Mathematical model of fluid flow along a straight through filled with a distributed flow from above

Rustem Tazetdinov [1], *Gennady* Tibrin[1], *Yuri* Deniskin[1*], and *Svetlana* Lapteva[2]

[1]Moscow Aviation Institute (National Research University), 4, Wolokolamskoe shosse, 125993 Moscow, Russia
[2]Moscow State University of Civil Engineering, 26, Yaroslavskoe shosse, 129337 Moscow, Russia

Abstract. In a one-dimensional approximation and in the absence of friction forces, a mathematical model has been developed for the steady flow of an incompressible fluid along a straight line, for example, a drainage gutter, into which a distributed flow flows from above. The boundary condition missing for solving the system of equations of momentum and continuity is determined using the principle of minimum potential energy. For a rectangular chute, equations are obtained that allow one to calculate the distributions of the layer thickness and fluid velocity along the length of the chute with a plug at one end and without a plug, slope and without slope to the horizon, depending on the intensity of the incoming flow, the size of the chute, and the density of the liquid. This model, by means of a simple recalculation, can also be extended to the flow in a trough of a different cross-sectional profile. The results of the study can be applied to the calculation of external drainage systems. Keywords: mathematical model, fluid, flow, chute, rectangular chute, momentum equation, continuity equation, fluid flow rate, fluid flow velocity.

1 Introduction

This task relates mainly to systems for the drainage of rainwater from the roofs of buildings or other structures, although other applications are possible. The problem of choosing the optimal dimensions of drainage gutters (trays) and the angles of their inclination to the horizon is of great practical importance and was solved mainly in an experimental plan. There are no theoretical works on this problem in the literature. In this paper, a mathematical model is presented and a method is given for calculating the flow characteristics (flow velocity, liquid layer thickness) depending on the size of the chute, its angle of inclination, the presence or absence of a plug, and the intensity of the flow entering the chute [1].

2 Model and method

The problem is solved approximately analytically under the following assumptions, which slightly distort the essence of the process, but greatly simplify the calculations and make them more visual:

[*] Corresponding author: yurideniskin@gmail.com

– absence of friction;
– the steady-state one-dimensional flow (along the troughs);
– the kinetic energy of the incoming flow is negligible.

Calculation schemes are shown in fig. 1 and 2, which show: the origin of coordinates in the upper section of the gutter; the x axis is along the gutter; the angle of inclination of the chute to the horizon; Q [kg/(m·s)] – uniformly distributed in the horizontal direction vertical linear flow of liquid flowing into the chute

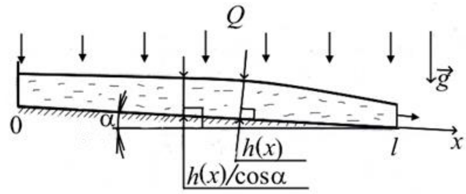

Fig. 1. Calculation scheme of fluid flow in a chute with a plug.

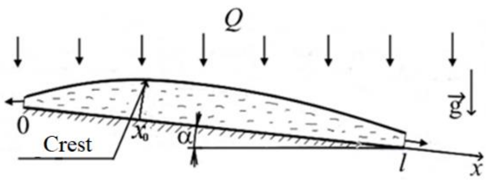

Fig. 2. Calculation scheme of fluid flow in a chute without a plug.

The flow of an incompressible fluid in a chute can be described by the following system of equations [1]:

– continuity

$$G = Q\, x \cos\alpha \tag{1}$$

where $G = G(x)$ is the flow rate of the liquid through the section x;

- amount of movement

$$d(Gv) = dP \tag{2}$$

where v is the fluid velocity averaged over the cross section;

P is the projection onto the x axis of the resultant of all external forces acting on the flow [2-3].

The fluid moves under the influence of gravity only. It can be conveniently divided into two parts:

1) hydrostatic pressure force caused by a change along the x-axis of the height of the liquid column;

2) longitudinal (along the x axis) component of the force of gravity arising from the inclination of the chute to the horizon

$$dP = -d\left(pF\right) + \rho g \sin\alpha \cdot F \cdot dx, \tag{3}$$

where ρ – liquid density;

p – height-average hydrostatic pressure of the liquid column;

F – cross-sectional area of the layer;

g – free fall acceleration 9.81 m/s2.

Combining (2) and (3) and taking into account that

$$G = \rho v F, \tag{4}$$

we get

$$\rho d\left(Fv^2\right) + d\left(pF\right) - \rho g \sin\alpha F dx = 0. \tag{5}$$

Integration gives

$$\rho F v^2 + pF - \rho g \sin\alpha \int_0^x F dx = C, \tag{6}$$

where C is the constant of integration.

3 Research and results

Rectangular gutter

Next, it is necessary to specify the values p and F, which depend on the shape of the cross section of the gutter. In practice, gutters of various cross sections are used: rectangular, V-shaped, U-shaped, semicircular, etc. Each of them has different formulas for calculating p and F[4].

To demonstrate the mathematical model, we restrict ourselves to a rectangular gutter. For him

$$\begin{cases} F(x) = bh(x); \\ p(x) \approx \dfrac{1}{2}\rho g h(x)\cos\alpha, \end{cases} \tag{7}$$

where $h(x)$ is the thickness of the liquid layer in the direction perpendicular to the x axis, b is the width of the inner cavity of the gutter [5].

Substituting equation (7) into (5) and (6), we obtain in general the momentum equation for a steady flow of liquid in the absence of hydraulic losses in a rectangular chute:

$$d\left(hv^2 + \frac{1}{2}gh^2\cos\alpha\right) = gh\sin\alpha dx; \tag{8}$$

$$h\left(v^2 + \frac{1}{2}gh\cos\alpha\right) = g\sin\alpha\int_0^x h dx + \mathrm{C}. \tag{9}$$

There are two different concepts for draining liquid through a chute:

1) with a plug at the upper end of the gutter;

2) without plug.

Gutter with plug

An example is shown in fig. 1.

In this case v(0) = 0. Then from (9) it follows

$$C = \frac{1}{2} g h_0^2 \cos\alpha, \tag{10}$$

where h_0 is the layer thickness in the initial section.

Instead of (9), one can write

$$\left(hv^2 + \frac{1}{2} g h^2 \cos\alpha \right) = g \sin\alpha \int_0^x h dx + \frac{1}{2} g h_0^2 \cos\alpha. \tag{11}$$

Since h_0 is still unknown, we get a problem without the necessary boundary condition.

To determine h_0, we use the well-known principle of minimum potential energy (in this case, only mechanical energy) for a liquid in a closed stationary system, which is the "chute-liquid" system [6].

We write (11) for the outlet section $x = l$, expressing $v(l)$ in terms of h using (1) and (7)

$$g \sin\alpha \int_0^l h dx + \frac{1}{2} g h_0^2 \cos\alpha = \frac{Q^2 l^2 \cos^2\alpha}{(\rho b)^2 h(l)} + \frac{1}{2} g h^2(l) \cos\alpha. \tag{12}$$

In equation (12), up to a factor ρb, on the left is the maximum linear potential energy accumulated by the flow, and on the right is the linear kinetic energy acquired by the flow, plus its residual linear potential energy at the output[7,8].

Since the quantities Q, ρ, b, l, α are given, the only variable can be only $h(l)$. As can be seen from (12), the potential energy does indeed have a minimum in $h(l)$, which is found from the condition

$$\frac{d}{dh(l)} \left(g \sin\alpha \int_0^l h dx + \frac{1}{2} g h_0^2 \cos\alpha \right) = -\frac{Q^2 l^2 \cos^2\alpha}{(\rho b)^2 h^2(l)} + g h(l) \cos\alpha = 0.$$

From here

$$h(l) = \left(\frac{Ql}{\rho b} \right)^{\frac{2}{3}} \left(\frac{\cos\alpha}{g} \right)^{\frac{1}{3}}. \tag{13}$$

Substituting (13) into (12), we obtain

$$\frac{1}{2} g h_0 \cos\alpha + g \sin\alpha \int_0^x h(x) dx = \frac{3}{2} \left(\frac{Ql}{\rho b} \right)^{1/3} g^{1/3} (\cos\alpha)^{5/3}. \tag{14}$$

Taking into account (14), (1), (4), we can transform equation (11) to the form

$$\left(\frac{Qx\cos\alpha}{\rho b}\right)^2\frac{1}{h(x)}+\frac{1}{2}gh^2(x)\cos\alpha+g\sin\alpha\int_x^l h(x)dx-\frac{3}{2}\left(\frac{Ql}{\rho b}\right)^{1/3}g^{1/3}(\cos\alpha)^{5/3}=0. \tag{15}$$

By solving numerically the semi-analytical equation (15), one can find the dependence h(x), then using (7) and (4) calculate v(x).

Let us consider two particular cases of flow in a chute with a plug [9].

Gutter without tilt to the horizon $(\alpha = 0)$

The flow occurs under the action of only the hydrostatic pressure gradient. From (13) and (14) it follows

$$h_0=\sqrt{3}\left(Ql/(\rho b)\right)^{2/3}\cdot g^{-1/3};$$

$$h(l)=\left(Ql/(\rho b)\right)^{2/3}\cdot g^{-1/3}.$$

Equation (15) reduces to

$$\left(Qx/(\rho b)\right)^2+gh^3(x)/2-\frac{3}{2}\left(Ql/(\rho b)\right)^{4/3}g^{1/3}h(x)=0. \tag{16}$$

Having calculated h(x) from (16), then we can calculate the dependence of v(x) using the equation

$$v(x)=Qx/(\rho b h(x)), \tag{17}$$

obtained from (1) and (4) at. In this case, the flow velocity at the outlet of the chute is equal to

$$v(l)=\left(Qlg/(\rho b)\right)^{1/3}. \tag{18}$$

Thus, in the absence of a chute inclination, the problem of calculating the flow characteristics in a rectangular chute with a plug is completely solved analytically [10].

For calculations, equation (16) is conveniently represented in a dimensionless form by introducing the notation

$$\bar{h}=h/h_0;\ \bar{x}=x/l. \tag{19}$$

Dividing all terms (16) by h_0^3, and transforming, we obtain an equation that does not explicitly contain physical parameters

$$\bar{h}^3-\bar{h}+2\cdot 3^{-3/2}\bar{x}^2=0. \tag{20}$$

Also, introducing the notation for the dimensionless flow velocity

$$\bar{v}(\bar{x}) = v(x)/v(l) = \bar{x}\left(\sqrt{3}\bar{h}(\bar{x})\right), \tag{21}$$

can be obtained from the distribution distribution. Both of these distributions are presented in Table 1.

Table 1. Distribution $\bar{h}(\bar{x})$ and $\bar{v}(\bar{x})$ for a gutter with a plug without inclination to the horizon.

$\bar{x}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\bar{h}$	1	0.9975	0.9916	0.9816	0.9671	0.9470	0.9210	0.8868	0.8401	0.7717	0.5773
$\bar{v}$	0	0.0579	0.1164	0.1764	0.2387	0.3048	0.3361	0.4557	0.5497	0.6732	1.0

These distributions for a given type and location of the gutter are universal, i.e. do not depend on physical parameters Q, ρ, b, l, g .

As an example for specific values (water): Q = 0.1…1.0 kg/(m·s); l = 10 m; b = 0.1 m; ρ = 103 kg/m3 basic parameters are: h0 = 0.037…0.1732 m; h(l) = 0.02154…0.1 m; v(l) =0.4640…1.0 m/s.

Chute with a critical angle of inclination

It can be seen from equation (14) that at small angles α it decreases h_0 with its increase. For some α , which we call critical $\alpha_{кр}$, $h_0 = 0$. Then from (14)

$$\sin\alpha_{кр} \Big/ \left(\cos\alpha_{кр}\right)^{5/3} = \frac{3}{2}\left(\frac{Ql}{\rho b}\right)^{4/3} \Big/ \left(g^{2/3}\int_0^l h(x)\,dx\right) \tag{22}$$

and from equation (22) we can calculate $\alpha_{кр}$.

However, to determine $\alpha_{кр}$ from equation (22) is difficult when the distribution is unknown $h(x)$ [11].

Another way is less labor intensive. From equation (15)

$$\frac{dh}{dx} = \left(\sin\alpha - \left(\frac{Q\cos\alpha}{\rho b}\right)^2 \frac{2x}{9h^2}\right) \Big/ \left(\cos\alpha - \left(\frac{Q\cos\alpha}{\rho b}\right)^2 \frac{x^2}{gh^3}\right). \tag{23}$$

From the analysis of this derivative one can see the nature of the change $h(x)$. At $\alpha = 0$, the function h(x) has a maximum at x = 0 and decreases α monotonically as it grows, the position of the maximum shifts towards larger x, and at some $\alpha = \alpha_{кр}$ $dh/dx = 0$ for x = l. It should be noted that with respect to the horizon, h continuously decreases with increasing x, otherwise there would be no flow [12,13]. Thus, from (23) one can obtain by putting in it $(dh/dx)_{x=l} = 0$,

$$\sin\alpha_{кр} \Big/ \cos^2\alpha_{кр} = \left(Q/(\rho b)\right)^2 2l \Big/ \left(gh^2(l)\right),$$

or, replacing *h(l)* by its expression in equation (13),

$$\frac{\sin\alpha_{\text{кр}}}{\left(\cos\alpha_{\text{кр}}\right)^{4/3}} = 2\left(Q/\left(\rho b\right)^{2/3}\right)\cdot\left(lg\right)^{-1/3}. \tag{24}$$

At $\alpha = \alpha_{\text{кр}}$, the hydrostatic pressure derivative is positive over the entire length of the chute, and the fluid moves only under the action of the longitudinal (along the *x*-axis) gravity [14]. However, with growth α, the amount of liquid stored in the chute decreases. If done $\alpha > \alpha_{\text{кр}}$, the nature of the flow will not change, but the amount of liquid will be even less, and the active length $l\cos\alpha$ the groove will shrink [15].

Comparing equations (22) and (24) one can obtain

$$\int_0^l h\left(x\right)dx = \frac{3}{4}lh\left(l\right). \tag{25}$$

The result obtained is interesting in that it can be used to find the amount of liquid stored in the chute at $\alpha = \alpha_{\text{кр}}$ without numerical integration *h(x)*. Indeed, the mass of the liquid in this case is equal to

$$M_{\text{кр}} = \rho b\int_0^l h(x)\,dx = \frac{3}{4}\rho b\, l\, h\left(l\right).$$

For example, for water without taking into account an insignificant correction for $\cos\left(\alpha_{\text{кр}}\right)^{1/3}$ in the same range of parameters as for the previous example, Mcr = 16.15…75 kg. For comparison, the mass of accumulated water in a gutter without slope for the same conditions, calculated using Table 1, is M = 33.92...157.49 kg, i.e. twice as much Mkr. Thus, a chute with an inclination $\alpha = \alpha_{\text{кр}}$ has the advantage of less loading compared to a chute without slope. In addition, at $\alpha > \alpha_{\text{кр}}$ stub is not needed. The critical angle itself calculated for the same parameters is $\alpha_{\text{кр}} = 0,247^0...1,146^0$. At the same time, the sagging of the lower end of the gutter in relation to the horizon is 0.043 ... 0.2 m.

Gutter without plug

In general, liquid can flow from both ends of the trough (Fig. 2). When the chute has no slope, two equal flows are formed, directed opposite to each other. Between them, exactly in the middle of the gutter, a fixed ridge (elevation) appears, which acts as a plug. Both streams move, pushing away from this crest under the action of only hydrostatic pressure forces [16].

If the chute is inclined, for example, to the right, then the ridge moves to the left. The left side is shortened and the right side is lengthened. The left flow moves only under the action of the pressure force, weakened due to the reverse action of the longitudinal force of gravity, and the right flow accelerates under the action of both of these forces acting in one direction. At a critical angle of inclination, the ridge is forced out of the trough and only one right-hand flow remains [17]. This case has already been considered in Section 1.2.

It is convenient to solve the problem separately for each section of the gutter.

Section 2 (x0 < x ≤ l)

In this case, equation (9) should be written in the form

$$h\left(v^2+\frac{1}{2}gh\cos\alpha\right)-g\sin\alpha\int_{x_0}^{x}hdx=C. \tag{32}$$

We also find the constant C for the boundary $x = x_0$:

$$C=\frac{1}{2}gh^2\left(x_0\right)\cos\alpha. \tag{33}$$

The continuity equation will be the same as (1*), only $\nu > 0$. Then, taking into account (33), (1*) and (4), equation (32) is transformed to the form

$$\left(\frac{Q\left(x-x_0\right)\cos\alpha}{\rho b}\right)^2\frac{1}{h}+\frac{gh^2\cos\alpha}{2}-g\sin\alpha\int_{x_0}^{x}hdx-\frac{gh^2\left(x_0\right)\cos\alpha}{2}=0. \tag{32*}$$

Also, minimizing the potential energy by, we find

$$h\left(l\right)=\left(Q\left(l-x_0\right)/\left(\rho b\right)\right)^{2/3}\left(\cos\alpha\right)^{1/3}g^{-1/3}$$

and we obtain a semi-analytical solution of the momentum equation in the final form

$$\left(\frac{Q\left(x-x_0\right)\cos\alpha}{\rho b}\right)^2\frac{1}{h\left(x\right)}+\frac{gh^2\left(x\right)\cos\alpha}{2}+g\sin\alpha\int_{x}^{l}hdx-\frac{3}{2}\left(\frac{Q\left(l-x_0\right)}{\rho b}\right)^{2/3}\left(\cos\alpha\right)^{5/3}g^{1/3}=0. \tag{34}$$

Equating the left parts of equations (31) and (34) with $x = x_0$, we obtain an equation for calculating the coordinate of the crest x_0 x_0 separating two flows

$$\frac{3}{2}\left(\frac{Q}{\rho b}\right)^2\left(\cos\alpha\right)^{5/3}g^{1/3}\left(x_0^{4/3}-\left(l-x_0\right)^{4/3}\right)=g\sin\alpha\int_{0}^{l}hdx. \tag{35}$$

When $\alpha = 0$ solving this equation $x_0 = l/2$. In this case, equations (31) and (34) are identical and have the form

$$\left(\frac{Q\left(x-l/2\right)}{\rho b}\right)^2\frac{1}{h\left(x\right)}+\frac{gh^2\left(x\right)}{2}-\frac{3}{2}\left(\frac{Ql}{\rho b}\right)^{1/3}g^{1/3}=0, \tag{36}$$

that is, they coincide with the equation for gutters with a plug at $\alpha = 0$ only l/2 is taken instead of l, and the argument in the first term is not x, but x - l/2 (or l/2 - x).

By introducing dimensionless variables:

$$\overline{h}=h\left(x\right)/h\left(l/2\right);\quad \overline{x}=\frac{l/2-x}{l/2}$$

and dividing all the terms of Eq. (36) by $h^3(x_0)$, we obtain an equation in dimensionless form that exactly coincides with Eq. (20). For dimensionless speed $\overline{v} = v(x)/v(l)$ (wherein $v(l) = v(0)$) we obtain an equation coinciding with equation (21). Thus, the data in Table 1 are also valid for this variant of the gutter at half the length. The values of the basic (boundary) values calculated for the same values of the parameters as in paragraph 1.1 are: *h(l/2) = 0.0235…0.1094 m; h(0) = h(l) = 0.0136…0.0630 m; v(0) = v(l) = 0.363…0.7937 m/s,* those. less (but not 2 times, but less) than for a gutter with a plug. The mass of water accumulated in each half of the gutter M1/2 = 21.37…99.48 kg. However, for the entire gutter, the mass of water is 2 times greater, which significantly exceeds the mass of water accumulated in the gutter with a plug without a slope. That is, the gutter without a plug is more loaded [18].

It follows from the analysis that, of all the options, the least loaded at a given Q is the chute with a critical angle of inclination. However, it is far from optimal for practice due to the highly uneven loading along the length. Also, since Q can change, a fixed slope angle will not always be critical. Therefore, in practice, it is advisable to use gutters with small angles of inclination (less than the critical one for the operating range Q). The accepted plumbing norms for the inclination of water gutters are no more than 3–5 mm per 1 m [19].

It should be noted that, although the model was brought to calculations only for a rectangular trough, it can also be applied by a simple recalculation for the flow in a trough of a different profile: semicircular, trapezoidal, triangular, etc [20].

The use of the mathematical model presented in this study will make it possible to more reasonably choose the optimal parameters of the gutters in practice.

4 Conclusion

The use of the principle of minimum potential energy made it possible to develop a mathematical model of a one-dimensional steady flow of an ideal incompressible fluid in a straight trough into which a distributed flow flows from above.

For a rectangular chute, analytical and semi-analytical solutions of the momentum equation were obtained to calculate the distributions of the velocity and thickness of the liquid layer along the length of the chute with and without inclination to the horizon, the presence of a plug at one end and without a plug, depending on the intensity of the incoming flow, the size of the chute and density liquids.

An equation is obtained for calculating the value of the angle of inclination, at which the value of the layer at the upper end of the gutter with a plug becomes equal to zero (critical angle) and the need for a plug disappears.

It is shown that at angles less than the critical one, in a trough without a plug between two flows flowing from both ends, a fixed ridge (elevation) is formed, the coordinates of which depend both on the angle of inclination and on the physical parameters of the flow.

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