Modeling the deformation of an elastic element of a small spacecraft in its plane during temperature shock

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Abstract. A basic mathematical model of the deformation of a large elastic element of a small spacecraft in its plane is constructed. Deformations are caused by a temperature shock after a small spacecraft leaves the Earth's shadow on the solar portion of the orbit. The model is used to conduct a computational experiment with the aim of assessing perturbations acting on a small spacecraft due to temperature shock. The temperature distribution during thermal shock is described by a one-dimensional model of thermal conductivity. The classical theory of thin plates is used to determine the deformations. The results of the estimation of disturbing factors are obtained as a result of a computational experiment for a model small spacecraft. These results indicate the need to compensate for the impact of temperature shock for small technological spacecraft. The data obtained can be used in the design of small space-craft for technological purposes. **Keywords:** small spacecraft, temperature shock, disturbing factor.

1 Introduction

Current trends associated with the miniaturization of space technology contribute to the widespread use of small spacecraft in various fields of science, engineering and technology. The small spacecraft have proven themselves in remote sensing of the Earth from space (for example, «Aist – 2D» [1, 2]), scientific experiments in space (for example, Lomonosov [3]) and other areas due to the low cost and short term of the space project. The small spacecraft plan to use for the development of space technology in the near future. The middle-class technological spacecraft series (for example, Foton [4, 5], Bion [6, 7], SJ [8, 9]) can soon be supplemented with small spacecraft (for example, the Vozvrat – MKA project [10]).

An important requirement for technological spacecraft is the observance of microacceleration conditions [11, 12]. These conditions determine the feasibility of implementing one or another gravitationally sensitive process on board the spacecraft., Fig. 1 is presented [13, 14] to compare the level of micro-accelerations required and achievable on board the international space station. It shows how the capabilities of modern space technology lag behind the needs of space materials science. Therefore, ensuring the required

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level of microacceleration is an important and urgent task when creating new space technology for over forty years.

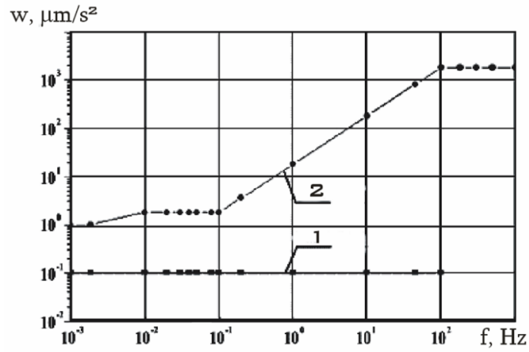

Fig. 1. There are the required (curve 1) and attainable at the international space station (curve 2) level of micro-acceleration (cited from [13]).

The use of small spacecraft for the needs of space materials science will be associated with the solution of a specific problem. The relatively small mass of a small spacecraft in the presence of large elastic elements in its composition is the reason for the significant influence of oscillations of elastic elements on the orbital motion of the spacecraft. Tests of promising new ROSA solar panels on the international space station [15, 16] confirmed the seriousness of this problem. Periodic immersion of a small spacecraft in the shadow of the Earth and exit from it contribute to the occurrence of temperature shock. This blow causes disturbing factors. Their action violates the favorable conditions for micro-acceleration. Therefore, in this paper, we develop a model for the deformation of the elastic element of a small spacecraft in its plane.

It is precisely the longitudinal force that makes the largest contribution to the field of micro-accelerations during temperature shock (Fig. 2) as shown by preliminary studies [17].

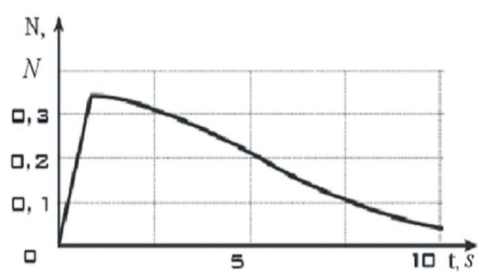

Fig. 2. There is the dependence of the longitudinal force at the attachment point of the elastic element to the casing of a small spacecraft on time during temperature shock (cited from [17]).

In this case, the dynamics of the temperature field for various layers of the elastic element when a small spacecraft leaves the Earth's shadow is shown in Fig. 3 [17].

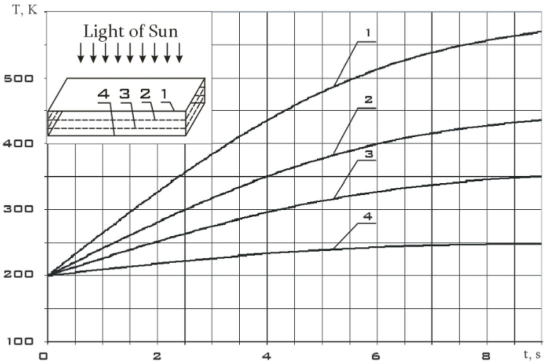

Fig. 3. There is the temperature change dynamics of the layers (1–4) of the elastic body of a small spacecraft (cited from [17]).

There is the question remained open of the need to take into account the longitudinal inertial force arising from the comprehensive expansion of the elastic element of a small spacecraft after it leaves the Earth's shadow in paperwork [17]. Therefore, we will retain the validity of the results of solving the one-dimensional heat conduction problem under thermal shock (Fig. 3) and estimating the internal longitudinal force N (Fig. 2) and analyze the temperature strain of the elastic element in its plane more carefully

2 The deformation model of an elastic element in its plane

Let us consider the effect of temperature shock on a large elastic element of length a when a small spacecraft leaves the Earth's shadow on a part of the orbit illuminated by the Sun (Fig. 4).

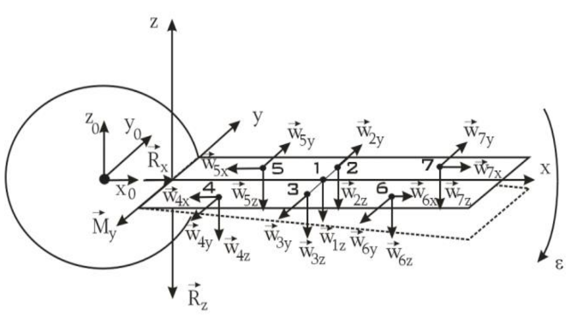

Fig. 4. There is a scheme of thermal shock of a large elastic element when leaving the Earth's shadow (x_0 y_0 z_0 is the main connected coordinate system).

We believe that at the moment when the small spacecraft left the Earth's shadow, the elastic element had a flat undeformed equilibrium shape and was uniformly heated up to 200 K. This formulation fully corresponds to the one-dimensional heat conduction problem solved in [17], the results of which are presented in Fig. 3. There is a comprehensive expansion of a large elastic element and the loss of stability of a rectilinear undeformed flat shape from due to temperature shock, when leaving the Earth's shadow. Let us analyze the motion of various points of the elastic element. We will choose 7 different points from the following considerations (table 1) for this purpose.

Table 1. There is Features of the Coordinates of the Points Selected for the Analysis of the Elastic Element of a Small Spacecraft.

Point	x coordinate	y coordinate
1	a/2	0
2	a/2	"+"
3	a/2	"–"
4	< a/2	"–"
5	< a/2	"+"
6	> a/2	"–"
7	> a/2	"+"

The point 1 has a single acceleration component $\vec{w}_{1z} = \vec{w}_1$ and is associated with the loss of stability of the flat shape of the elastic element. The points 2 and 3 lying on the midline have two components of acceleration associated with loss of stability and comprehensive expansion in the direction of the y axis. A similar picture would be observed for points lying on the x axis, except for the point 1. They have two acceleration components associated with loss of stability and comprehensive expansion in the direction of the x axis. The points that do not lie on the midlines have three components of acceleration associated with loss of stability and comprehensive expansion in the directions of the x and y axes.

We will build a basic model of deformation of the elastic element of a small spacecraft in its plane (Fig. 5).

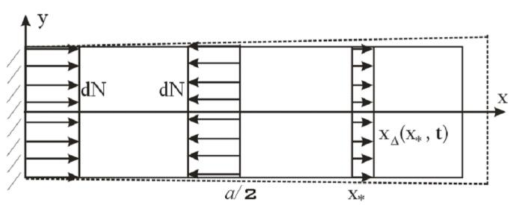

Fig. 5. There is the deformation model of an elastic element during temperature shock in the xy plane. The following simplifying assumptions were made.

1. The elastic element is a uniform rectangular plate.
2. Boundary conditions: three free edges and one edge are rigidly embedded in the body of a small spacecraft.
3. The points of the elastic element located to the left of $x = a/2$ (Fig. 5) do not move.
4. The points of the elastic element located to the right of $x = a/2$ move freely.
5. The temperature of the points of the elastic element depends only on the z coordinate (Fig. 4).
6. The deformation of the points of the elastic element in the direction of the y axis is symmetric with respect to the x axis.

Fig. 5 shows the distribution of the elementary volumetric force dN acting on each point of the elastic element to the left of the line $x = a/2$. It is the same for each point and, according to [15, 16], is equal to:

$$dN(t) = \alpha\, E\big(T(z,\, t) - T_0(z,\, 0)\big)dx\, dy\, dz\,, \tag{1}$$

Where α is the coefficient of linear expansion of the material of the elastic element; E is Young's modulus; $T(z, t)$ and $T_0(z, t)$ are respectively, the final and initial temperature distribution of the elastic element.

This part of the elastic element experiences a longitudinal compression deformation in the x axis direction due to the rigidly fixed left edge of the elastic element.

There are no internal forces in the direction of the longitudinal axis x in this part since the points of the elastic element located to the right of $x = a/2$ move freely. Fig. 5 shows the geometric location of the points of the elastic element having an arbitrary coordinate $a/2 < x_* < a$, as well as the diagram of the displacements $x_\Delta(x, t)$ of these points in the direction of the x axis. This plot represents a uniform distribution of the displacements of the indicated points in the direction of the x axis since $x_\Delta(x, t)$ does not depend on the y coordinate. In the general case, $x_\Delta(x, t)$ should also depend on z. However, the thickness of the elastic element is significantly less than its other two sizes in the problem under consideration. Therefore, the dependence on z can be neglected. The values $x_\Delta(x, t)$ increase from 0 at $x = a/2$ to the maximum value for the points of the free right edge of the elastic element ($x = a$) as the x coordinate increases. We use the equation of static equilibrium [17, 18] to evaluate them:

$$N(x, t) = Eb\left[x_\Delta(x, t) - \alpha \int_{-h/2}^{h/2} \left(T(z, t) - T_0(z, 0)\right) dz \right], \tag{2}$$

Where b is the width and h is the thickness of the elastic element.

$$x_\Delta(x, t) = \left(\frac{2x}{a} - 1\right)\alpha \int_{-h/2}^{h/2} \left[T(z, t) - T_0(z, 0)\right] dz, \; for\; a/2 \le x \le a. \tag{3}$$

Then, we can obtain the deformation field $x_\Delta(x, t)$ of the points of the elastic element of the small spacecraft in the x axis direction using the temperature distribution (Fig. 3) and expression (3). This field is shown in Fig. 6.

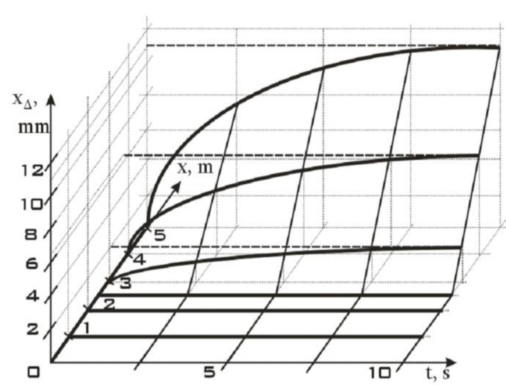

Fig. 6. There is the dynamic field of displacements in the x-axis direction of the points of the elastic element of a small spacecraft in the process of thermal shock.

An elastic element 5 m long, 0.5 m wide and 6 mm thick was considered. Fig. 6 shows that the displacements of the points of the elastic element the coordinate of which is less than $x = 2{,}5\ m$ are equal to zero. The lines $x = 5\ m$ (the right free edge of the elastic element in Fig. 5) correspond to the maximum displacements. A linear dependence $x_\Delta(x, t)$ on x is observed in each section of the figure perpendicular to the time axis. It corresponds to expression (3). A nonlinear dependence is observed corresponding to the temperature distribution (Fig. 3) in cross sections perpendicular to the x axis.

3 Evaluation of the longitudinal force of inertia

Let us further evaluate the inertia force $\vec{\Phi}_x$ arising due to the expansion of the elastic element in the direction of the x axis caused by thermal shock. Its module is equal to:

$$\Phi_x = \frac{m}{a} \int_{a/2}^{a} \ddot{x}_{\Delta}(x, t) dx \quad , \tag{4}$$

Where m is the mass of the elastic element; $\ddot{x}_{\Delta}(x, t)$ is acceleration of its points in the direction of the longitudinal axis x.

We obtain $\ddot{x}_{\Delta}(x, t)$ using the expression (3) to use the expression (4):

$$\ddot{x}_{\Delta}(x, t) = \frac{\partial^2 x_{\Delta}}{\partial x^2} \dot{x}^2 + 2 \frac{\partial^2 x_{\Delta}}{\partial x \partial t} \dot{x} + \frac{\partial^2 x_{\Delta}}{\partial t^2} , \tag{5}$$

Where $\dot{x}$ is the velocity of the points of the elastic element in the direction of the x axis.

There is in the expression (5), according to (3): $\frac{\partial^2 x_{\Delta}}{\partial x^2} = 0$. Therefore:

$$\ddot{x}_{\Delta}(x, t) = 2 \frac{\partial^2 x_{\Delta}}{\partial x \partial t} \dot{x} + \frac{\partial^2 x_{\Delta}}{\partial t^2} . \tag{6}$$

Imagine the x coordinate of the points of the elastic element taking into account displacements in the following form:

$$x(x, t) = x_0 + x_{\Delta}(x, t),$$

where x_0 is the initial value of the coordinates of the points before deformation.

Then we have:

$$\dot{x}(x, t) = \dot{x}_{\Delta}(x, t) = \frac{\partial x_{\Delta}(x, t)}{\partial x} \dot{x}(x, t) + \frac{\partial x_{\Delta}(x, t)}{\partial t}$$

Therefore:

$$\dot{x}(x, t) = \frac{1}{1 - \frac{\partial x_{\Delta}(x, t)}{\partial x}} \frac{\partial x_{\Delta}(x, t)}{\partial t} . \tag{7}$$

We obtain substituting the expression for $\dot{x}$ (7) into the expression for acceleration

$\ddot{x}_\Delta(x, t)$ (6):

$$\ddot{x}_\Delta(x, t) = 2\frac{\partial^2 x_\Delta}{\partial x \partial t}\frac{1}{1-\dfrac{\partial x_\Delta}{\partial x}}\frac{\partial x_\Delta}{\partial t} + \frac{\partial^2 x_\Delta}{\partial t^2}. \tag{8}$$

Further, we find the partial derivatives of the displacements of the points of the elastic element in the direction of the x axis using expression (3):

$$\frac{\partial x_\Delta}{\partial x} = \frac{2}{a}\alpha \int_{-h/2}^{h/2} \left[T(z, t) - T_0(z, 0)\right] dz;$$

$$\frac{\partial x_\Delta}{\partial t} = \alpha\left(\frac{2x}{a} - 1\right)\int_{-h/2}^{h/2} \frac{\partial T(z, t)}{\partial t} dz; \tag{9}$$

$$\frac{\partial^2 x_\Delta}{\partial t^2} = \alpha\left(\frac{2x}{a} - 1\right)\int_{-h/2}^{h/2} \frac{\partial^2 T(z, t)}{\partial t^2} dz;$$

$$\frac{\partial^2 x_\Delta}{\partial x \partial t} = \frac{2}{a}\alpha \int_{-h/2}^{h/2} \frac{\partial T(z, t)}{\partial t} dz.$$

We substitute these expressions in (8) and use the temperature field distribution shown in Fig. 3. As a result, we obtain the dynamic acceleration field in the x axis direction which is shown in Fig. 7.

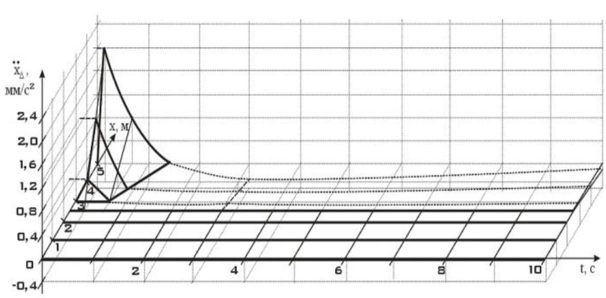

Fig. 7. There is the dynamic acceleration field of the points of the elastic element of a small spacecraft in the direction of the x axis during temperature shock.

Fig. 7 shows a pronounced dynamic part of the transition process. It is expressed in a quasilinear spasmodic increase in accelerations $\ddot{x}_\Delta(x, t)$ followed by a less intense decrease and transition through zero. At the same time, a temperature balance is practically reached by the end of the tenth second characterized by constant values of displacements x_Δ relative to the initial flat undeformed position and practically zero values of accelerations $\ddot{x}_\Delta$.

We will further evaluate the inertia force Φ_x using expression (4) and the dynamic acceleration field shown in Fig. 7. A graph of the dependence of the inertia force Φ_x on time for the mass of an elastic element of 50 kg is shown in Fig. 8.

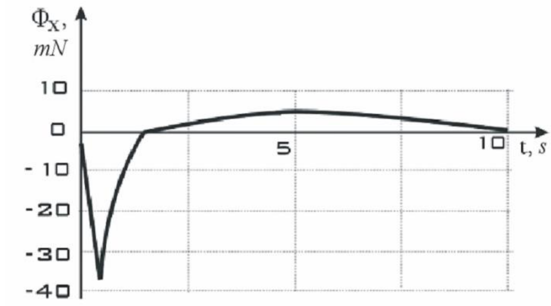

Fig. 8. There is the dependence of inertia Φx on time during temperature shock.

The inertia force Φ_x in contrast to the force N (Fig. 2) changes sign passing through zero. It is explained by the fact that the sign of the force N depends on the sign of the temperature difference (expression (1). The temperature difference was always positive and did not change its sign since the elastic element only heated during the considered time interval of 10 s. The sign of inertia Φ_x depends on whether accelerated or delayed expansion is observed. If accelerated expansion is observed in the most dynamic part of the transient process, then its speed constantly decreases. Therefore, at some time intervals the force N and Φ_x are directed in different directions and in others – in one direction. However, they are directed in different directions in the most dynamic part of the transition process.

4 Conclusions

The basic deformation model of the elastic element of a small spacecraft in its plane is constructed taking into account the accepted simplifying assumptions. The longitudinal inertia force arising from temperature shock was estimated with-in the framework of the constructed model. The obtained values of the longitudinal inertia force make up about 10% of the values of the longitudinal internal force N which was adopted as the main perturbing factor for developing the co-trol law in [17]. This control law is designed to compensate for the effect of temperature shock on the level of micro-accelerations. The estimates carried out in the presented work still leave open the question of taking into account the longitudinal inertia force. On the one hand, favorable conditions can be violated at its maximum value. However, on the other hand, its influence is an extremely short-term phenomenon that fits into the framework of the most dynamic part of the transition process.

The issue of accounting has a meaning from a theoretical point of view be-cause a more minimal value of microaccelerations will be achieved than without taking it into account in this case. This accounting depends on the capabilities of the real executive bodies of the orientation and motion control system of a small spacecraft from a practical point of view. It most likely will be possible and sufficient to adjust the initial value of the thrust of the executive body in most cases. At the same time, the real spread of this thrust may not even allow compensating even at the initial moment of time not to mention its dynamics.

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