

# Dynamic behavior of earth dams under different kinematic impacts

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**Abstract.** The paper provides a detailed analysis of the current state of the problem. A mathematical model is presented to determine the dynamic behavior of earth dams, considering the viscoelastic properties of soil, using the hereditary Boltzmann-Volterra theory with the A.R. Rzhantsyn kernel under periodic kinematic impacts. To solve the problems considered, the finite element method and complex arithmetics were used to reduce integrodifferential equations to a high-order complex algebraic equation. The accuracy of the methods was verified by solving test problems. Steady-state forced vibrations of the Pskem earth dam 195 m high are studied considering the real geometry and soil properties under resonant vibration modes. It was stated that the largest stress amplitudes in the body of the dam occur not only under the first resonance, but they can occur under other dense spectra of the eigenfrequencies of the dam, due to the interaction between close natural modes of vibration. The strength of various sections of the dam body was tested under kinematic impact using the Coulomb-Mohr theory of strength; the most dangerous sections of the dam were identified in terms of the highest stress. **Keywords:** earth dam, dynamic behavior, kinematic impact, viscoelasticity, resonant mode, stresses, strength, frequency response.

## 1 Introduction

Ensuring the safety of hydro-technical structures and protecting the population is of paramount importance worldwide, therefore it requires a very serious attitude to safety during the design, construction, and operation of hydro-technical structures.

This, in turn, requires the development and improvement of mathematical models, calculation methods that account for the real features of both the structure and its material, as well as the study of the strength of the structure [1-11] under various impacts.

At the same time, ensuring the strength and seismic resistance of the dam is one of the urgent tasks since dams, with reservoirs and accumulation of large volumes of water, pose a threat to the downward territories [1 – 13].

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Recently, a number of studies were published where the stress-strain state (SSS) and dynamic behavior of various dams were considered.

These scientific works include:

- the studies conducted in [14] consider the mitigation to the dam damage during strong earthquakes using a damping protective layer (a layer consisting of river sand) between the foundation and the base in the model of an earth dam. The eigenfrequencies of such a system were determined using the ANSYS program. The study of the physical model of such a system was conducted on a vibrating table under various resonant oscillation modes. Without an insulating layer, the dam model was significantly damaged. The best test results were obtained when the thickness of the layer was  $\frac{1}{4}$  of the height of the dam;

- in [15], the results of studies of natural oscillations and harmonic responses for the power plant structure of the Kenyir dam in Terengganu (Malaysia) were presented. The dynamic characteristics of the power plant were determined using the ANSYS software. The frequency response of the points of the structure was obtained in a large frequency range by applying a force to the structure. A real-scale 3D model of the Kenyir Dam power plant was built using Solid Works software and ANSYS software. The maximum deviation of 0.90361 m in the direction of the z-axis was obtained at a resonant frequency of 5.4 Hz;

- reference [16] describes the vibration tests conducted in Iran to evaluate the dynamic behavior of the structures of the Masjed-Soleiman (MS) earth dam. During the tests, the response of the dam body in three directions was measured under triaxial excitation. Eigenfrequencies, vibration modes and damping of the dam body were estimated. In addition to dynamic tests, a two-dimensional and three-dimensional analysis of the natural oscillations of the dam body was performed using the ANSYS software, taking into account the filling of the reservoir with water. It was determined that the influence of water on eigenfrequencies of the dam for a completely filled reservoir reduces the eigenfrequency of the dam by approximately 10%;

- in [17], some aspects of the influence of foundation flexibility on the seismic response of concrete gravity dams were considered. The influence of the foundation flexibility on the seismic response was investigated, and the eigenmodes of the dam-foundation model were analyzed. A finite element model (FEM), a model with continuous parameters, and a model with three degrees of freedom were used to evaluate the dynamic behavior of concrete gravity monoliths on flexible foundations. The accuracy of the discrete model was evaluated by comparing the frequency response functions of the relative displacement of the dam crest;

- the dynamic characteristics of the dam, which presents a reinforced concrete structure of a cylindrical shape with a crest length of 170 m and a height of 46 m, were experimentally studied in [18]. The dam oscillations were excited with a 32 kN servo-hydraulic shaker and its response was measured at 270 points in three directions. 20 eigenmodes with frequencies  $f = 3.6 \dots 12.9$  Hz were determined and the calculation model of the dam was updated in accordance with the results of these tests;

- in [19], when determining the eigenfrequencies of an earth dam, the results of two methods (an analytical shear-beam method and a FEM) were compared. The results obtained differ significantly from each other. The analytical method leads to better results since it does not consider the strain in the dam. This difference becomes even more significant when determining higher eigenfrequencies since the FEM captures the frequencies that the analytical method did not capture;

- in [20], an equation was obtained using the shear-beam theory to describe the dynamic behavior of an earth dam erected from homogeneous isotropic linearly elastic materials. The partial differential equation was reduced to an ordinary differential equation of the Bessel type, solved with the appropriate boundary conditions; an infinite number of eigenfrequencies and vibration modes of the considered dams were obtained. The stresses arising at different oscillation frequencies were estimated;

- in [21], full-scale tests were conducted to assess the dynamic behavior of the concrete arch dam Shahid Raji (Iran). The dynamic behavior of the dam-reservoir-foundation system was studied by the FEM. The main eigenfrequencies of the system for symmetric and antisymmetric shapes were obtained under different vibration modes. Experimental and theoretical results obtained were compared;

- the dynamic characteristics of arch dams were studied in [22] using external vibration. For this purpose, a prototype model of an arch dam was built under laboratory conditions, taking into account the reservoir and the foundation. Eigenfrequencies of the dam were defined under external and forced vibrations, at various levels of reservoir filling. An impact hammer excited the forced vibrations of the dam. The results of the study of dynamic characteristics obtained for different levels of reservoir filling were compared. It was determined that a completely filled reservoir significantly changes the behavior of the dam;

- in [23], the effect of the foundation and the reservoir on the eigenfrequencies of the Pine Flat gravity dam (USA) was studied. Using the ANSYS APDL software, the dynamic characteristics of the models of the "dam-foundation-reservoir" systems were studied. The analysis of the results obtained showed that an account for the foundation, base, and the level of filling of the reservoir reduces the eigenfrequencies of the considered systems;

- in [24], a complete solution to the FEM eigenvalue problem for a number of earth dams in Southern Italy is given. The interaction of the dam with the foundation and ground base was taken into account. The results obtained showed that to accurately determine the behavior of the dam, it is necessary to consider the foundation and ground base. The obtained numerical results were compared with the analytical ones;

- in [25], numerical studies and evaluation of the seismic behavior of earth dams were performed. The FEM, with account for the ideal plastic properties of soil and the damping was used to study the nonlinear dynamic behavior of the dam. The numerical model constructed was tested using a centrifuge for frequency parameters. A good agreement between the calculated and measured results was observed. A parametric study was conducted to identify the influence of the height of the dam and input excitation characteristics on the seismic response of earth dams;

- in [26], an equation was obtained to determine the natural frequency of earth dams by analytical methods. The advantages of this method include a more accurate assessment of seismic parameters, and an account for the flexibility of the earth dam foundation. The results obtained using this technique were compared with the ones obtained using the GeoStudio-2007 finite element software;

- in [27], studies on the dynamics of the concrete spillway dam of a river hydroelectric power station were presented. A detailed review of the method to determine the dynamic characteristics of structures and the results of assessing the response of structures to time-varying excitation and earthquake accelerograms were given. The resulting response spectra made it possible to estimate the maximum horizontal accelerations;

- in [28], using three-dimensional boundary elements, the dynamic behavior of the Soria arch dam (Gran Canary, Spain) was studied, under its interaction with the ground base, the filling of the reservoir, and the actual geometry of the dam wall. It was determined that the effect of the dynamic characteristics of the dam is significantly influenced by the correct consideration of the properties of the structure material, the level of water filling and the correctness of the accounting for the geometry of the dam;

- in [29], to assess the reliability of the spillway structure of the Chenderoh dam (Malaysia), the results of experimental spectral analysis and operational modes of deviation were used. Along with the experimental study, a numerical simulation of a 3D physical model was conducted using the ANSYS software. The values of natural and operating frequencies of this dam were: 220.87 Hz for the spillway, and operating frequency - 45 Hz. It was determined that the value of the operating frequency differs from the natural frequency of the

systems under consideration; it has a minimal impact on the design of the spillway, and the spillway is considered safe for operation;

- in [30], five natural frequencies of the construction of the Indirasagar gravity dam, located in the state of Andhra Pradesh, were determined taking into account only the weight of the dam and the hydrostatic pressure of water using the ANSYS software package. From the analysis of the results, the maximum tensile stresses that occur in the lower part of the dam were revealed. It was determined that the value of the first natural frequency is 5.81 Hz, and the maximum frequency and offset are obtained at the fifth natural frequency;

- reference [31] gives the results of vibration tests to determine the structural behavior of the Deriner arch dam. The dynamic characteristics of the dam were determined using the FEM. As a result, the values of the first few natural frequencies were determined, which were in the range of 1.60-4.10 Hz;

- in [32], the dynamic characteristics of a modular building were studied using operational spectral analysis. Various operational-spectral methods were used to analyze the registered structure response acceleration and extract natural frequencies, mode of vibrations and damping coefficients;

- in [33], a method for estimating the first natural frequency using the vibration of earth dams with higher levels of strain caused by seismic events was proposed. The natural frequency was determined from the dam responses in different frequency ranges;

- the eigenfrequencies of the Koyna gravity dam with a rigid and flexible base with and without mass were estimated in [34]. The results showed that the first natural frequencies of the dam with an increase in the mass of the structure and a decrease in the rigidity of the foundation lead to a slight increase in frequency;

- the research given in [35] is devoted to the evaluation of the SSS and dynamic characteristics of various dam structures. With the proposed model and the FEM, the natural frequencies and modes of vibrations of an earth dam with a height of 296 m were studied.

- references [36-38] gives the development of a mathematical model and a method for solving the problem of contact interaction of various multilayer structures with a combined base. As a result of solving specific problems, a number of new mechanical effects associated with the manifestation of internal force factors in three-layer beam systems interacting with combined bases were revealed;

- in [39], using the ANSYS software, the response of the Chenderoh dam (Malaysia) to the vibration generated by the spill of water was studied. The results of the frequency domain response and waveforms from the impact of vibration caused by the flow of water were compared with natural frequencies and waveforms of the dam. The analysis showed that the transient oscillations caused by the water flow occurred at a frequency of 13.3 Hz, while the eigenfrequency of the left bank section of the dam was 52.3 Hz, which indicated the absence of a resonance phenomenon for this spill event;

- in [40], the dynamic behavior of the dam was studied by the FEM using the ANSYS program, taking into account the soil foundation. At that, the damping properties and the boundary state of soil, the nonlinear relationship between the soil and the dam were taken into account. It was determined that the damping properties of soil reduces the response of the structure and increases the seismic resistance of the system as a whole;

- the study in [41] investigates the displacement and acceleration in the upper part of the earth dam, and the stress state in the body of the dam, with and without considering the water saturation of soil. The calculations were conducted using the FEM and the FLAC program. The results obtained were compared with the ones obtained by the conventional method for homogeneous dams. It is argued that a consideration of the soil conditions leads to more complex analyses, when determining the dynamic characteristics;

- in [42], the tailing dam model was tested on a vibrating table, with horizontal peak ground acceleration. Test results showed that the tailing dam is stable, while the entire dam tends to slide forward;

- in [43], the nonlinear seismic analysis of earth dams was conducted by the FEM using the Geo-Studio software. In a numerical study, a nonlinear FEM analysis was used, taking into account the linear and elastic-plastic components of the model to describe the properties of soil. The analysis made it possible to give a general pattern of the dam behavior, i.e., the change in the contours of displacements and stresses of the dam;

- the study in [44] is devoted to assessing the effect of soft soil foundation on the dynamic behavior of earth dams under strong ground motion. The FEM was used to study the reactions of earth dams during an earthquake. The presence of a soft base layer increases the main oscillation period of the dam;

- in [45,47], the stress state and natural oscillations of two earth dams in a spatial setting were studied. When assessing the strength, some dangerous sections of the dam were identified and natural frequencies were found that were not taken into account using a plane model;

- in [46], the SSS of earth dams under harmonic load was studied numerically by the FEM. A two-dimensional problem for the cross-section of the dam was considered using the equation of state and structural changes in the moisture properties of soil;

- in [48,49], the assessment of the strength of earth dams was studied, as well as the elastic-plastic properties of soil at different levels of reservoir filling. An account for the elastoplastic properties of soil with a completely filled reservoir significantly changes the pattern of shear stresses distribution in the body of the dam, i.e. they increase in the upper retaining prism and in the lower prism.

Here is a review of only a few scientific articles published over the last 15-20 years; each of these articles has its own approach when solving specific problems, and its merits and shortcomings, however, all of them are used in solving specific practical problems.

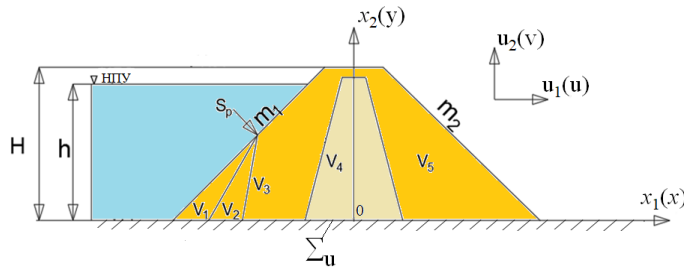
Based on this analysis, it can conclude that the development of mathematical models, and methods of solving and studying the dynamics of earth dams is a very urgent and necessary problem today all over the world and it needs to be solved.

## 2 Methods

### 2.1 Mathematical model

Consider a nonhomogeneous system (an earth dam) (Figure 1) that occupies volumes  $V=V_1+V_2+V_3+V_4+V_5$ ; and is in interaction with the water. The body  $V$  is a model of an earth dam ( $V_1, V_2, V_3, V_5$  are the retaining prisms,  $V_4$  is the core); the material properties of the dam are assumed nonhomogeneous in volume. The lower part of the dam rests on a rigid foundation  $\Sigma_u$ .

Periodic kinematic impact  $\vec{u}_0(x, y, t)$  is applied to surface  $\Sigma_u$ , and hydrostatic water pressure  $\vec{p}_s(x, y)$  acts on  $S_p$ ,  $\vec{f}$  are the body forces, and the soil in some sections of the dam has viscoelastic properties.



**Fig. 1.** Scheme of a nonhomogeneous system.

It is necessary to investigate the steady-state forced oscillations of the system in a plane strain state (Figure. 1) under the action of periodic kinematic influences  $\vec{u}_0(x, y, t)$ , body forces  $\vec{f}$  and hydrostatic water pressure  $\vec{p}_s(x, y)$ .

The study of this type of oscillations of the system makes it possible to determine the effect of the maximum amplitudes of displacements and stresses at any point of the dam on the parameters of the system and external impacts. In this case, the intensity of the value of resonant amplitudes of displacements and stresses are quantitative estimates.

To simulate dynamic processes in the system (Figure 1), the principle of virtual displacements based on the d'Alembert principle is used, i.e.:

$$\delta A = - \int_V \sigma_{ij} \delta \varepsilon_{ij} dV - \int_V \rho_n \ddot{u} \delta u dV + \int_V \vec{f} \delta u dV + \int_{S_p} \vec{p}_s \delta u dS = 0, \quad (1)$$

Kinematic boundary conditions are

$$x \in \Sigma_u : \vec{u}_o(\vec{x}, t) = \vec{\psi}_1(t), \quad (2)$$

Here  $\vec{u}$ ,  $\varepsilon_{ij}$ ,  $\sigma_{ij}$  are the components of the displacement vector, strain and stress tensors, respectively;  $\delta \vec{u}$ ,  $\delta \varepsilon_{ij}$  - are the isochronous variations of displacements and strains;  $\rho_n$  is the density of the material of the elements of the considered system ( $n=1,2,3,4,5$ );  $\{u_1, u_2\} = \{u, v\}$ ,  $\{x\} = \{x_1, x_2\} = \{x, y\}$  are the displacement vector components and body coordinate point, respectively;  $\vec{f}$  is the vector of body forces;  $\vec{\psi}_1(t)$  is the periodic function of time;  $i, j, k = 1, 2$ .

Hydrostatic water pressure is determined by the following formula [50].

$$\vec{p}_c = \rho_o g (h - y), \quad (3)$$

Where  $\rho_o$  is the density of water;  $(h - y)$  is the depth of a point on the upstream face of the dam.

Physical relations between stresses  $\sigma_{ij}$  and strains  $\varepsilon_{ij}$  of the form [51,52] are also used:

$$\sigma_{ij} = \tilde{\lambda}_n \varepsilon_{kk} \delta_{ij} + 2\tilde{\mu}_n \varepsilon_{ij} \quad (4)$$

Volterra integral operators are used to describe the viscoelastic properties of the material [51,52] :

$$\left. \begin{aligned} \tilde{\lambda}_n \varphi &= \lambda_n \left[ \varphi(t) - \int_{-\infty}^t \Gamma_{\lambda_n}(t-\tau) \varphi(\tau) d\tau \right] \\ \tilde{\mu}_n \varphi &= \mu_n \left[ \varphi(t) - \int_{-\infty}^t \Gamma_{\mu_n}(t-\tau) \varphi(\tau) d\tau \right] \end{aligned} \right\} \quad (5)$$

Where  $\lambda_n, \mu_n$  are the Lamé constants;  $\Gamma_{\lambda_n}, \Gamma_{\mu_n}$  are the relaxation kernels;  $\varphi(t)$  is an arbitrary function of time.

The Cauchy relation [51] is used:

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad i, j = 1, 2. \quad (6)$$

Now for the system (Fig. 1) it is necessary to determine  $\vec{u}(\vec{x}, t)$ ,  $\sigma_{ij}(\vec{x}, t)$  in the structure, appearing under the effects of (2), (3), and  $\vec{f}$ , satisfying equations (1), (4), (5), (6) and the periodicity conditions for any virtual displacement  $\delta \vec{u}$ .

## 2.2. Solution method

The solution of the variational problem (1)–(6) is sought in the following form

$$\vec{u}(\vec{x}, t) = \vec{u}_o(\vec{x}, t) + \vec{u}^*(\vec{x}) \cdot \exp(-i\Omega t), \quad (7)$$

Where  $\Omega$  is the specified frequency of external influences;  $\vec{u}^*(\vec{x})$  is the vector of sought-for displacement amplitudes,  $i$  is the complex number.

In this case, (5) can exactly be replaced by an expression in the following form

$$\left. \begin{aligned} \tilde{\lambda}_n \varphi &= \lambda_n \left[ 1 - \Gamma_{\lambda_n}^C(\Omega) - i\Gamma_{\lambda_n}^S(\Omega) \right] \varphi \\ \tilde{\mu}_n \varphi &= \mu_n \left[ 1 - \Gamma_{\mu_n}^C(\Omega) - i\Gamma_{\mu_n}^S(\Omega) \right] \varphi \end{aligned} \right\}, \quad (8)$$

Where  $\Gamma_{\lambda_n}^S(\Omega)$ ,  $\Gamma_{\lambda_n}^C(\Omega)$ ,  $\Gamma_{\mu_n}^S(\Omega)$ ,  $\Gamma_{\mu_n}^C(\Omega)$  are the sines and cosines of the Fourier image of kernels  $\Gamma_{\lambda_n}(\tau)$ ,  $\Gamma_{\mu_n}(\tau)$ .

Substitution of (7) into (1) reduces the variational equation (1) to the following form:

$$\begin{aligned} - \int_V \sigma_{ij}^* \delta \varepsilon_{ij} dV + \Omega^2 \int_V \rho_n \bar{u}^* \delta \bar{u}^* dV = - \int_V f \delta \bar{u}^* dV + \\ + \int_V \rho_n \ddot{u}_o \delta \bar{u}^* dV - \int_{S_p} \bar{p}_c(\bar{x}) \delta \bar{u}^* dV, \end{aligned} \quad (9)$$

Where  $\sigma_{ij}^*$  is the complex amplitude of the stress tensor components;  $\bar{u}^*(\bar{x})$  is the sought-for complex amplitude.

The procedure of the finite element method [53] allows us to reduce the variational equation (9) to a complex nonhomogeneous algebraic system of the following form

$$([K] - \Omega^2 [M]) \{u\} = \{f\} \quad (10)$$

Where  $[K]$  is the complex stiffness matrix;  $[M]$  is the matrix of masses of the system (Fig. 1);  $\{u\}$  is the vector of sought-for complex displacement amplitudes;  $\{f\}$  is the load vector from kinematic impact, hydrostatic pressure and body forces. Elements  $k_{ij}(\Omega)$  of matrix  $[K]$  are functions of the impact frequency  $\Omega$ .

Now the resulting complex system of algebraic equations (10) at fixed frequencies of external impacts is solved by the Gauss method or the square root method.

To implement the developed mathematical model and method for solving the problems under consideration, a program was developed on the IBM PC using complex arithmetic.

### 3 Results and discussion

In this section, the steady-state forced vibrations of the projected Pskem earth dam, 195 m high, are studied, taking into account their actual geometry and nonhomogeneous features of the structure.

According to the design data, the Pskem dam has the following characteristics: dam height = 195 m, upstream slope coefficient  $m_1=2.4$ , downstream slope coefficient  $m_2=2.0$ , crest width  $b = 12.0$  m, width of the lower part of the core = 130.0 m. Core material properties are: deformation modulus  $E_{\partial e \phi} = 15$  MPa, Poisson's ratio  $\nu=0.32$ ; specific gravity of

soil  $\gamma=1.7$  tf/m<sup>3</sup>, angle of internal friction  $\varphi = 24^\circ$ , soil cohesion coefficient

$c = 20$  kPa. Prism material properties are: deformation modulus  $E_{\partial e \phi} = 95$

MPa, Poisson's ratio  $\nu=0.27$ ; soil specific gravity  $\gamma=1.97$  tf/m<sup>3</sup>, internal friction angle

$\varphi = 42^\circ$ , soil cohesion coefficient  $c = 7$  kPa.

To describe the viscoelastic properties of soil, the kernels of the form [54] were used:

$$\Gamma(t) = A e^{-\beta t} t^{\alpha-1} \quad (11)$$



Parameters  $A, \alpha, \beta$  of the kernel (11) for various types of soil were determined in [55] from the experimental data of soil using the technique described in [51].

Then, the steady-state forced oscillations of the dam were studied under two-component periodic kinematic effects at the base of the structure, i.e.:

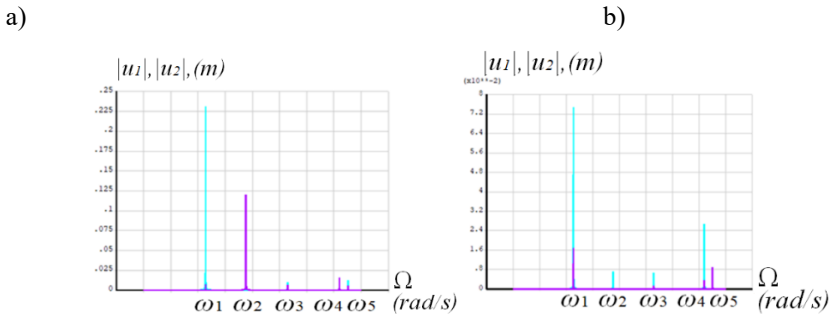
$$\vec{x} \in \sum_u \begin{cases} u_{10}(t) = B \exp(-i\Omega t) \\ u_{20}(t) = C \exp(-i\Omega t) \end{cases} \quad (12)$$

Where  $B, C, \Omega$  are the amplitudes and frequency of the kinematic impact, respectively.

Based on the results obtained, the frequency response for a number of characteristic points of dams, i.e.: displacements -  $(u_1, u_2)$ , stresses  $\sigma_{11}, \sigma_{22}, \sigma_{12}$ , principal stresses -  $\sigma_1, \sigma_2, \tau_{max}$ , and intensity of normal stresses  $\sigma_i$  at various kinematic impacts " $\Omega$ " (12) were built in the range from 1.0 to 20.0 rad/sec. At that,  $B/C = 2.0$  ( $B = 0.01m$ ).

To check the adequacy of the developed mathematical models and the accuracy of the solution method, amplitude-frequency response characteristics (AFC) of displacements  $(u_1, u_2)$  were constructed for various points of the dam. Figure 2 shows the frequency response for two points of the dam, i.e., for **A** ( $x_1=-7.0$  m,  $x_2=195.0$  m) and for **F** ( $x_1=-102.9$  m,  $x_2=70.4$  m), without considering body forces and viscoelastic properties of soil.

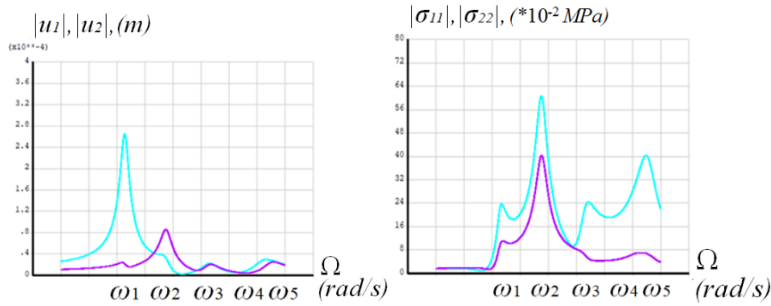
If the impact frequency « $\Omega$ » coincides with any of the frequencies of the natural vibrations of the dam, i.e.  $\omega_1=0.2138$  Hz;  $\omega_2=0.2880$  Hz;  $\omega_3=0.3630$  Hz;  $\omega_4=0.3649$  Hz;  $\omega_5=0.4600$  Hz, then, elastic resonance occurs; this confirms the reliability of the developed technique (Figure 2).



**Fig. 2.** Amplitude-frequency response of displacements ( $|u_1|, |u_2|$ ) of points **A** - (a) and **F** - (b) of the elastic Pskem dam, obtained without considering body forces: ( ) - horizontal displacements ( $u_1$ ) ( ) - vertical displacements ( $u_2$ ).

Figure 3. a shows the frequency response of horizontal  $u_1$  and vertical  $u_2$  displacements of point **F** ( $x_1=-102.9$  m,  $x_2=70.4$  m), and Figure 3. b shows the frequency response of horizontal and vertical normal stresses at point **F1** ( $x_1=-36.9m, x_2=163.4m$ ) of the Pskem dam, considering the viscoelastic properties of the material and ignoring the body forces.

a) b)



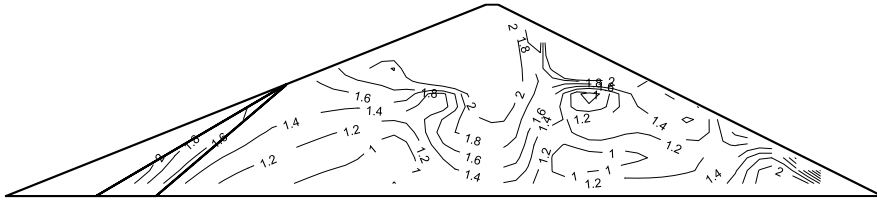
**Fig. 3.** Amplitude-frequency characteristics of displacements -  $|u_1|, |u_2|$  for point F, and horizontal and vertical stresses  $|\sigma_{11}|, |\sigma_{22}|$  or point F1 of the Pskem earth dam, considering the viscoelastic properties of the material: ( ) - horizontal displacements ( $u_1$ ); ( ) - vertical displacements ( $u_2$ ); ( ) - stress  $\sigma_{11}$ ; ( ) - stress  $\sigma_{22}$ .

Analysis of Fig. 3, a shows that for many points of the dam at the first resonance, the amplitudes of horizontal displacements  $|u_1|$  are much higher than the amplitudes of vertical displacements  $|u_2|$ . At the second resonance, on the contrary,  $|u_2|$  has a sufficiently commensurate amplitude in comparison with  $|u_1|$  (Figure 3, a). This is explained by the fact that the first mode of natural vibrations of the structure is a shift of the central section, and the second mode is the vertical strain of the dam, etc.

Analysis of Figure 3, b (i.e., frequency response by stresses), shows that the maximum stress amplitudes at the points of the dam occur when frequency  $\Omega$  coincides with  $\omega_1$ . The maximum amplitudes of vibrations for various points of the dam can arise when frequency  $\Omega$  coincides with  $\omega_1$  and with a dense spectrum of eigenfrequencies, i.e.,  $\omega_3 - \omega_4$  (Figure 3. b). This is due to the interaction of eigenmodes of vibration with similar frequencies, which create a single peak of a high amplitude; this also can be dangerous in terms of structure strength.

The main normal and maximum shear stresses in the body of the dam were studied under steady-state forced vibrations under different frequencies  $\Omega$  of impact, i.e., before resonance, near resonance, and after resonance. At the same time, separate dangerous areas in the structure were identified where the maximum stresses occur.

Along with this, the isolines of the distribution of equal values of the safety factor  $K$  in the section of the Pskem dam are shown; they were obtained using the Coulomb-Mohr theory of strength [56], considering the viscoelastic properties of soil and ignoring the body forces, at impact frequency  $\Omega$  between the first and second resonant frequencies (Figure 4.). It is assumed that for  $K > 1$  - in this section of the structure the soil is in a pre-limit state and has a margin of safety; for  $K = 1$  - the soil is in the condition of limit equilibrium; and for  $K < 1$  - the soil strength in this section of the structure is violated and an instability zone is formed.



**Fig. 4.** Isolines of the distribution of equal values of the safety factor  $K$  in the section of the Pskem dam, obtained at an impact frequency  $\Omega$  between the first and second resonant frequencies, considering the viscoelastic properties of soil and ignoring the body forces.

An analysis of the results (Figure 4.) shows that at this frequency of the impact, the sections of the dam located in the middle of the lowest part of the upstream and downstream prisms of the dam have the minimum strength. The remaining sections of the dam have adequate strength.

## 4 Conclusion

1. The article provides a detailed overview of known publications related to the evaluation of the dynamic behavior of various dams.
2. A mathematical model was developed that takes into account the viscoelastic properties of soil using the Boltzmann-Volterra hereditary theory of viscous-elasticity.
3. A method and algorithms for solving dynamic problems for earth dams in a plane-strain state using the Boltzmann-Volterra hereditary theory under stationary kinematic effects were presented.
4. Steady-state forced vibrations of a high earth dam were studied, considering the viscoelastic properties of soil under resonant vibration modes.
5. The maximum amplitudes of vibrations of the points of the structure and dangerous sections of the dam were revealed in terms of the highest stress occurring under various resonant frequencies.
6. The strength of the dam was tested at various frequencies of kinematic impact using the Coulomb-Mohr theory of strength; the safety factors for various points of the dam were determined.

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