Numerical methods to calculate magnetic fields of magnetostriction level converter

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Abstract. This article discusses the method for obtaining a system of finite-difference equations to calculate the magnetic fields of magnetostrictive level transducers (MLT) of the applied type. An optimal numerical method for their solution is presented. The formulated problem of finding the optimal width of the non-magnetic wall of the reservoir of the applied MLT is solved by using the finite-difference approximation of the Maxwell system. The article shows the method for obtaining such equations based on the method of grids and considers in detail the optimal method for their numerical solution. The results of mathematical modeling of the magnetic field of the applied MLT using the developed program make it possible to determine the optimal value of the non-magnetic wall width of the tank of the applied MLT, at which the strength of the longitudinal magnetic field of the permanent magnet will be sufficient to form an ultrasonic torsion wave in medium of magnetostrictive sound line. **Keywords**: applied MLT, numerical methods, mathematical modeling

1 Introduction

Modern industrial conditions have led to the availability of a wide variety of instruments for level measurement and control. The requirements for them are very different and depend on the application area. However, the main ones are high accuracy and resolution, the ability to work with aggressive media, low cost and relative simplicity of design. All these requirements are met by the MLT, in particular, a new subclass of devices - the applied MLT on torsional waves.

A distinctive feature of the applied MLT is the use of a non-contact method for level measuring. The transfer of information in them occurs through the interaction of the reservoir of the magnetic field of a permanent magnet with the intensity H_o with the magnetic field of a magnetostrictive acoustic conduit with current through the non-magnetic wall. After this interaction, an ultrasonic torsion wave is formed in the latter's medium, which is then read by a signal electro-acoustic transducer [1].

The choice of the width H of the non-magnetic wall of the tank, where the MLT is installed, affects the efficiency of its operation, which is an important task, the solution of which makes it possible to improve the characteristics of the applied MLT.

In order to improve the technical and operational characteristics of the applied MLT, the

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problem of finding the optimal width H of the non-magnetic tank wall appears during their use, at which the strength of its magnetic bias field will be sufficient to form an ultrasonic torsion wave in the medium of its acoustic duct. In order to identify such a dependence, the calculation of the magnetic field of an applied MLT in this article is carried out using numerical methods and implemented in the form of a computer program.

2 Materials and Methods

To solve this problem, it is proposed to apply numerical methods for solving the Maxwell system of equations describing the distribution of the magnetic field at any point in space [2,3]. It is known that the electromagnetic field is determined by the vectors of magnetic induction $\overline{B}$, the strengths of electric $\overline{E}$, magnetic $\overline{H}$ fields and electric displacement $\overline{D}$, interconnected by the following system [4]:

$$\begin{cases} \operatorname{rot}\overline{H} = \overline{j} + \dfrac{\partial \overline{D}}{\partial t}; \\ \operatorname{rot}\overline{E} = -\dfrac{\partial \overline{B}}{\partial t}; \\ \operatorname{div}\overline{D} = \rho; \\ \operatorname{div}\overline{B} = 0, \end{cases} \tag{1}$$

Where $\overline{D} = \varepsilon_0 \varepsilon \overline{E}$, $\overline{B} = \mu_0 \mu \overline{H}$, $\overline{j} = \gamma \overline{E}$ – the conduction current density, ε_0, μ_0 – the electric and magnetic constants, ε, μ – the dielectric and magnetic permeabilities of the medium, γ – the specific conductivity of the substance, ρ – the volumetric density of the electric charge.

The considered magnetic field of the applied MLT magnetization is stationary; therefore, we can take the following in (1):

$$\frac{\partial \overline{B}}{\partial t} = \frac{\partial \overline{D}}{\partial t} = 0. \tag{2}$$

Taking this into account, the system (1) can be reduced to one of the following partial differential equations [3]:

$$\operatorname{div}(\mu \operatorname{grad} u_{\text{м}}) = -\rho_{\text{м.ст}}; \tag{3}$$

$$\operatorname{div}(\mu^{-1} \operatorname{grad} \overline{A}) = -\operatorname{rot}\overline{H}, \tag{4}$$

Where $u_{\text{м}}$ – the generalized scalar magnetic potential ($\overline{H} = -\operatorname{grad} u_{\text{м}}$), $\overline{A}$ – the vector magnetic potential ($\overline{B} = \operatorname{rot}\overline{A}$), $\rho_{\text{м.ст}} = -\operatorname{div}\mu\overline{H}$ – the density of external sources of the magnetic field.

The equations (3), (4) are valid at any point of the computational domain, so that their direct solution, taking into account the boundary conditions, accurately determine the potential $\overline{A}$ and the magnetic field H_{o} strength of the applied MLT at the point under consideration. The analysis of these equations shows that it is more convenient to solve one partial differential equation (3). However, the introduction of a scalar potential u_{M} is possible only in the regions with no conduction currents $\overline{j}$, which makes it unsuitable for the magnetic field calculation of an applied MLT.

Due to the nonlinearity and anisotropy of ferromagnetic material properties, such characteristics are nonlinear and depend on various parameters. This significantly complicates analytical integration (4). However, there are many different methods for the numerical solution of such equations to any given accuracy using a computer [3].

The most effective is the transition from the considered partial differential equation to the difference one, which is its discrete analog, for example, by the method of grids [2, 5]. To do this, it is necessary to select a system of nodes (a grid) that fills the computational domain based on the following conditions: obtaining smaller errors during the transition to the difference equation and a simple difference equation. The computational error, in this case, will be most influenced by the distance between the nodes (h step) of the grid.

As was shown in [3], the difference equations take on a simpler form during a regular grid selection. The introduction of irregular grids is justified when solving problems with the boundaries of objects of complex geometric shapes. In this case, the objects are limited by straight lines, so the use of a regular grid will be effective.

Let us transform the equation (4) in partial derivatives into the corresponding difference equation. In this case, the difference equations for the potential inside the media are supplemented by the equations of a different type at their interfaces (boundary conditions), which will lead to the solution algorithm complication due to the need to identify each grid point for its belonging to the boundary. This can be avoided by applying the balance method [3], the essence of which is that when you calculate the static magnetic field in a piecewise inhomogeneous medium, one should express the vector $\overline{B}$ through the vector potentials of the grid nodes and calculate the resulting integral approximately using Maxwell's postulate.

Using the balance method, we obtain a difference equation corresponding to the equation (4) in partial derivatives. To do this, let's consider a fragment of four adjacent grid cells (Figure 1) and select the points a, b, c, d in the centers of each of them.

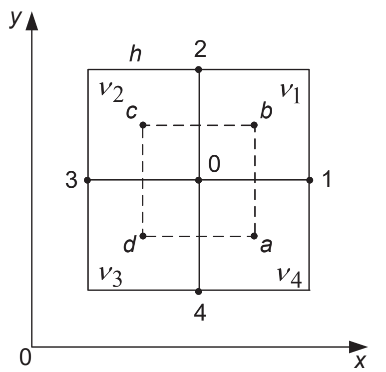

Fig. 1. The contour of abcd integration for obtaining the difference equations. $\nu_1, \nu_2, \nu_3, \nu_4$

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– the coefficients inverse to the mean values of magnetic permeability of each cell with the nodes 0,1,2,3,4 of the computational domain

The equation of the system (1), taking into account the expression (2), can be rewritten as follows:

$$\oint_l \overline{H} dl = \oint_l \nu \overline{B} dl = i\,, \tag{5}$$

Where l – the contour of abcd, $\nu = \mu^{-1}$, i – medium conduction current.

The integral (5) along the contour abcd, we represent as the following sum:

$$\oint_l \nu \overline{B} dl = \int_a^b \nu B_y dy - \int_b^c \nu B_x dx - \int_c^d \nu B_y dy + \int_d^a \nu B_x dx\,. \tag{6}$$

Let us choose the grid step sufficiently small, which allows us to consider the tangent component of the induction $\overline{B}$ constant within the integration intervals. In this case, the expression (6) will be written as follows:

$$B_{yab}\int_a^b \nu dy - B_{xbc}\int_b^c \nu dx - B_{ycd}\int_c^d \nu dy + B_{xda}\int_d^a \nu dx = jh = i\,, \tag{7}$$

Where j – the current density averaged among four cells.

Since $\nu = \nu(x, y)$ is some function, then one can use approximate calculations to calculate the definite integrals in (7), for example, the parabola method [2, 5]. Its application to the first integral of the expression (7) allows us to put down the following:

$$\int_a^b \nu dy \approx \frac{h}{2}(\nu_1 + \nu_4). \tag{8}$$

Let's note also that the tangential component of the induction vector B_n on each segment of integration, taking into account (4), is expressed through the vector magnetic potentials A_i of the adjacent cells in accordance with the expressions [3]:

$$\begin{gathered} B_{yab} = -\frac{A_1 - A_0}{h};\ B_{xbc} = \frac{A_2 - A_0}{h}; \\ B_{ycd} = -\frac{A_0 - A_3}{h};\ B_{xda} = \frac{A_0 - A_4}{h}. \end{gathered} \tag{9}$$

We obtain the approximations for the remaining integrals similar to (8), taking into ac-

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count the expressions (4) - (6) and (9) from (7), grouping the coefficients of similar terms:

$$A_1 \frac{\nu_4+\nu_1}{2} + A_2 \frac{\nu_1+\nu_2}{2} + A_3 \frac{\nu_2+\nu_3}{2} + A_4 \frac{\nu_3+\nu_4}{2} - A_0 \frac{\nu_1+\nu_2+\nu_3+\nu_4}{2} = \\ = A_1k_1 + A_2k_2 + A_3k_3 + A_4k_4 - A_0\left(k_1+k_2+k_3+k_4\right) = -i, \tag{10}$$

where $k_1 = \dfrac{\nu_4+\nu_1}{2}$, $k_2 = \dfrac{\nu_1+\nu_2}{2}$, $k_3 = \dfrac{\nu_2+\nu_3}{2}$, $k_4 = \dfrac{\nu_3+\nu_4}{2}$.

The resulting expression (10) is a finite-difference equation for the node 0. Such equations can be written for all grid nodes, except for those that lie on the boundary of the computational domain, since the behavior of the magnetic field at such points is known and is determined by the boundary conditions. This approach makes it possible to obtain a system of algebraic equations, which is a difference approximation of the solution (4) of Maxwell's equation system (1) for the magnetic field of the applied MLT. The system of equations (10) connects the potentials at the nodes of a uniform grid with a given step h. Its analysis allows us to highlight some features that must be taken into account when choosing a solution method:

• the number of boundary conditions determines the accuracy (error) of the magnetic field potential calculation;

• the method of grid node numbering determines the type of the coefficient matrix;

• for a large number of unknowns, the matrix of the system coefficients is sparse, ill-conditioned and symmetric. However, its symmetry can be violated near the boundaries of the computational domain (the presence of boundary conditions).

In the latter case, it will be expedient to perform algebraic transformations of the matrix in order to bring it to its previous form, due to the optimality of processing and storage on a computer [2, 5].

To solve the systems of algebraic equations with the indicated properties, it is advisable to use iterative numerical methods [3]. Their essence lies in the fact that the value of the desired indicator obtained at the previous step allows you to calculate another, more accurate one, at the current step. The process is repeated until some accuracy criterion is met. The advantages of these methods are the relative simplicity of iterative formulas, ease of implementation on a computer, guaranteed achievement of the result with the required accuracy, regardless of the accepted initial values of the desired quantities.

Taking this into account, we put down the system of equations of the form (10) in a matrix form:

$$Au = P, \tag{11}$$

Where $A = \left\| a_{i,j} \right\|$ – the matrix of system coefficients, u – the matrix of unknowns, P – the column of the right-hand elements.

To solve the equations (11), we use the Seidel method [2, 5], the high rate of convergence of which is explained by the fast iterative convergence of the computational process.

With regard to the considered system (11), the Seidel formula can be written in the following form [3]:

$$\widetilde{u}_{i,j}^{n+1} = \frac{a_{i,j-1}u_{i,j-1}^{n+1} + a_{i-1,j}u_{i-1,j}^{n+1} + a_{i,j+1}u_{i,j+1}^{n} + a_{i+1,j}u_{i+1,j}^{n} - P_i}{\sum_{i,j} a_{i,j}}, \quad (12)$$

Where $u_{i,j}^{n}$ – the values of the unknowns $u_{i,j}$, calculated at the n-th step, $\widetilde{u}_{i,j}^{n+1}$ – the adjusted values of the unknowns $u_{i,j}$.

Hereinafter, we assume that the initial approximation $u_{i,j}^{0}$ is known.

Seidel's method allows one to obtain a solution to the system of equations (11) with any predetermined accuracy ε. The condition [2, 5] is used as a criterion to achieve a given accuracy:

$$\max\left|\widetilde{u}^{\,n+1} - \widetilde{u}^{\,n}\right| \le \varepsilon. \quad (13)$$

The use of the Seidel method makes it possible to reduce the amount of computer memory for storing the initial data and calculated results in the form of a single array.

An even higher rate of convergence is provided by the methods of upper or lower relaxation [3], which are the modification of the Seidel method. Their iterative process is based on using the following expression:

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \overline{\omega}\left(\widetilde{u}_{i,j}^{n+1} - u_{i,j}^{n}\right), \quad (14)$$

Where $\overline{\omega}$ – the coefficient of convergence acceleration.

The algorithm of relaxation method use provides for calculating the potential at the node $\widetilde{u}_{i,j}^{n+1}$ in accordance with the Seidel formula (12) and its correction to the value $u_{i,j}^{n+1}$ according to the expression (14). The criterion for achieving accuracy ε is the expression similar to (13).

The choice of the value $\overline{\omega}$ affects the rate of convergence and is carried out from the condition of the minimum number of iterations. As is known [3], the optimal value of the convergence acceleration coefficient depends on the χ - the condition number of system coefficient matrix. Since it is related to the parameters of each specific problem, it is not possible to choose the optimal value of the convergence acceleration factor $\overline{\omega}_0$ in the general case.

The optimal value of the convergence acceleration coefficient $\overline{\omega}_0$ can be determined approximately. For example, for a rectangular mesh of the size $(N+1)\times(M+1)$, where N>14, M>14 and the coefficient matrix with $\chi >> 1$, the following expression is valid [3]:

$$\bar{\omega}_{\text{o}} = 2\left(1 - \pi\sqrt{\frac{1}{M^2} + \frac{1}{N^2}}\right). \quad (15)$$

Thus, in accordance with the expression (15), the value of the convergence acceleration coefficient can be selected for a mesh of any size, which, when substituted into the expression (14), will allow to determine the values of the potentials at all nodes of the computational domain using the upper relaxation method ($\bar{\omega}_{\text{o}} > 1$) with a given accuracy ε in the least number of iterations.

Let's note that there are other, more complex iterative methods to solve the systems of equations of the form (11), for example, the Richardson method, the alternating triangular method, and others [3-5]. However, their use to solve this system of equations is inappropriate due to their orientation towards the problems of three-dimensional field calculation with complex geometry, which will only introduce additional difficulties during a computer program compilation for their implementation.

3 Results

The result of this work is a program that allows you to obtain a picture of the magnetic field strength H_{o} of applied MLT magnetization field at the points of the computational domain (Figure 2). The advantage of this program over existing analogues is the possibility of obtaining a continuous dependence of the magnetic field strength H_{o} of a permanent magnet 3 at the point of the magnetostrictive acoustic duct 5 near the float 4 on the width of the non-magnetic wall H of the reservoir 1. The program allows you to find the specified dependencies for any geometrical dimensions of the computational domain and the materials of the applied MLT elements. To calculate the characteristics of the field, a system of finite-difference equations (10) is compiled using the balance method and solved by the upper relaxation method with the choice of the optimal value $\bar{\omega}_{\text{o}}$ according to the formula (15).

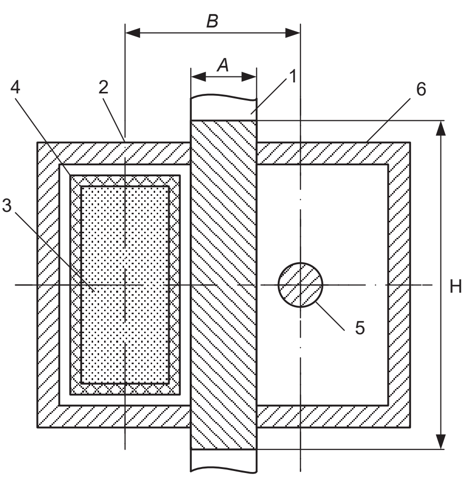

Fig. 2. Design scheme of the applied MLT.

1 - reservoir with a non-magnetic wall and the width H, 2 - guide groove, 3 - permanent

magnet, 4 - float body, 5 - sound conduit, 6 - electromagnetic shield, - the thickness of non-magnetic reservoir wall 1, B - the axial distance between permanent magnet 3 and sound conductor 5

The program is focused on integration into the MATLAB system and the result of its work is an m-file containing the program and numerical data for the specified system.

4 Discussion

After calculating the magnetic field of the applied MLT using the developed program, they obtained the calculated dependences of the magnetic field strength H_o of the permanent magnet 3 on the width H of the non-magnetic wall of the tank 1 for different magnetically hard materials (Figure 3).

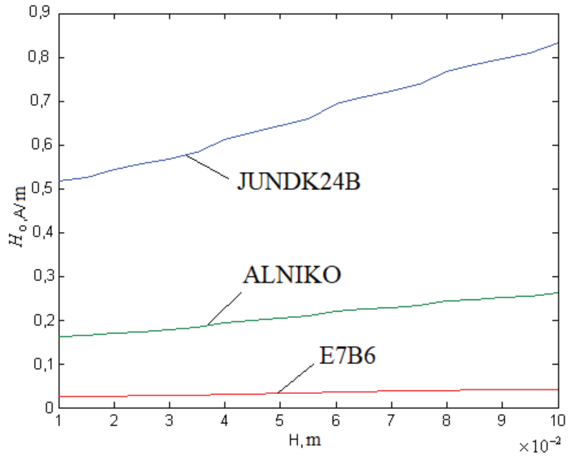

Fig. 3. Dependence of the magnetic field strength H_o of a constant magnet 3 on the width H of the non-magnetic wall of the tank 1.

Fig. 4. shows the results of the magnetic field calculations of an applied MLT, when YUNDK24B alloy is selected as a permanent magnet.

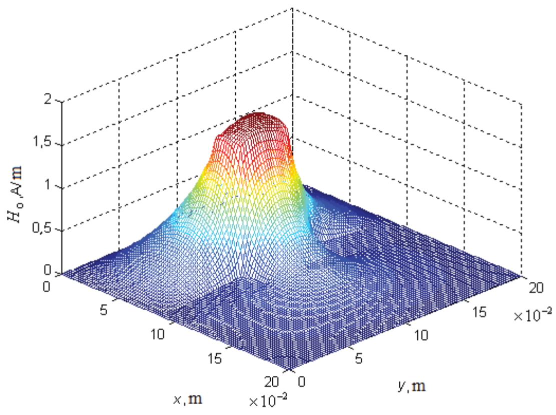

Fig. 4. The picture of the magnetic field strength H_o of the applied MLT.

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To test the efficiency of the upper relaxation method with the value of the convergence acceleration coefficient $\overline{\omega}_o$, calculated by the formula (15), the number of iterations necessary to achieve the required accuracy ε at various values $\overline{\omega}$ was calculated. In the course of this computational experiment, the coefficient $\overline{\omega}$ from the expression (14) took the following values: 0.5; 0.9; 1; 1.5; (the coefficient $\overline{\omega}_o$ was calculated by the formula (15)). Thus, a dependence was obtained, the graph of which is shown on Figure 5.

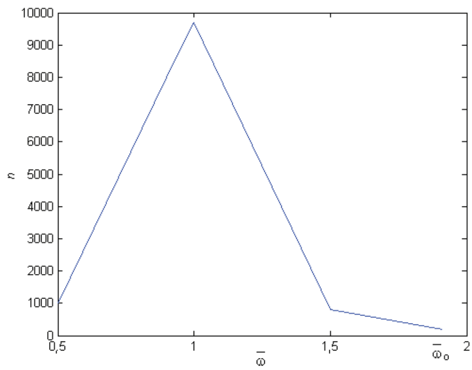

Fig. 5. Dependence of the number of iterations n on the convergence acceleration factor $\overline{\omega}$.

As can be seen from Fig. 5, the introduction of the convergence acceleration factor $\overline{\omega}$ makes it possible to reduce significantly the number of required iterations in comparison with the Seidel method (at $\overline{\omega} = 1$). However, the best result can be achieved if the optimal value $\overline{\omega}_o$ is chosen, that allows to solve the problem in the minimum number of iterations.

5 Conclusions

Thus, the formulated problem of finding the optimal width H of the non-magnetic wall of the reservoir 1 of the applied MLT is solved by using the finite-difference approximation of the Maxwell system. The article shows the method for obtaining such equations based on the method of grids and considers in detail the optimal method for their numerical solution.

The results of mathematical modeling of the magnetic field of the applied MLT using the developed program make it possible to determine the optimal value of the non-magnetic wall H width of the tank 1 of the applied MLT, at which the strength of the longitudinal magnetic field H_o of the permanent magnet will be sufficient to form an ultrasonic torsion wave in medium of magnetostrictive sound line.

The introduction of the coefficient $\overline{\omega}_o$, calculated by the approximate formula (15) allows us to reduce the number of required iterations by 50 times approximately. This significantly reduces the requirements of the program for computer resources, reduces the time for the problem solution and allows to obtain the results with high accuracy.

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