily small probability of a bit error at a given signal-to-noise ratio (SNR). The theorem is valid under the condition of a large codeword length [2]. In practice, this theorem led to the introduction of two classes of codes permitting to get as close as possible to the Shannon limit (the maximum theoretical bandwidth of the channel).

LDPC codes are linear block codes. As in other block codes, the encoding procedure can be presented as the message vector multiplied by generator matrix G. Decoding can be performed by multiplying the vector of symbols at the demodulator output by check matrix H. The result of decoding is vector of syndromes S. Using the vector of syndromes, it is possible to determine the error positions and correct them.

An increase in the length of the codeword in classical block codes results in expanding both matrices (G and H), as well as the vector of syndromes. This leads to storage expansion and an increase in the computational complexity of encoding and decoding algorithms. The solution to this problem is the use of LDPC codes.

The idea behind the LDPC codes is that large matrices with a small (relative to the size) number of non-zero elements are used, which eliminates the need to store the entire matrices, because only the positions of non-zero elements can be stored. In addition, it becomes possible to use iterative decoding algorithms: their computational complexity is less than that of classical block code decoding algorithms. Gallager proposed two such algorithms: Bit Flipping for “hard” decoding and Belief Propagation for “soft” decoding.

After Gallager's works were published, the LDPC codes were forgotten for a long time, because it was impossible to apply them in real systems due to high computational complexity.

In practice, the LDPC codes were first introduced in the DVB-X2 digital television standards. Currently, the LDPC codes are widely used in communication systems, such as Wi-Fi 802.11n, 802.11ac, WiMAX 802.16e, and in 5G NR.

3 Multipath propagation

Multipath propagation is a phenomenon in which not only a direct signal arrives at the receiving antenna, but also its copies with a certain time delay and attenuation in amplitude. In the case of multipath propagation, the received signal can be described by the following formula:

$$X(t)=\sum_{n} G_{n}\left(t\right) S\left(t-\tau_{n}(t)\right), \tag{1}$$

where n is beam number, $G_n(t)$ is attenuation of the n-th beam, $\tau_n(t)$ is the delay of the n-th beam [3].

Multipath propagation leads to intersymbol interference.

If the duration of delays is small relative to the duration of a symbol, then multipath propagation results in signal fading.

The use of orthogonal frequency division multiplexing (OFDM) allows overcoming intersymbol interference. When OFDM is generated, a guard interval (cyclic prefix) is added to the beginning of each symbol, which eliminates the influence of intersymbol interference [4].

To overcome frequency fading, equalization is applied. Equalization is the estimation of the channel transfer function and spectral restoration of the signal received from it. It can be implemented using pilot subcarriers or pilot symbols.

In the first case, on the transmitting side, pilot signals are put on certain subcarriers. The transmitter can evaluate the transmission characteristics of the channel based on these subcarriers. Interpolating the received characteristic makes it possible to obtain the frequency response of the channel, which allows restoring the spectrum of the received signal.

In the second case, an OFDM symbol with pilot signals is located at certain positions in the frame. Knowing these symbols, the receiver can evaluate the frequency response of the channel and restore the spectrum of the received signal.

4 Methods

To study the efficiency of the LDPC codes in multipath channels, we have developed a simulation model of a communication system with OFDM. Its block diagram is shown in Figure 1.

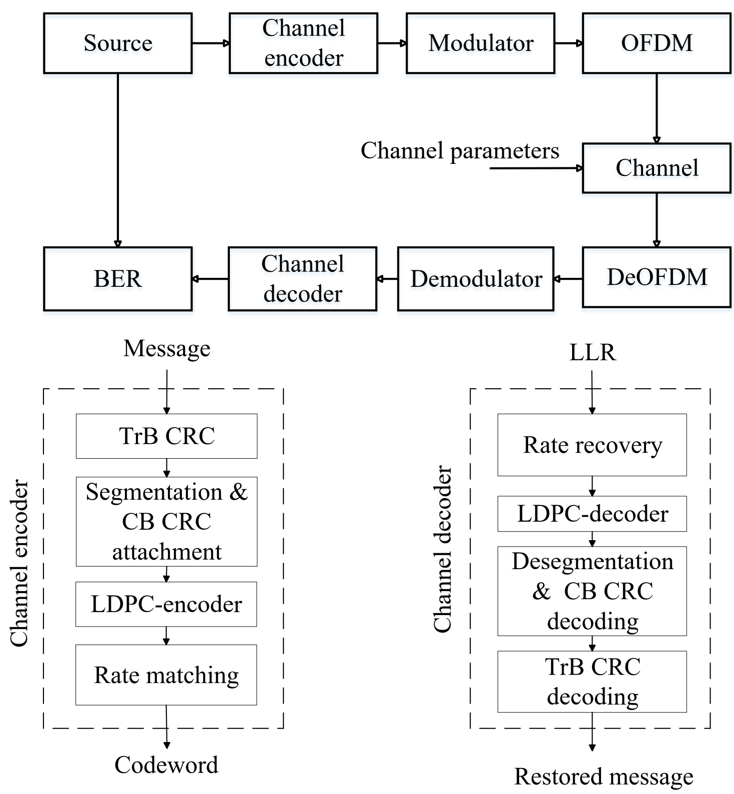

Fig. 1. Block diagram of the developed model.

In the OFDM block, OFDM symbols are generated, a pilot signal and cyclic prefix are added, and an inverse Fourier transform is performed. In the deOFDM block, reverse operations together with equalization are performed.

4.1 Channel codec

A codec developed in accordance with the specifications of 3GPP (3rd Generation Partnership Project) for 5th generation communication systems (5G NR) is used [5] as a channel codec. The LDPC codes in 5G are presented in a quasi-cyclic form. This code structure allows for the use of fast encoding and decoding algorithms [6-7]. As in other QC-LDPC codes, the check matrix is formed on protographs. The standard [6] defines two protographs as the base graphs: **BG1** and **BG2**.

The check matrix structure, as well as the sizes of input and output blocks, are completely specified by the base graph.

The procedure of channel coding using the LDPC codes in 5G consists of the following steps:

- Transport block Cyclic Redundancy Check (CRC) attachment (TrB CRC),
- Segmentation and code block CRC attachment (CB CRC),
- LDPC encoding,
- Rate matching [6].

The first stage is transport block CRC attachment. Also at this stage, the base graph is selected.

After that, if the transport block size is less than the maximum size of the input block, then the transport block goes directly to the LDPC encoder; otherwise, the transport block is segmented into code blocks and the CRC is added to each of them. Also, at the segmentation stage, parameters to be used later are calculated and determined.

After the segmentation, LDPC encoding takes place. To form a check matrix, each element $V_{i,j}$ of the base graph is replaced by a matrix of dimension Zc by Zc, where Zc is the lifting size of the base graph determined at the segmentation stage. If the element is -1, then the matrix will be zero; otherwise, the element is replaced by an identity shift matrix. The columns in this matrix are shifted by $V_{i,j}$ to the right (if $V_{i,j}>Z_c$, then by the remainder of dividing $V_{i,j}$ by Z_c), where $V_{i,j}$ is an element in the i-th row and j-th column of the base graph. If the element is zero, then it is replaced by an identity diagonal matrix [6].

After encoding, rate matching takes place: it means the process of matching the available resources (the number of antennas, the length of time segments, the number of subcarriers, etc.) and the length of the codewords. If there are more resources than it is required for transmission, some bits can be duplicated, which leads to the increase in redundancy; otherwise, some of the test bits may be punctured [8].

The Belief Propagation algorithm proposed by Gallager has a high computational complexity and is of little use in its original form. Instead, simplified variations of this algorithm based on MPA (Message Passing Algorithm) and SPA (Sum-Product Algorithm) are used. The developed model uses SPA [3,9–11].

This algorithm works with “soft” demodulator solutions and is executed over a number of iterations.

The “soft” solution of the demodulator refers to the log-likelihood ratio (LLR). LLR is a natural logarithm of the likelihood ratio of the fact that the received symbol corresponds to the bit “0" value to the fact that the received symbol corresponds to a “1” value:

$$LLR = \ln\left(\frac{\Pr(r_i \mid u=0)}{\Pr(r_i \mid u=1)}\right), \tag{2}$$

where Pr is occurrence probability of a certain bit, u is transmitted bits, and r is received symbol values.

To describe the algorithm, let us introduce the following notation:

$\mathbf{M} = \{m_i\}$ is the received message, i =1,2...N, N is the message length.

L is the LLR vector; $l\left(cw_i\right)$ — elements of the LLR vector;

$l\left(r_{ij}\right)$ — a posteriori probabilities calculated from the matrix check nodes; $l\left(q_{ij}\right)$ — a posteriori probabilities calculated from the matrix variable nodes;

$l(Q_i)$ — updated LLR values.

For convenience, let us illustrate the check and variable nodes on the Tanner graph (Figure 2) for the following check matrix:

$$\mathbf{H} = \begin{vmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{vmatrix}$$

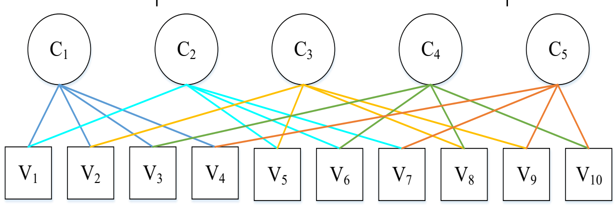

Fig. 2. Tanner Graph.

The Tanner graph is a bipartite graph divided into check nodes marked with circles and variable nodes marked with squares. The check nodes correspond to the rows of matrix **H**, and the variable nodes — to the columns of the matrix. Check node c_i is connected by an edge to variable node v_j if, and only if the matrix element in the i-th row and j-th column is not zero [12].

At the initialization stage, LLR is calculated using formula 3:

$$l\left(m_i\right) = \ln\left(\frac{P(m_i = 0)}{P(m_i = 1)}\right) \tag{3}$$

Further, at each iteration, the LLR vector is updated according to the following formulas:

$$l(r_{ij}) = 2\operatorname{atanh}\left(\prod_{i' \in V_{j \setminus i}} \tanh\left(\frac{l\left(q_{i'j}\right)}{2}\right)\right) \tag{4}$$

Before the first iteration $l\left(q_{ij}\right) = l\left(m_i\right)$, then

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$$l(q_{ij}) = l\left(m_i\right) + \sum_{j' \in C_{i \backslash j}} l\left(r_{j'i}\right) \tag{5}$$

Explanations of the indices are given below.

The "soft" output of the decoder is presented as the updated LLR values:

$$l\left(Q_i\right) = l\left(m_i\right) + \sum_{j' \in C_i} l\left(r_{j'i}\right) \tag{6}$$

The "hard" output of the decoder *c(i)* is equal to:

$$\begin{cases} l\left(Q_i\right) < 0 \rightarrow c_i = 1 \\ l\left(Q_i\right) \geq 0 \rightarrow c_i = 0 \end{cases} \tag{7}$$

After each iteration, the syndrome is checked.

$$\mathbf{S} = \mathbf{H}^{\mathrm{T}}\mathbf{c} \tag{8}$$

Decoding continues until the syndrome is zero or the number of iterations reaches its maximum.

Index sets C_i and V_j correspond to all non-zero elements in column *i* and row *j*, respectively. Whereas, set $C_{i \backslash j}$ corresponds to non-zero elements in the *i*-th column, with the exception of the *j*-th element of the column, and $\mathrm{V}_{j \backslash i}$ corresponds to non-zero elements in the *j*-th row, with the exception of the *i*-th element of the row.

The calculation of these index sets is shown in Figure 3.

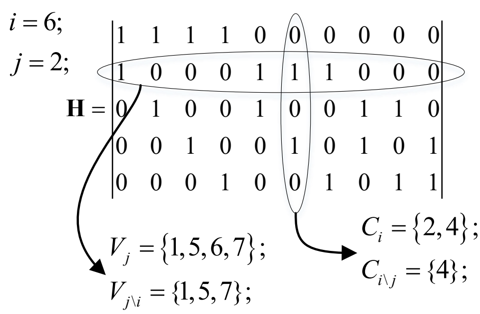

Fig. 3. Demonstration of calculating index sets.

4.2 Transmission channel

The channel in the developed model is the proposed 3GPP multipath channel models for 5G NR communication standards (TDLA, TDLB, TDLC) [13]. In these models, there is the limited number of beams; each has its own delay and attenuation.

An additive interference in the form of additive white Gaussian noise (AWGN) is also superimposed on the signal. The block diagram of the transmission channel is shown in Figure 4.

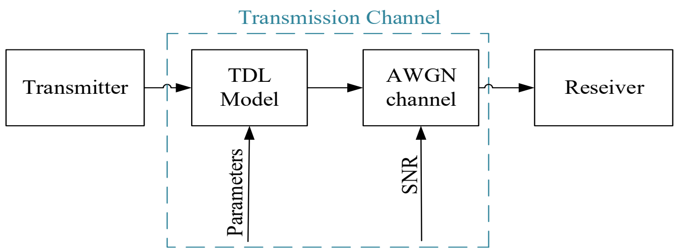

Fig. 4. Block diagram of the transmission channel.

4.3 Equalization

To overcome fading in the channel, we use the method for estimating the frequency response by the pilot symbol. The equalization method used is Zero Forcing [14]. Signal restoration by this method is described by the following formula:

$$x_{eq} = x\frac{\mathrm{conj}(h_i)}{\left|h_i\right|^2}, \tag{9}$$

where $\hat{x}_{eq}$ is the restored signal, $\hat{x}$ — the received signal, $\hat{h}_i$ — th the channel frequency response, and conj is the operation of complex conjugation.

5 Simulation results

The main parameters of the developed model are given in Table 1.

Table 1. Model parameters.

FFT size	1024
Cyclic prefix length	12
Transport block size	20480 bits
Number of blocks	1000
Equalization method	Zero Forcing
Type of modulation	QPSK
Code rate	128/1024, 512/1024
Decoding algorithm	SPA, maximum number of iterations is 8
Channel type	TDLA, TDLB, TDLC + AWGN with SNR [1:30] dB
Sampling frequency	15 MHz

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Time and frequency mismatch, as well as Doppler shift are not introduced.

The results are the dependencies of BER on SNR.

Figures 5–6 show the obtained dependences at R=128/1024 and R=512/1024, respectively. Also, for clarity, the charts show dependencies for a system without coding.

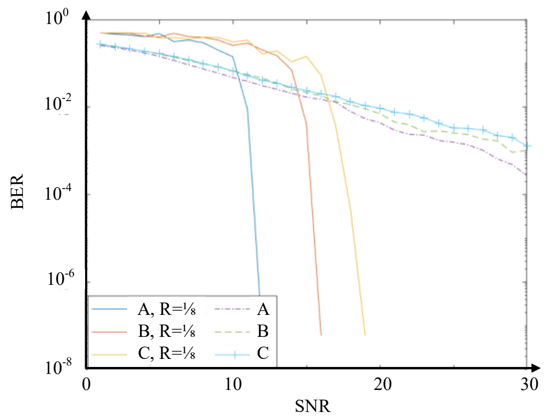

Fig. 5. Dependencies of BER on SNR for TDL A, B and C channels at R=128/1024, as well as system without coding.

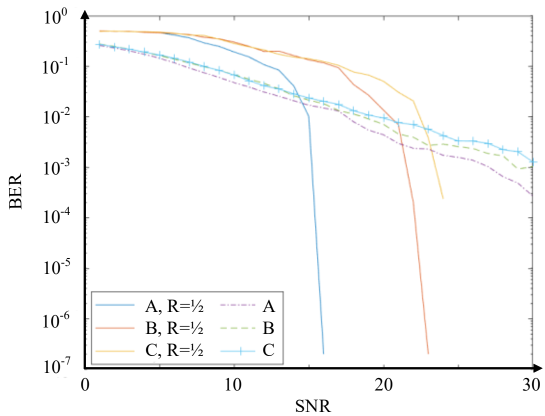

Fig. 6. Dependencies of BER on SNR for TDL A, B and C channels at R=512/1024, as well as system without coding.

Based on the results obtained, it is possible to estimate the effeciency of the LDPC codes use in multipath channels; as shown in the charts, at a code rate of *R*=128/1024, BER reaches the threshold level when

- SNR=12 dB for the TDLA channel,
- SNR=16 dB for the TDLB channel,
- SNR=19 dB for the TDLC channel;

with the code rate of *R*=512/1024, BER reaches the threshold level when:

- SNR=16 dB for the TDLA channel,
- SNR=23 dB for the TDLB channel,
- SNR=26 dB for the TDLC channel;

in a system without coding, BER does not reach the threshold level in the specified SNR range.

The threshold level for efficiency estimation is BER = 10^{-6}.

Following this, it can be concluded that the use of the LDPC codes can significantly reduce the probability of bit error, even in conditions of difficult signal propagation, which makes them a potential candidate for the role of a noise-immune code in next generations of communication systems.

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