Cooling of an infinite rectangular plate with an adiabatically insulated side

Aleksandr Kanareykin[1*]

[1]Sergo Ordzhonikidze Russian State University for Geological Prospecting, Miklouho-Maclay St. 23., 117997, Moscow, Russia

Abstract. The work is devoted to the issues of non-stationary heat transfer. The article presents a solution for the distribution of the temperature field in an infinite rectangular plate with an adiabatically isolated side. As a result, an analytical expression of the plate temperature distribution is obtained in the form of a series containing trigonometric and exponential functions. The paper also considered special cases when the internal thermal resistance of thermal conductivity is greater and when the external resistance of heat output is less. Special cases were interpreted physically. One of the special cases leads the problem to a problem with boundary conditions of the first kind, when the surface temperature is constant, which indicates the reliability of the results obtained.

1 Introduction

The creation of installations that are optimal in terms of energy consumption for today is unthinkable without a deep study of the thermophysical processes that take place in these installations.

An impressive layer of work is devoted to the study of heat exchange processes. In particular, works describing non-stationary heat exchange in modern heat exchange elements of heat exchange equipment are of particular scientific interest [1-3].

It is known that the propagation of heat in solids is described by a system of differential equations of thermal conductivity. Finding a solution to problems of this class is associated with many mathematical difficulties. At the same time, there are various methods for solving classical boundary value problems of non-stationary thermal conductivity and generalized type problems [4-6].

The paper considers the case of an adiabatically isolated wall. Which leads to the fact that the task is asymmetric. Several papers have been devoted to the calculation of temperature fields in the presence of adiabatic isolation [7, 8]. The solution of non-symmetric problems has a very complex and large result, the practical application of which is associated with great difficulties. Most of the solutions found in the literature are obtained for the so-called symmetric heat exchange conditions, when the maximum (minimum) temperature of the system is in the geometric centre of the body. In this paper, by applying the method of separation of the transients, the nonstationary problem of

---

* Corresponding author: kanareykins@mail.ru

temperature distribution in an unconstrained plate under boundary conditions of the third kind was solved. As a result, a fairly simple analytical solution of the temperature field distribution is obtained. At the same time, it was assumed that the thermophysical characteristics of the substance do not change during the cooling process.

2 Main Part

Consider a homogeneous plate with a thickness δ with constant physical characteristics (fig. 1). At the same time, at the initial moment of time t = 0, the temperature in the plate is evenly distributed and equal to To.
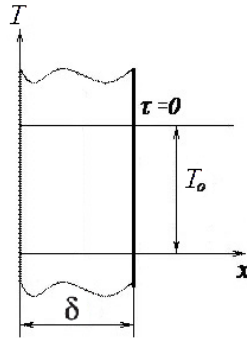

Fig. 1. Rectangular plate with adiabatically insulated wall.

To find a solution to the problem, it is necessary to solve a one-dimensional differential equation of thermal conductivity

$$\frac{\partial T}{\partial t} = \frac{\lambda}{c\rho} \frac{\partial^2 T}{\partial x^2} \qquad (1)$$

satisfying the following conditions: initial

$$T\big|_{t=0} = T_0 \qquad (2)$$

and boundary: there is a heat exchange on the right

$$\frac{\partial T}{\partial x} + hT\big|_{x=\delta} = 0 \qquad (3)$$

but not on the left

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (4)$$

To begin with, let's introduce a new variable

$$\tau = \frac{\lambda}{c\rho}t \tag{5}$$

In this case, equation (1) is simplified

$$\frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial x^2} \tag{6}$$

We will look for a solution in the form of a product of two functions: one of which $X(x)$ is a coordinate function, the other is $Y(\tau)$ –time

$$T(x,\tau) = X(x)Y(\tau) \tag{7}$$

Applying the Fourier method, we equate both functions to the constant k^2. As a result of this action, we obtain linear differential equations

$$X'' + k^2 X = 0 \tag{8}$$

$$Y'' + k^2 Y = 0 \tag{9}$$

The solution of equation (8) will be found in the form

$$X(x) = A\cos kx + B\sin kx \tag{10}$$

The solution of the second equation (9) is found in the form

$$Y(x) = Ce^{-k^2\tau} \tag{11}$$

It follows from condition (3) that

$$X'(\delta) + hX(\delta) = 0 \tag{12}$$

and from condition (4) it follows that

$$X'(0) = 0 \tag{13}$$

First, let's use condition (13)

$$-Ak\sin 0 + Bk\cos 0 = 0 \tag{14}$$

It follows from it that B = 0. Then the solution of equation (10) will be found in the following form

$$X(x) = A\cos kx \tag{15}$$

Now we apply the second boundary condition (3)

$$-Ak\sin k\delta + Ah\cos k\delta = 0 \tag{16}$$

Where from

$$ctgk\delta = \frac{k}{h} \tag{17}$$

Transform the right part

$$ctgk\delta = \frac{\delta k}{\delta h} \tag{18}$$

Denote the product kδ by μ. Then the expression (18) will take the form

$$ctg\mu = \frac{\mu}{Bi} \tag{19}$$

Whe re

$$Bi = \delta h = \frac{\alpha\delta}{\lambda} \tag{20}$$

The dimensionless number of Bio.

Equation (19) itself with constant coefficients is characteristic or transcendental. Thus, we obtain a set of functions satisfying the boundary condition

$$X_n(x) = A_n\cos\left(\frac{\mu_n x}{\delta}\right) \tag{21}$$

As a result, we get a set of temperature functions

$$T_n = M_n\cos\left(\frac{\mu_n x}{\delta}\right)e^{-\left(\frac{\mu_n}{a}\right)^2\tau} \tag{22}$$

To satisfy the boundary condition (2), it is necessary to put the numbers Mn equal to the generalized Fourier coefficients

$$M_n = \frac{\int_0^\delta T_0\cos\left(\frac{\mu_n x}{\delta}\right)dx}{\int_0^\delta \cos^2\left(\frac{\mu_n x}{\delta}\right)dx} = \frac{T_0\frac{\delta}{\mu_n}\sin\mu_n}{\frac{\delta}{2}+\frac{\delta}{4\mu_n}\sin 2\mu_n} = T_0\frac{2\sin\mu_n}{\mu_n+\sin\mu_n\cos\mu_n} \tag{23}$$

Substituting now the values of M_n in (22), we obtain a formula for determining the temperature field in an asymmetrically cooled homogeneous plate

$$T(x,\tau) = \sum_n T_0 \frac{2\sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \cos\left(\frac{\mu_n x}{\delta}\right) e^{-\left(\frac{\mu_n}{a}\right)^2 \tau} \qquad (24)$$

In dimensionless form, equation (24) is written as

$$\theta(x, F_0) = \frac{T}{T_0} = \sum_n \frac{2\sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \cos\left(\frac{\mu_n x}{\delta}\right) e^{-\mu_n^2 F_0} \qquad (25)$$

Where

$$F_0 = \frac{a\tau}{\delta^2} \qquad (26)$$

The Fourier criterion.

We will conduct a study of the behavior of the temperature field of the plate. In the case when the number Bi tends to infinity, it means that the intensity of external heat transfer is infinitely high. Which leads to the fact that the surface temperature of the plate is equal to the ambient temperature. In this case, we obtain a problem with boundary conditions of the first kind when the surface temperature is constant.

3 Conclusions

In this paper, an analytical expression was obtained for finding the temperature field in a plate of infinite length with an adiabatically isolated side. According to the obtained analytical expression, the temperature field of the plate during cooling at any time has the form of an asymmetric curve in the form of a cosine and decreases in time according to the exponential law. Special cases were also considered. To do this, the resulting solution was investigated for small and large values of the Bio number. The reliability of the results is confirmed by the fact that one of the particular cases leads the problem to a problem with boundary conditions of the first kind, when the surface temperature is constant.

References

1. Ji. Li, Journal of Thermal Science **9(2)**, 152 – 157 (2000)
2. C.Y. Yang, International Journal of Heat and Mass Transfer **48**, 3506 – 3515 (2005)
3. L. Xuefang, D.M. Christofer, Chinese Science Bulletin **59(12)**, 1334 – 1339 (2014)
4. S.A. Rukolaine, International Journal of Heat and Mass Transfer **78**, 58 – 63 (2014)
5. M. Fabrizio, Heat and Mass Transfer **53(9)**, 2785 – 2797 (2017)
6. W.A. Shen, Heat and Mass Transfer **39**, 499 –507 (2003)
7. S.C. Mishra, International Journal of Heat and Mass Transfer **61**, 41 – 55 (2013)
8. A. Kanareykin, E3S Web of Conferences **258**, 09071 (2021)