E3S Web of Conferences **376**, 01082 (2023) https://doi.org/10.1051/e3sconf/202337601082
ERSME-2023

Predictive modeling of wave hydrodynamics and relief formation in the presence of multi-scale turbulent exchange

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Abstract. Introduction. Reliable prediction of indicators of turbulent flows is a very difficult task, which is explained by the exceptional physical complexity of turbulence, in particular its probabilistic nature, a wide space-time spectrum and a fundamentally three-dimensional non-stationary nature. Despite conducting a wide range of studies focused on the problem under consideration, they did not fully reflect the totality of various factors and processes affecting the structure and parameters of vertical turbulent mixing. Materials and methods. The article is devoted to the study of spatial-three-dimensional wave processes in shallow water bodies, taking into account the features of turbulent exchange depending on the source and localization in the column of liquid, as well as the study of the influence of regular wave processes on turbulent exchange and vertically using a mathematical model of wave processes based on the system of Navier-Stokes equations, including three equations of motion in the with dynamically changing geometry of the computational domain. Results. Based on the developed software package, a scenario of changes in hydrodynamic wave processes of the coastal zone is constructed. Discussions and conclusions. The separation of the wave flow into a near-surface macroturbulent layer caused by wave motion and a lower layer with background hydrodynamic turbulence is proved, the strength and intensity of turbulence changed synchronously with wave oscillations, demonstrating a pronounced asymmetry of turbulence generation throughout the water column.

1 Introduction

The tendencies of combining modern mathematical apparatus and computing technologies to solve practical problems in various areas of human-environment interaction under conditions of natural and man-made impacts are actively manifested. Computational technologies based on modern mathematical methods are becoming the main tools in the development of fundamentally new technologies, while mathematical modeling is an integral part of unique physical experiments and forecasting. On the other hand, the development of methods of

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 https://doi.org/10.1051/e3sconf/202337601082

mathematical modeling of these processes should, of course, be accompanied by a wide range of verification experiments. The current state of developing methods of numerical modeling of hydrodynamic processes does not allow, in some cases, to provide an acceptable level of their description even when using the most modern models and supercomputer capacities. This is due to the extreme complexity, multi-scale and variety of physical processes occurring in marine systems [1-4]. In this regard, one of the main directions of increasing the effectiveness of research related to the study of the dynamics of the development of coastal and shallow-water systems, using mathematical modeling, is the development and development of technologies for conducting model and field studies taking into account the large-scale effect [5-6, 8].

Thus, the task of constructing and researching a spatial-three-dimensional model of wave hydrodynamics designed for modeling hydrodynamic processes in the presence of multi-scale turbulent exchange, based on the coordination of analytical, numerical, experimental approaches and comparisons using field data, is relevant. The article is devoted to the study of the influence of wave processes on the turbulent vertical exchange. The initial conditions for modeling are obtained based on the processing of remote sensing data. The article uses multispectral satellite images as sounding data. Based on the obtained images, the initial conditions for a mathematical model of hydrodynamics can be determined, on the basis of which predictive calculations are performed. Remote sensing data allows, having a series of processed images of the same water area for different time points (dates), to determine the dynamics of changes in the coastline.

Modern numerical models such as SWAN, SWASH, FINLAB, H2Ocean and XBeach models are constantly being improved thanks to new scientific discoveries obtained as a result of research involving laboratory and field experiments. Flow velocities, turbulence properties and forces acting on objects can be determined and used to interpret observed phenomena, for example, erosion, and the data can be used to validate the model. At the same time, the complexity of obtaining full-scale data in the real area indicates the need to involve 3D models of hydrodynamics that take into account the specifics of coastal systems [4-5]. Also, the development of these algorithms should lead to seamless implementation on peta and exascale architectures.

2 Materials and methods

3D wave hydrodynamics mathematical model

3D wave hydrodynamics mathematical model includes [7]:

– equations of motion (Navier - Stokes):

$$u_t' + uu_x' + vu_y' + wu_z' = -\frac{1}{\rho}P_x' + \left(\mu u_x'\right)_x' + \left(\mu u_y'\right)_y' + \left(\nu u_z'\right)_z',$$

$$v_t' + uv_x' + vv_y' + wv_z' = -\frac{1}{\rho}P_y' + \left(\mu v_x'\right)_x' + \left(\mu v_y'\right)_y' + \left(\nu v_z'\right)_z', \quad (1)$$

$$w_t' + uw_x' + vw_y' + ww_z' = -\frac{1}{\rho}P_z' + \left(\mu w_x'\right)_x' + \left(\mu w_y'\right)_y' + \left(\nu w_z'\right)_z' + g$$

– continuity equation:

$$\rho_t' + (\rho u)_x' + (\rho v)_y' + (\rho w)_z' = 0, \tag{2}$$

where $\mathbf{V} = \{u, v, w\}$ is the velocity vector of the water flow of a shallow water body; ρ is the density of the aquatic environment; P is the hydrodynamic pressure; g is the gravitational acceleration; μ,ν are coefficients of turbulent exchange in the horizontal and vertical directions; $\mathbf{n}$ is the normal vector to the surface describing the boundary of the computational domain.

Add boundary conditions to system (1)-(2):

– the entrance (left border): $\mathbf{V} = \mathbf{V}_0$, $P_\mathbf{n}' = 0$,

– the bottom border: $\rho\mu(\mathbf{V}_\tau)_\mathbf{n}' = -\boldsymbol{\tau}$, $\mathbf{V}_\mathbf{n} = 0$, $P_\mathbf{n}' = 0$,

– the lateral border: $(\mathbf{V}_\tau)_\mathbf{n}' = 0$, $\mathbf{V}_\mathbf{n} = 0$, $P_\mathbf{n}' = 0$,

– the upper border: $\rho\mu(\mathbf{V}_\tau)_\mathbf{n}' = -\boldsymbol{\tau}$, $w = -\omega - P_t'/\rho g$, $P_\mathbf{n}' = 0$,

– the surface of the structure: $\rho\mu(\mathbf{V}_\tau)_\mathbf{n}' = -\boldsymbol{\tau}, w = 0$, $P_\mathbf{n}' = 0$,

where ω is the intensity of evaporation of a liquid, $\mathbf{V}_\mathbf{n}$, $\mathbf{V}_\tau$ are the normal and tangential component of the velocity vector, $\boldsymbol{\tau} = \{\tau_x, \tau_y, \tau_z\}$ is the vector of tangential stress. Fig. 1 shows the geometry of the water body.

2.1 Raster model of the bottom

The basis for the representation of spatial information in both vector and raster data models are discrete operational territorial units (OTU). Each OTU is a spatial object for which the homogeneity of the attribute information available about it is assumed from the point of view of the phenomenon being studied. In the raster data model, OTU correspond to cells of a regular grid that completely covers the entire territory of the characteristic size of the represented spatial objects and phenomena (Fig. 1).

The initial data for constructing a raster model are measurements of variable values at points with known coordinates. These data are written to an ASCII–encoded text file consisting of three or more columns (1 – X coordinate, 2 – Y coordinate, Z_1 ... Z_n – the remaining 4 columns are the measured values using which the values in the nodes of the regular grid will be calculated). The structure of this file is as follows: X_1 Y_1 Z_1 … Z_n … X_m Y_m Z_m … Z_{nm}.

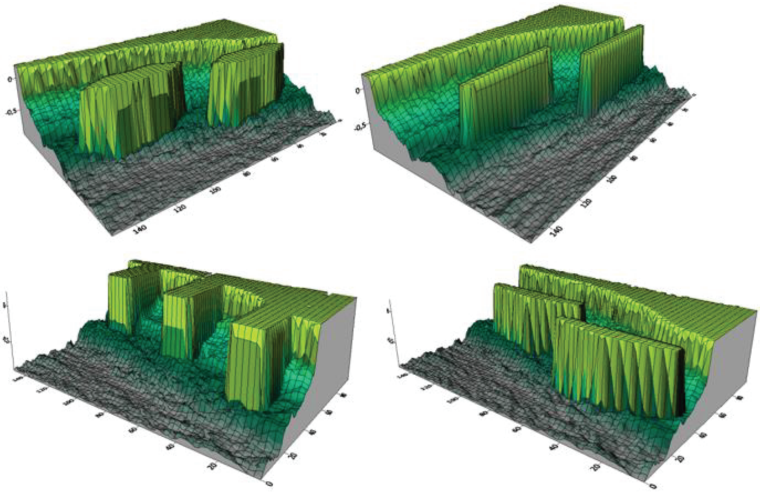

Fig. 1. Computational domain depth map.

2.2 The discrete model of hydrodynamics

The main requirement for a discrete model is the implementation of conservation laws that are valid in the initial physical and mathematical formulation of the problem, the so-called principle of conservativeness. The Navier-Stokes equations for an incompressible fluid are the laws of conservation of mass and momentum written in differential form. There is no equation for kinetic energy, and the balance of kinetic energy is a consequence of the laws of conservation of mass and momentum. In the difference case, the fulfillment of the laws of conservation of mass and momentum is achieved by approximating the divergent form of the original differential equations by the integro-interpolation method. In the difference case, the fulfillment of the mass and momentum balances, generally speaking, does not follow the fulfillment of the kinetic energy balance. It is possible to construct schemes satisfying the laws of conservation of mass and momentum, but not preserving kinetic energy.

As applied to the Navier-Stokes equations for incompressible fluid, the need to fulfill the principle of conservativeness was first demonstrated by A. Arakawa. If the energy conservation law is not fulfilled in the discrete model, either excessive dissipation or an increase in kinetic energy is observed in the system. In the case of excessive dissipation, the scheme is stable, but the results obtained with its help at high values of the Reynolds number are not physical: there is a strong attenuation of energy in the regions of high gradients. Such, for example, are schemes with approximation of convective terms against the flow of the first order. The increase in kinetic energy leads to instability of the numerical procedure. Schemes that are stable at small values of the Reynolds number and unstable at large ones are especially dangerous.

The advantage of circuits on spaced grids is the connection between speed and pressure, which does not give non-physical, grid oscillations characteristic of circuits on combined grids.

The computational domain inscribed in a parallelepiped. For the numerical realization of the discrete mathematical model of the hydrodynamic problem posed, a uniform grid is introduced:

$$\overline{w}_h = \left\{ t^n = n\tau, x_i = ih_x, y_j = jh_y, z_k = kh_z;\ n = \overline{0..N_t}, i = \overline{0..N_x}, j = \overline{0..N_y}, k = \overline{0..N_z};\right.$$

$$N_t\tau = T, N_x h_x = l_x, N_y h_y = l_y, N_z h_z = l_z\},$$

where τ is the step by the time, h_x, h_y h_z are steps in space, N_t is the number of time layers, T is the upper bound on the time coordinate, N_x, N_y N_z are the number of nodes by spatial coordinates, l_x, l_y l_z are the boundaries along the parallelepiped in the direction of the axes Ox, Oy and Oz accordingly.

The method of correction to pressure was used to solve the hydrodynamic problem. The variant of this method in the case of a variable density will take the form [9-10]:

$$\frac{\tilde{u}-u}{\tau} + u\bar{u}'_x + v\bar{u}'_y + w\bar{u}'_z = \left(\mu\bar{u}'_x\right)'_x + \left(\mu\bar{u}'_y\right)'_y + \left(\nu\bar{u}'_z\right)'_z,$$

$$\frac{\tilde{v}-v}{\tau} + u\bar{v}'_x + v\bar{v}'_y + w\bar{v}'_z = \left(\mu\bar{v}'_x\right)'_x + \left(\mu\bar{v}'_y\right)'_y + \left(\nu\bar{v}'_z\right)'_z,$$

$$\frac{\tilde{w}-w}{\tau} + u\bar{w}'_x + v\bar{w}'_y + w\bar{w}'_z = \left(\mu\bar{w}'_x\right)'_x + \left(\mu\bar{w}'_y\right)'_y + \left(\nu\bar{w}'_z\right)'_z + g, \quad (4)$$

$$P''_{xx} + P''_{yy} + P''_{zz} = \frac{\hat{\rho}-\rho}{\tau^2} + \frac{\left(\hat{\rho}\tilde{u}\right)'_x}{\tau} + \frac{\left(\hat{\rho}\tilde{v}\right)'_y}{\tau} + \frac{\left(\hat{\rho}\tilde{w}\right)'_z}{\tau},$$

$$\frac{\hat{u}-\tilde{u}}{\tau} = -\frac{1}{\hat{\rho}}P'_x, \frac{\hat{v}-\tilde{v}}{\tau} = -\frac{1}{\hat{\rho}}P'_y, \frac{\hat{w}-\tilde{w}}{\tau} = -\frac{1}{\hat{\rho}}P'_z,$$

where $V = \{u, v, w\}$ are the components of the velocity vector, $\{\hat{u}, \hat{v}, \hat{w}\}$, $\{\tilde{u}, \tilde{v}, \tilde{w}\}$ are the components of the velocity vector fields on the «new» and intermediate time layers, respectively, $\bar{u} = \left(\tilde{u} + u\right)/2$, $\hat{\rho}$ and ρ are the distribution of the density of the aqueous medium on the new and previous time layers, respectively.

When running in Direct Numerical Simulation (DNS) mode, it is assumed that the grid is accurate enough to resolve Kolmogorov turbulence scales. Thus, in DNS mode, the previously described equations are solved directly, without the need to use additional models. When running in the Large Eddy simulation mode (LES), it is assumed that Kolmogorov scales are not resolved.

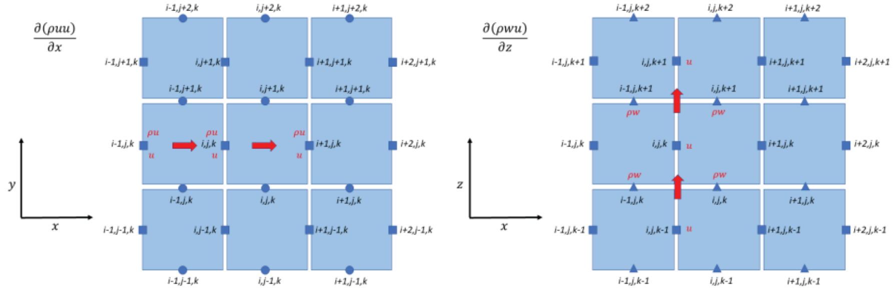

Fig. 2. Contributions of u momentum.

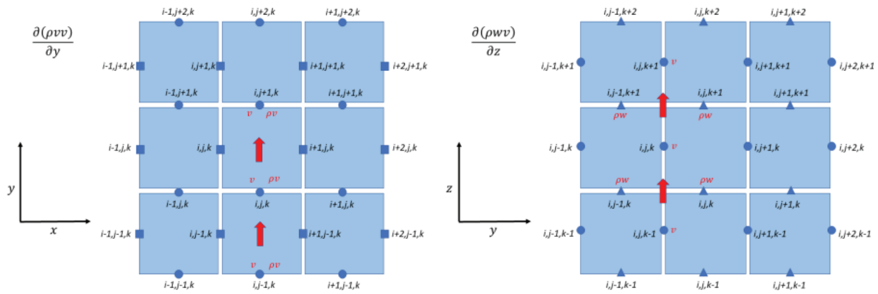

Fig. 3. Contributions of v momentum.

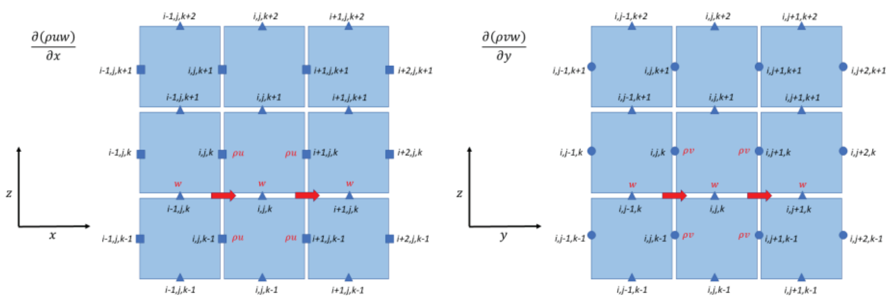

Fig. 4. Contributions of w momentum.

As a result, the numerical discretization acts as a filter for the governing equations, resulting in the following set of filtered equations: When the code is executed in LES mode, the set of equations remains the same, but all variables alternate as the corresponding filtered version and turbulent transfer coefficients are used. Consider the time levels n and n+1, they should be considered as the initial and final states of each stage. Arakawa C grids are presented, indicating where the various variables are located. Density, temperature, potential temperature and pressure are determined at the cell centers (i, j, k) at the time level n.

3 Results and Discussions

As an example of using the developed software package, the problem of calculating the hydrodynamic effect of waves on the bottom of the Taganrog Bay in the Pushkin Embankment area was solved, the modeling area has dimensions of 150 by 100 m and a depth of 1.8 m. The calculations used a grid of 300×400×40 calculation nodes, the time step was 0.01 seconds.

The results of numerical experiments on modeling the propagation of wave hydrodynamic processes based on a 3D model of the motion of an aqueous medium, taking into account the inhomogeneity of turbulent mixing in the vertical direction at various points in time, are presented in Fig. 6. On the basis of the developed software package, a forecast of changes in hydrodynamic wave processes of the coastal zone is constructed, the formation of vortex structures is predicted. In Fig. 7 presents a raster model of a dynamically changing bottom relief of a reservoir due to wave action, in the presence of a multi-scale turbulent exchange.

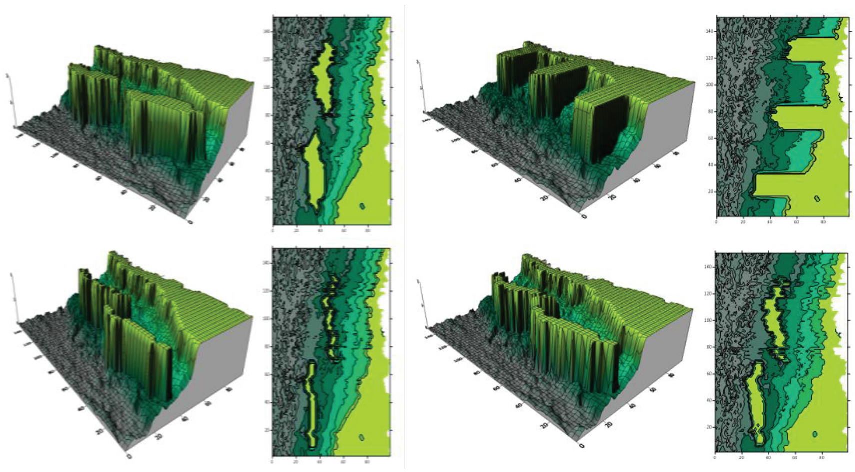

Fig. 5. The aquatic environment movement velocity vector.

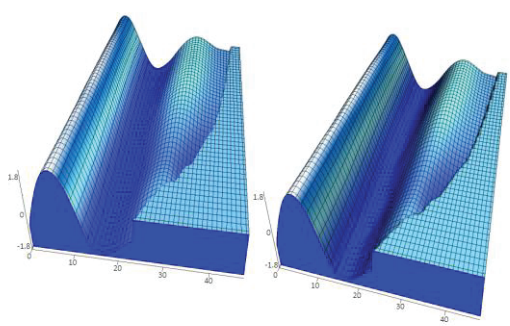

Fig. 6. The aquatic environment movement velocity vector.

The development of a three-dimensional model of wave hydrodynamic processes based on field data made it possible to describe the movement of the aquatic environment of a shallow reservoir, taking into account the presence of bank protection structures and the propagation of waves to the coast.

4 Summary

The article solves the problem of constructing and researching a spatial-three-dimensional model of wave hydrodynamics designed for modeling hydrodynamic processes in the presence of multi-scale turbulent exchange, based on the coordination of analytical, numerical, experimental approaches and field data. The article describes the construction and adaptation to climatic conditions and geographical features of precision mathematical models of hydrodynamics of wave processes and relief formation. The software package described in this paper makes it possible to reduce the forecast time of hydrophysics processes, including dangerous and catastrophic phenomena. Based on the developed software package, prognostic calculations of the processes of coastal erosion and changes in the bottom relief were performed. Modeling of the state of a water body takes into account the characteristic features of natural processes, among which it is worth noting the spatial and temporal variability of relief formation and sedimentation, and, as a consequence, changes in the coastline.

Acknowledgments

The research was carried out at the expense of the grant of the Russian Science Foundation No. 23-21-00210, https://rscf.ru/project/23-21-00210/.

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