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Mathematical modeling of temperature stresses of heat-generating elements of elliptical cross-section

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Abstract. The work is devoted to improving the reliability of nuclear power plants. During operation, the fuel elements of nuclear reactors operate at sufficiently high temperatures. The heterogeneity of the temperature field leads to the appearance of so-called thermal stresses. The main objective of this work is mathematical modeling of temperature stresses of fuel elements of cylindrical and elliptical cross-sections reactors. Based on mathematical modeling, it was shown that the level of thermal stresses arising during the transition from a circular cross-section to an elliptical one decreases.

1 Introduction

Nuclear reactors are power plants that generate energy based on nuclear fission reactions using fuel elements in which nuclear fission reactions occur, which release energy in the form of heat, while energy is extracted from the fuel elements through heat exchange with a coolant that cools these fuel elements.

In this case, the fuel rod is a tube of small diameter (or narrowed), muffled at both ends, forming a fuel element of a nuclear reactor and containing nuclear fuel. In this case, a fuel element with nuclear fuel is formed [1-7].

The fuel element must exhibit the following basic qualities:

- the density of its fissionable atoms must correspond to the parameters of the effect of neutrons and the energy density per unit volume of the reactive volume,

- it should transfer heat between the nuclear fuel and the coolant,

- it must retain solid and gaseous products of nuclear fission released by fuel during reactor operation. Nuclear fission reactions occurring inside nuclear fuel generate solid and gaseous fission products, which cause a possible significant swelling of nuclear fuel. The process of swelling, in particular gas swelling, is activated by heat, which also activates mechanisms by which gaseous fission products are released beyond the limits of nuclear fuel. Therefore, it is necessary that the shell of the fuel element be able to compensate for these deformations and gaseous emissions from the fuel without loss of integrity [8-10].

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The intensity of nuclear fission reactions inside the fuel is directly related to the amount of thermal power per unit volume, which should be diverted to the coolant through the shell of the fuel element.

Therefore, it is necessary to minimize the thermal resistance between the heat source and the cooling coolant in order to limit the maximum fuel temperature and the effects caused by the heat flow: the gradient in the nuclear fuel and various fuel and shell expansions.

Figure 1 shows an elliptical fuel rod.

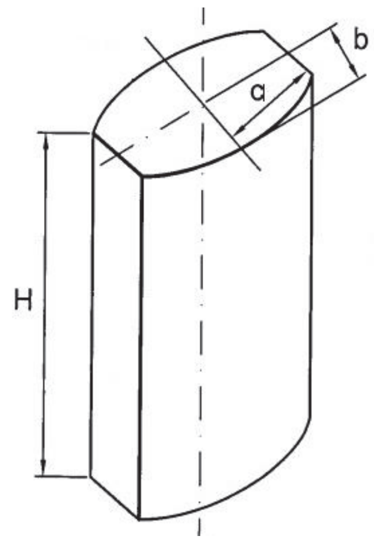

Fig. 1. Fuel rod.

The purpose of the work is to improve the design of the fuel element.

2 Main Part

The magnitude of the resulting thermal stresses depends on the temperature difference. In an elliptical cylinder , the components of the stress tensor have the form [11]

$$\sigma_{xx} = \frac{4A}{b^2}\left(\frac{x^2}{a^2} + \frac{3y^2}{b^2} - 1\right) \quad (1)$$

$$\sigma_{yy} = \frac{4A}{a^2}\left(\frac{3x^2}{a^2} + \frac{y^2}{b^2} - 1\right) \quad (2)$$

where

$$A = \frac{\alpha E q_v}{\lambda(1-\nu)\left(3\frac{a^4+b^4}{a^2b^2}+2\right)}\frac{a^2b^2}{8} \quad (3)$$

the axial voltage itself depends on the temperature distribution

$$\sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy}) - \alpha E T \quad (4)$$

We will conduct a comparative analysis of the corresponding components of the thermal stress tensor for circular or elliptical cylinders, taking into account R^2=ab (equality of cross-sectional areas). The ratio of the considered thermal stresses is equal to

$$\frac{\sigma_{xx}^{ell}}{\sigma_{xx}^{cir}} = \frac{8\left(\frac{x^2}{a^2}+\frac{3y^2}{b^2}-1\right)}{\left(3\left(\frac{b}{a}\right)^3+2\left(\frac{b}{a}\right)+3\left(\frac{a}{b}\right)\right)\left(\frac{x^2+3y^2}{R^2}-1\right)} \quad (5)$$

where: σ_{xx}^{ell} - thermal stresses in a cylindrical body of elliptical cross-section, σ_{xx}^{cir} - thermal stresses in a cylindrical body of circular cross-section.

Using the relation (5), it can be shown that the ratio of the considered thermal stresses takes the maximum value when the semi-axes are equal (a = b).

To determine the temperature field inside concrete structures, we will look for the temperature distribution in an infinitely long body, the section of which is an ellipse with semi-axes *a* and *b*. The body in question is located in an environment with an ambient temperature of T_0. The same heat source q_v operates inside the body.

To find the temperature distribution, it is necessary to solve the Poisson equation

$$\Delta T + \frac{q_v}{\lambda} = 0 \quad (6)$$

In this case, the boundary condition on the surface has the form

$$\left.\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right|_{T=T_c} \quad (7)$$

To obtain a formula describing the temperature field, we use the system of elliptic coordinates α, β, 0≤α<∞, -π≤β≤π. If $\alpha=\alpha_0$ is the equation of the body surface, then the Poisson equation in elliptic coordinates will take the form

$$\frac{1}{c^2(ch^2\alpha+\cos^2\beta)}\left(\frac{\partial^2 T}{\partial\alpha^2}+\frac{\partial^2 T}{\partial\beta^2}\right)=-\frac{q_v}{\lambda} \tag{8}$$

Finally, the desired temperature distribution is described by the equation [12]

$$T=T_c+\frac{q_v c^2}{4\lambda}\left(sh^2\alpha_0-sh^2\alpha\right) \tag{9}$$

Transform the expression (9), bringing it to a dimensionless form

$$\theta=\frac{4\lambda q_v(T-T_c)}{q_v c^2 sh^2\alpha_0}=1-\left(\frac{sh^2\alpha}{sh^2\alpha_0}\right)^2 \tag{10}$$

Thus, the temperature distribution in an elliptical body obeys a parabolic law, as shown in figure 2.

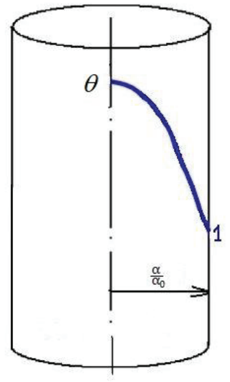

Fig. 2. Dimensionless temperature field of the element in the section.

We will conduct a comparative analysis of the temperature difference for circular or elliptical cylinders. As is known, the temperature difference between the center of the cylinder and its surface is equal to

$$T_0-T_R=\frac{q_v R^2}{4\lambda} \tag{11}$$

From formula (12) we find the temperature difference between the center of an elliptical body and its surface

$$T_0-T_c=\frac{q_v c^2}{4\lambda}sh^2\alpha_0 \tag{12}$$

their attitude takes the form

$$\frac{T_0 - T_c}{T_0 - T_R} = \frac{c^2 sh^2 \alpha_0}{R^2} \tag{13}$$

Based on mathematical modeling, it is obtained that when the geometric shape of the cross-section changes (during the transition to an elliptical section), the level of thermal stresses arising decreases. The last relation (13) shows that the temperature difference between the center and the surface decreases with the transition to an elliptical cross section. This is due to the fact that with an increase in the length of the outer boundary, the heat exchange surface also increases, which affects the intensity of heat removal to the environment. This leads to a decrease in the temperature difference between any center of the body and its surface. Thus, by varying the geometric shape of the cross-section, it is possible to reduce the level of thermal stresses that occur, and, consequently, to increase the durability of the reactor design.

3 Conclusions

Thus, the work was devoted to improving the reliability of nuclear power plants. The study of the basic physical and mechanical properties of fuel elements directly related to the mechanics of a deformable solid is carried out. The method of mathematical modeling was used to determine temperature fields and thermal stresses. Based on it, it was shown that it is necessary to determine the level of temperature stresses of the structures of fuel elements arising from the unevenness of the temperature field. Based on mathematical modeling, it was found that when switching to an elliptical cross-section of the fuel elements, the level of thermal stresses arising decreases. Which confirms the choice of the section of the fuel elements in the form of an ellipse.

References

1. S. A. Panfilov, O.V. Kabanov, International Journal of Advanced Biotechnology and Research **7(4)**, 1395–1402 (2016)
2. A.I. Kanareykin, IOP Conference Series: Earth and Environmental Science **990**, 012012 (2022)
3. R.L. Williamson, Journal of Nuclear Materials **423(1)**, 149-163 (2012)
4. J.D. Hales, Computational Materials Science **99**, 290-297 (2015)
5. Y. Rashid, R. Dunham, R. Montgomery, *Fuel analysis and licensing code: FALCON MOD0. EPRI Report 1011308* (2004)
6. A.I. Kanareykin, IOP Conference Series: Earth and Environmental Science **1045**, 012070 (2022)
7. H.C. Kim, Y.S. Yang, Y.H. Koo, KSME **38**, 157–166 (2014)
8. C. Newman, G. Hansen, D. Gaston, Journal of Nuclear Materials **392(1)**, 6-15 (2009)
9. J.C. Ramirez, M. Stan, P. Cristea, Journal of nuclear materials **359(3)**, 174-184 (2006)
10. B. Mihaila, Journal of Nuclear Materials **394(2)**, 182-189 (2009)
11. A.I. Kanareykin, Journal of Physics: Conference Series 012013 (2021)
12. A.I. Kanareykin, AIP Conference Proceedings **2762**, 020007 (2022)