On the question of the smallest rolling resistance coefficient value of elastic wheel on a rigid horizontal surface

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Abstract. As is known, rolling resistance of an elastic wheel on a rigid support surface is determined by internal friction losses (hysteresis) in the wheel material and friction losses in the contact of wheel with support surface (friction losses in the hub bearings and aerodynamic losses are neglected due to their smallness compared to with the above). To assess rolling resistance, the rolling resistance coefficient is used, which is defined as ratio of rolling resistance force applied to the wheel axle to the normal reaction on the wheel. Its value can be determined experimentally or by analytical dependencies. In this article, an analytical derivation of equation for calculating rolling resistance coefficient is given, followed by determining conditions under which this coefficient will be minimal.
Keywords: wheel, rolling, resistance, contact.

1 Introduction

Studies of elastic wheels rolling process (both theoretical and experimental) in order to determine their kinematic and power parameters are reflected in numerous publications, for example, in [1-18].

According to [19-23], when idle wheel is rolling, loaded only with a normal load, due to imperfect material elasticity, there are losses due to internal friction in the wheel material (hysteresis), which cause resistance moment occurrence M_f and rolling resistance force appearance F_f - a longitudinal tangential force, acting in contact of wheel with the base in direction opposite to wheel movement. A similar rolling resistance force also occurs on brake wheel loaded, in comparison with idle wheel, with an additional braking torque M_m. Presence of this force leads to angular velocity loss of wheel and to elements slippage of its treadmill relative to the base in contact zone. Angular velocity relative loss in this case can be represented as ratio of angular velocity absolute loss of wheel under the action of tangential force to angular velocity ω_0 during free rolling, determined from condition that tangential force acting in contact is equal to zero:

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$$\xi_{iw.} = \frac{\omega_0 - \omega}{\omega_0} = \frac{r_r - r_r^f}{r_r^f}$$

When drive wheel rolls, movement of which occurs under the action of torque M_K, a driving (traction) force appears in contact, directed along the wheel. Presence of this force causes slippage of wheel treadmill elements in the zone of contact with base and to the loss of wheel axis linear speed, which will be estimated by value of the relative loss of speed

$$\xi_{dr.} = \frac{V_0 - V}{V_0} = \frac{r_r^f - r_r}{r_r}$$

Where $V_0 = \omega r_r^f$ – wheel axle speed during its free rolling, in which there is no tangential force in contact with the base; $V = \omega r_r$ – is wheel axle speed in presence of force in contact.

Ratio $\frac{V}{\omega} = r_r$ is called rolling radius. In accordance with this r_r^f - is wheel free rolling radius in the absence of longitudinal tangential force in contact. For driving wheel $r_r^f = \frac{V_0}{\omega}$, for brake and idle wheels – $r_r^f = \frac{V}{\omega_0}$.

Rolling resistance of an elastic wheel on a rigid support surface is determined by internal friction losses (hysteresis) in wheel material and friction losses in the contact of wheel with support surface (we neglect friction losses in hub bearings and aerodynamic losses due to their smallness in terms of compared to the above).

2 Materials and methods

To assess rolling resistance, the rolling resistance coefficient is used, defined as ratio of the rolling resistance force F_f applied to the wheel axle (for driving wheel, this force is conditional) to normal reaction to the wheel F_n.

Rolling resistance force can be found from wheel power balance equation:

$$F_f V = P_H + P_{fr},$$

Where P_H and P_{fr}- are power losses for hysteresis and friction in contact, respectively, determined by the following formulas [19]:

$$P_H = F_n a_H \omega P_{fr}^{dr} = F_x \xi^{dr} V_0 P_{fr}^{br} = F_x \xi^{br} V ,$$

Where a_H- displacement of normal reaction of the supporting surface, due to hysteresis, relative to wheel axis; ω — is wheel angular velocity; F_x — longitudinal force acting in contact of wheel with supporting surface; V — is wheel axis speed, V_0 - is wheel axis speed during free rolling (when its axis is free from longitudinal force, and hence tangential force in contact is zero); ξ - relative loss of speed; index «dr» refers to driving wheel, index «br»- to brake and to idle.

For idle wheel, taking into account the above dependencies

$$F_{f_0} V = F_n a_H \omega + F_{f_0} \xi^{iw} V = F_n a_H \omega + F_{f_0} V - F_{f_0} \frac{r_r^f}{r_r} V \quad (1)$$

Where F_{f_0} — rolling resistance force of driven wheel. From this equation, we obtain formula that is convenient for experimentally determining displacement of normal reaction a_H, due to hysteresis, according to experimentally determined wheel rolling resistance coefficient $f_0 = \frac{F_{f0}}{F_z}$ in driven mode: $a_H = f_0 r_r^f$.

For driving wheel, rolling resistance force is determined from equality

$$F_f V = F_n a_H \omega + F_x \xi^{dr} \omega r_r^f \quad (2)$$

After transformations, taking into account the expressions for a_H and ξ^{dr}, we obtain

$$f = f_0 \frac{r_r^f}{r_r} + \frac{F_x}{F_z} \frac{r_r^f - r_r}{r_r} \quad (3)$$

Formula for determining rolling resistance in braking mode of wheel rolling will be obtained in a similar way from equation

$$F_f V = F_n a_H \omega + F_x \xi^{br} V \quad (4)$$

$$f = f_0 \frac{r_r^f}{r_r} + \frac{F_x}{F_z} \frac{r_r - r_r^f}{r_r} \quad (5)$$

If we assume that longitudinal force$F_x$or brake wheel is negative, then equation (3) will be common for all rolling modes.

Obtained dependencies allow us to consider such question as finding the minimum value of rolling resistance coefficient.

There is an opinion that the minimum value of f_{min} occurs when wheel is free to roll, when longitudinal tangential force in contact of the wheel with support surface $F_x = 0$ and rolling resistance moment due to hysteresis is completely overcome by driving moment supplied to wheel. According to another point of view,f_{min} corresponds to case of idle wheel rolling, when moment from hysteresis is overcome by longitudinal force F_0; applied to wheel axis; in this case, tangential force $F_x = -F_0$ acts in the contact of wheel with supporting surface.

If we put in expression (3) $F_x = 0$ (which corresponds to case of free rolling, for which $r_k = r_k^f$), then we get $f = f_0$. Assuming $F_x = -F_{f0}$,i.e. considering idle rolling mode, and taking into account that $\frac{F_{f0}}{F_n} = f_0$, we also obtain that in this case $f = f_0$.

This agreement between results is explained as follows. On the one hand, in idle mode, due to need to implement in contact the tangential force necessary to overcome the moment of rolling resistance due to hysteresis, there is some loss of wheel angular velocity compared to free rolling at the same wheel axle speed. Consequently, power loss due to hysteresis, under assumption that torque does not change from hysteresis, decreases somewhat. On the other hand, implementation of tangential force causes occurrence of a certain slip area and associated friction losses in contact. At a constant wheel axle speed, the sum of these power losses in idle mode is the same as power of hysteresis losses in free mode.$f = f(F_x)$

Since graph$f = f(F_x)$ is a parabolic curve (Fig. 1), it should be assumed that the minimum value of f_{min} is less than f_0 and is located on graph between points with abscissa $x = F_x = -F_0$ and $x = F_x = 0$.

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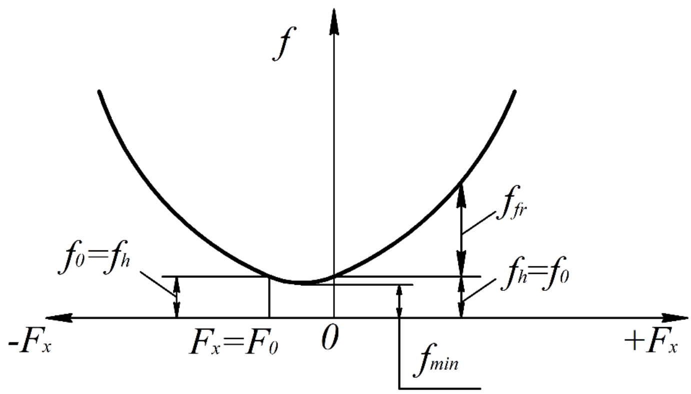

Fig. 1. Dependence of rolling resistance coefficientfon longitudinal tangential force F_x.

To determine extreme value of f_{min} we differentiatefwith respect to F_x and equate the resulting expression to zero, i.e.$\frac{df}{dF_x}= 0$.

For considered region of small forces$F_x$dependence of rolling radius on longitudinal tangential forces can be represented by the formula proposed by E.A. Chudakov [19]:

$$r_r = r_r^f \pm \gamma_F F \ , \tag{6}$$

whereγ_F- tangential elasticity coefficient.

After some transformations of expression$\frac{df}{dF_x}$we arrive at equation:

$$F_x^2 - \frac{2r_r^f}{\gamma} F_x - \frac{f_0 F_n r_r^f}{\gamma} = 0, \tag{7}$$

solving which with respect to F_x we get longitudinal tangential force, at which $f = f_{min}$:

$$F_x = \frac{r_r^f}{\gamma}\left(1 - \sqrt{1 + \frac{f_0 F_z \gamma}{r_r^f}}\right). \tag{8}$$

Rolling radius corresponding tof_{min}, taking into account expression (6), will be equal to:

$$r_r = r_r^f \sqrt{1 + \frac{f_0 F_z \gamma}{r_r^f}} \tag{9}$$

Substituting dependences (8) and (9) into equation (3), we obtain a formula that allows us to find the minimum value of rolling resistance coefficient:

$$f_{min} = \frac{2f_0}{\sqrt{1+\frac{f_0 F_n \gamma}{r_r^f}}} + \frac{2r_r^f}{\gamma F_z}\left(\frac{1}{\sqrt{1+\frac{f_0 F_n \gamma}{r_r^f}}}\right). \tag{10}$$

3 Results

For example,$f_{min}$and the corresponding values of$F_x$and$r_r$were calculated from obtained dependences for wheel with pneumatic tire of size 15.00–20 model Ya–190; r_r^f= 613 [mm], F_z= 32 [kN], γ = 0,0028 [mm/N], f_0= 0,019 [19]. Receivedf_{min}= 0,018988, F_x= –303,76[N], r_r= 613,581 [mm].

As can be seen from example, f_{min} differs very slightly fromf_0, and corresponding f_{min} value of F_x is almost equal to half force F_0=$f_0 F_n$= 608 [N]. It can be considered that the minimum value of rolling resistance coefficient corresponds to case when wheel is driven by a simultaneously acting longitudinal force equal to half force when wheel is rolling in idle mode, and driving moment equal to half the rolling resistance moment from hysteresis (i.e. this is the case corresponding to the middle of neutral regime).

Experimental confirmation of result obtained is not yet possible, because error of equipment available to researchers exceeds the value of difference between f_0 and f_{min}required for measuring.

4 Conclusion

The minimum value of rolling resistance coefficient corresponds to the case when wheel is driven by a simultaneously acting longitudinal force equal to half force when wheel is rolling in idle mode, and driving moment equal to half the rolling resistance moment from hysteresis (i.e. this is the case corresponding to the middle of neutral mode).

In practical calculations, difference between f_0 and f_{min} ,can obviously be neglected, i.e. assume that f_{min}=f_0.

References

1. J. Cho, S. Lee, H.Y. Jeong, Finite Elem Anal Des **105**, 26-32 (2015) DOI:10.1016/J.FINEL.2015.06.009
2. J. Li, Y. Zhang, J. Yi, J. DynSystMeas Control Trans ASME **135**, 359-370 (2012) DOI:10.1115/1.4006887.
3. S. Ozaki, W. Kondo, J Terramech **64**, 1-9 (2016) DOI:10.1016/j.jterra.2015.12.001
4. H. Dieter Kutzbach, A. Bürger, S. Böttinger, Journal of Terramechanics **82**, 13-21 (2019) https://doi.org/10.1016/j.jterra.2018.11.002
5. R.G. Longoria, R. Brushaber, A. Simms, Journal of Terramechanics **82**, 43-52 (2019) https://doi.org/10.1016/j.jterra.2018.12.004
6. W. Liang, J. Medanic, R. Ruhl, Veh. Syst. Dyn. **46**, 197–227 (2008)
7. Y. Wang, M.V. Blundell, G. Woodand, C. Bastien, Proc Inst Mech Eng Part K: J Multi-body Dyn **228**, 400–413 (2014)
8. H. Wang, I.L. Al-Qadi, I. Stanciulescu, Int. J. Pavement Eng **13**, 310–321 (2012)
9. M. Behroozi, O.A. Olatunbosun, W. Ding, Mater Des **35**, 810–819 (2012)
10. S. Kim, K. Kondo, T. Akasaka, Tire Sci Technol **28**, 2–32 (2000)
11. I. Vorozheikin, A. Marusin, I. Brylev, V. Vinogradova, Atlantis Highlights in Computer Sciences, Vol. 1. International Conference on Digital Transformation in Logistics and Infrastructure (ICDTLI 2019), 385–389 (2019) DOI: 10.2991/icdtli-19.2019.67

12. G. Ginzburg, S. Evtiukov, I. Brylev, S. Volkov, Transportation Research Procedia **20**, 212–218 (2017) DOI: 10.1016/j.trpro.2017.01.054
13. D. Fujiwara, N. Tsujikawa, T. Oshima, Journal of Terramechanics **94**, 1-12 (2021) https://doi.org/10.1016/j.jterra.2020.11.004
14. J. Kurjenluoma, L. Alakukku, J. Ahokas, Journal of Terramechanics **46(6)**, 267-275 (2009) https://doi.org/10.1016/j.jterra.2009.07.002
15. S.M. Shafaei, M. Loghavi, S. Kamgar, Soil and Tillage Research **211**, 104998 (2021) https://doi.org/10.1016/j.still.2021.104998
16. R.M. MakharoblidzeI, M. Lagvilava, B.B. Basilashvili, Z.K. Makharoblidze, Annals of Agrarian Science **16(1)**, 65-68 (2018) https://doi.org/10.1016/j.aasci.2017.12.010
17. J. Pytka, Soil and Tillage Research **82(1)**, 77-88 (2005) https://doi.org/10.1016/j.still.2004.06.005
18. S. Papamichael, Ch. Vrettos, Journal of Terramechanics **94**, 39-48 (2021)
19. R.V. Virabov, *Haulage capacity of friction gears* (Mashinostroenie, M., 1982)
20. T.A. Balabina, Y.I. Brovkina, A.N. Mamaev, Lecture Notes in Mechanical Engineering **9783319956299**, 2027-2035 (2019) DOI: 10.1007/978-3-319-95630-5_218
21. T.A. Balabina, D.S. Simonov, V.R. Rogov, A.N. Mamaev, A.S. Vorontsov, IOP Conference Series: Materials Science and Engineering **832**, 012077 (2020) DOI: 10.1088/1757-899X/832/1/012077
22. A. Mamaev, T. Balabina, M. Karelina, E3S Web of Conferences **363**, 01018 (2022) DOI: 10.1109/EMCTECH55220.2022.9934037.
23. M.Yu. Karelina, T.A. Balabina, A.N. Mamaev, Lecture Notes in Mechanical Engineering, Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019), 531–540 (2020) DOI: 10.1007/978-3-030-22063-1_56