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(S, d)Magic Labeling of some ladder graphs

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Abstract. Let $G(p,q)$ be a connected, undirected, simple and non-trivial graph with p nodes and q lines. Let f be an injective function f: V(G) →{ $s, s+d, s+2d, \ldots s+(q+1)d$ } and g be an injective function g: E(G) → {d,2d,3d,… 2(q-1)d}.Then the graph G is said to be (s,d) magic labeling if $f(u)+g(uv)+f(v)$ is a constant, for all $u, v \in V(G)$. A graph G is called (s,d) magic graph if it admits (s,d) magic labeling. In this paper the existence of (s,d) magic labeling in some ladder graphs are found.

1 Introduction

In graph labeling, a collection of integers are assigned to a set of nodes, lines, or both based on specific criteria. By applying the "magic" concept to graphs, we want the total number of labels associated with a vertex's or an edge's edges to remain constant across the graph. Sedlacek introduced the first magic-type labelling in 1963. He assigned real numbers to the edges of a graph and required the labelled sum of all edges incident to a vertex be constant.

2 Definitions

Definition 2.1:

A graph $G(p,q)$is said to be (s,d) magic graph if there exists a function $f: V(G) \rightarrow$ { $s, s+d, s+2d, \ldots s+(q+1)d$ } and $g: E(G) \rightarrow$ {d, 2d, 3d.….2(q-1)d} which are injective such that the sum of the labels on the vertices and the labels of its incident edge is a constant.

Definition 2.2: [5]

Let P_n denotes the path on n vertices, then the Cartesian product of $P_n \times P_2$,where n≥ 2, is called a ladder graph

Definition 2.3:

Two paths of length $n-1$ with $V(G) = \{u_i v_i : 1 \leq i \leq n$ and

$E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 2 \leq i \leq n-1\}$ form an Open ladder .

Definition 2.4:

The slanting ladder is a graph that consists of two copies of Pn with vertex set$\{u_i : 1 \leq i \leq n\} \cup$

$\{v_i : 1 \leq i \leq n\}$ and edge set is generated by linking u_i and v_{i+1}, $1 \leq i \leq n-1$.

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Definition 2.5

The graph TLn n≥ 2 is formed by adding the edges $u_i v_{i+1}$: 1 ≤ i ≤ n − 1, to the ladder, where Ln is the graph P2 × Pn.

Definition 2.6:

By adding the edges $u_i v_{i+1}$ $for\ 1 \leq i \leq n-1$,a triangular ladder is modified in to open triangular ladder and it is denoted as O(TL_n)

Definition 2.7: [5]

Circular ladder graph is a simple graph obtained by using Cartesian product of cycle graph C_n with n vertices and path graph P_2 , and is denoted by CL_n.(i.e $C_n \times P_2 = CL_n$).This is isomorphic to the graph obtained by linking the end vertices of the ladder by two new edges in cyclic form.

Definition 2.8 :

By eliminating the edges $u_i v_i$ for i=1 and n ,a diagonal ladder graph is modified in to open diagonal ladder and it is denoted as O(DL_n)

Definition 2.9: [3]

A Mobius ladder graph M_n is a graph obtained from the ladder $P_n \times P_2$ by linking the opposite end points of the two copies of P_n.

3 Main result

Theorem 3.1 An open ladder graph O(L_η) is (s, d) magic labeling .

Proof: Let $V\big(O(L_\eta\big) = \{u_i v_i : 1 \leq i \leq \eta\}$ and $E\big(O(L_\eta\big) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq \eta - 1\}$

$$\cup\ \{u_i v_i:\ 2 \leq i \leq \eta - 1\}$$

Here $p = \eta$ and $q = 2\eta + (\eta - 4)$

By definition $f: V(G) \rightarrow \{s, s + d, s + 2d \ldots . s + (q + 1)d\}$ to label the vertices.

$$f(u_1) = s$$

$$f(u_{i+1}) = s + id, \qquad 1 \leq i \leq \eta\text{-}1$$

$$f(v_i) = s + \eta d \qquad i = 1$$

$$f(v_{i+1}) = s + (\eta + i)d, \qquad 1 \leq i \leq \eta - 1$$

We define $g: E\ (G) \rightarrow$ {d, 2d, 3d.....2(q-1)d}

$$g(u_i u_{i+1}) = 2(s + (q-1)d) - (f(u_{i+1}) + f(u_i)) \quad 1 \leq i \leq \eta - 1$$

$$g(v_i v_{i+1}) = 2(s + (q-1)d) - (f(v_{i+1}) + f(v_i)) \quad 1 \leq i \leq \eta - 1$$

$$g(u_i v_i) = 2(s + (q-1)d) - (f(v_i) + f(u_i)) \quad 1 \leq i \leq \eta$$

Therefore $f(u_i) + g(u_i u_{i+1}) + f(u_{i+1})$, $f(v_i) + g(v_i v_{i+1}) + f(v_{i+1})$ and $f(u_i) + g(u_i v_i) + f(v_i)$

are constant, equals to $2(s + (q-1)d)$. Hence an open ladder O(L_η) admits (s, d)magic labeling.

Table 1. Labeling of vertices and edges for the graph O(L_η).

	Labeling of vertices		Labeling of edges		
Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$g(u_i u_{i+1})$	$g(v_i v_{i+1})$	$g(u_i v_i)$
$i = 0$	s	$s+\eta d$	-	-	-
, $1 \le i \le \eta - 1$	$s + id,$	$s+(\eta+i)d$	$2(s+(q-1)d)-(f(u_{i+1})+f(u_i))$	$2(s+(q-1)d)-(f(v_{i+1})+f(v_i))$	-
$i = \eta$	-	-	-	-	$2(s+(q-1)d)-(f(v_i)+f(u_i))$

Theorem 3.2: A triangular ladder T(L_η) is (s, d) magic labeling.

Proof: Let V (T(L_η))= $\{u_i v_i;\ 1 \le i \le \eta\}$ and E (T(L_η)) = $\{u_i u_{i+1}, v_i v_{i+1}, 1 \le i \le \eta\text{-}1\}$ $\cup$ $\{u_i v_i;\ 1 \le i \le \eta\} \cup \{u_i v_{i+1}\ 1 \le i \le \eta\text{-}1\}$

By definition $f: V(G) \rightarrow \{s, s+d, s+2d \ldots . s+(q+1)d\}$ to label the vertices.

Hence $p = \eta$ and $q = 4\eta - 3$

$$f(u_1) = s$$

$$f(u_{i+1}) = s + 2id, \qquad 2 \le i \le \eta$$

$$f\ (v_1) = s + d$$

$$f\ (v_{i+1}) = s + (2i+1)d \quad 2 \le i \le \eta$$

We define $g: E\ (G) \ \rightarrow\ \{\mathrm{d}, 2\mathrm{d}, 3\mathrm{d} \ldots . . 2(\mathrm{q}\text{-}1)\mathrm{d}\}$

$$g\ (u_i v_i) = 2(s+(q-1)d) - \ (f(u_i) + f(v_i))$$

$$g\ (v_i v_{i+1}) = \ 2(s+(q-1)d) - (f(v_{i+1}) + f(v_i)) \quad 1 \le i \le \eta - 1$$

$$g\ (u_i u_{i+1}) = \ 2(s+(q-1)d) - (f(u_{i+1}) + f(u_i)) \quad 1 \le i \le \eta - 1$$

$$g\ (u_i v_{i+1}) = \ 2(s+(q-1)d) - (f(u_{i+1}) + f(v_i)) \quad 1 \le i \le \eta - 1$$

Therefore, $f\ (u_i) + g\ (u_i u_{i+1}) + f\ (u_{i+1}),\ f\ (v_i) + g\ (v_i v_{i+1}) + f\ (v_{i+1})$, $f(u_i) + g\ (u_i v_{i+1}) + f(v_{i+1})$and $f\ (u_i) + g\ (u_i v_i) + f\ (v_i)$ are a constant, equals to $2\ (s+(q-1)d)$. Hence triangular ladder T(L_η) admits (s, d)magic labeling.

Table 2. Labeling of vertices and edges for the graph T(L_η)

	Labeling of vertices	Labeling of edges

Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$g(u_iv_i)$	$g(v_iv_{i+1})$	$g(u_iu_{i+1})$	$g(u_iv_{i+1})$
$i=0$	s	$s+d$	-	-	-	-
$1 \le i \le \eta - 1$	-	-	-	$2(s+(q-1)d)-(f(v_i)+f(v_{i+1}))$	$2(s+(q-1)d)-(f(u_i)+f(u_{i+1}))$	$2(s+(q-1)d)-(f(v_i)+f(u_{i+1}))$
$2 \le i \le \eta - 1$	$s+2id$	$s+(2i+1)d$	-	-	-	-
$1 \le i \le \eta$	-	-	$2(s+(q-1)d)-(f(u_i)+f(v_i))$	-	-	-

Theorem 3.3: A slanting ladder SL_η is (s,d) magic labeling

Proof: Let V(SL_η) = $\{v_1u_i \ \ 1\le i \le \eta\}$ and E(SL_η) = $\{v_iu_{i+1}, (v_1v_{i+1}), (u_iu_{i+1}) \ 1\le i \le$ η-1

By definition $f: V(G) \to \{s, s+d, s+2d \dots . s+(q+1)\, d\}$ to label the vertices.

Here $|\mathrm{V}(SL_\eta)| = \eta$ and $|\mathrm{E}(SL_\eta)| = 3(\eta-1)$

$$f\ (v_1) = s+d$$

$$f\ (u_{i+1}) = s+2id, \qquad 2\le i\le \eta-1$$

$$f\ (v_{i+1}) = s+(2i+1)\,d \quad 2\le i\le \eta-1$$

We define $g: E\ (G)\ \to\ \{\mathrm{d, 2d, 3d \dots . 2(q\text{-}1)d}\}$

$$g\ (u_iu_{i+1}) = \ 2(s+(q-1)d)-(f(u_{i+1})+f(u_i))\ \ 1\le i\le \eta-1$$

$$g\ (v_iv_{i+1}) = \ 2(s+(q-1)d)-(f(v_{i+1})+f(v_i))\ \ 1\le i\le \eta-1$$

$$g\ (u_iv_{i+1}) = \ 2(s+(q-1)d)-(f(v_{i+1})+f(u_i))\ \ 1\le i\le \eta-1$$

Therefore $f(u_i)+g\ (u_i\ u_{i+1})+f\ (u_{i+1})$, $f(u_i)+g\ (u_i\, u_{i+1})+f\ (v_{i+1})$ and $f(v_i)+g\ (v_i\, v_{i+1})+f\ (v_{i+1})$ are constant, equals to 2 $(s+(q-1)d)$
Hence slanting ladder SL_η admits (s,d) magic labeling .

Table 3. Labeling of vertices and edges for the graph S(L_η)

	Labeling of vertices		**Labeling of edges**		
Value of *i*	$f(u_{i+1})$	$f(v_{i+1})$	$g(u_iu_{i+1})$	$g(v_iv_{i+1})$	$g(u_iv_{i+1})$
$i=0$	s	$s+d$	-	-	-
$1 \le i \le \eta - 1$	-	-	$2(s+(q-1)d)-(f(u_i)+f(u_{i+1}))$	$2(s+(q-1)d)-(f(v_i)+f(v_{i+1}))$	$2(s+(q-1)d)-(f(u_i)+f(v_{i+1}))$

$2 \leq i \leq \eta - 1$	$s + 2id$	$s + (2i + 1)d$	-	-	-

Theorem 3.4: An open triangular ladder O(TL_η) is (s,d) magic labeling $\eta \geq 2$.

Proof: Let V(O(TL_η)) = $\{u_i v_i \quad 1 \leq i \leq \eta\}$ and E(O(TL_η)) = $\{u_i u_{i+1}, (v_i v_{i+1})\}$, $1 \leq i \leq \eta$-1}
$\cup \{u_i v_i \ \ 2 \leq i \leq \eta - 1\} \cup \{u_i v_{i+1} \ \ 1 \leq i \leq \eta - 1\}$

By definition $f: V(G) \to \{s, s+d, s+2d \ldots . s(q+1)d\}$ to label the vertices.

$$\text{let } f\ (u_1) = s$$

$$f\ (v_1) = s + d$$

$$f\ (u_{i+1}) = s + 2id, \qquad 2 \leq i \leq \eta - 1$$

$$f\ (v_{i+1}) = s + (2i+1)\ d \quad 2 \leq i \leq \eta - 1$$

We define $g: E\ (G) \to$ {d, 2d, 3d.....2(q-1)d}

$$g\ (u_i u_{i+1}) = 2(s + (q-1)d) - (f\ (u_{i+1}) + f\ (u_i)) \qquad 1 \leq i \leq \eta - 1$$

$$g\ (v_i v_{i+1}) = 2(s + (q-1)d) - (f\ (v_{i+1}) + (f\ (v_i)) \qquad 1 \leq i \leq \eta - 1$$

$$g\ (u_i v_i) = 2(s + (q-1)d) - (f\ (v_i) + f\ (u_i)) \qquad 2 \leq i \leq \eta - 1$$

$$g\ (u_i v_{i+1}) = 2(s + (q-1)d) - (f\ (v_{i+1}) + f\ (u_i)) \qquad 1 \leq i \leq \eta - 1$$

Therefore

$f\ (u_i) + g\ (u_i\, u_{i+1}) + f\ (u_{i+1})$, $f(v_i) + g\ (v_i\, v_{i+1}) + f\ (v_i + 1)$, $f\ (u_i) + g\ (u_i v_i) + f\ (v_i)$ and $f\ (u_i) + g\ (u_i\, v_{i+1}) + f\ (v_{i+1})$ are constant, equals to $2(s + (q-1)d)$

Hence O(TL_η) admits (s, d)magic labeling.

Table 4. Labeling of vertices and edges for the graph O(TL_η)

	Labeling of vertices		Labeling of edges			
Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$g(u_i v_i)$	$g(v_i v_{i+1})$	$g(u_i u_{i+1})$	$g(u_i v_{i+1})$
$i = 0$	s	$s + d$	-	--	-	-
$1 \leq i \leq \eta - 1$	-	-	-	$2(s + (q-1)d) - (f\ (v_{i+1}) + (f\ (v_i))$	$2(s + (q-1)d) - (f\ (u_{i+1}) + f\ (u_i))$	$2(s + (q-1)d) - (f\ (v_{i+1}) + f\ (u_i))$
$2 \leq i \leq \eta - 1$	$s + 2id$	$s + (2i + 1)\ d$	$2(s + (q-1)d) - (f\ (v_i) + f\ (u_i))$	-	-	-

Theorem 3.5: The Mobius ladder M_η is (s, d) magic labeling Proof: Let G = $M_\eta$with $V(M_\eta) = \{v_i u_i\ , 1 \leq i \leq \eta\}$ and $E(M_\eta) = \{(u_i u_{i+1}), (v_i v_{i+1}) \quad 1 \leq i \leq \eta - 1\}$

By definition $f: V(G) \rightarrow \{s, s+d, s+2d \ldots . s+(q+1)d\}$ to label the vertices.
Case 1: If η is odd
Let $f(u_i) = s + 2(i-1)d$ $f(v_i) = s + (2i-1)d$

Table 5. Subcase

η	$\eta = 3$	$\eta = 5$	$\eta \geq 7$
$f(u_\eta)$	$s + (\eta + 2)d$	$s + 2\eta d$	$s + 2\eta d$
$f(v_\eta)$	$s + (\eta + 5)d$	$s + (2\eta + 4)d$	$s + (2\eta + 5)d$

Case 2: If η is even Let $f\ (u_i) = s + 2(i-1)d \quad 1 \leq i \leq \eta$

$$f\ (v_i) = s + (2i-1)d; \quad 1 \leq i \leq \eta - 1$$

$$f\ (v_\eta) = s + (2\eta + 3)d$$

We define $g: E\ (G) \rightarrow$ {d, 2d, 3d.....2(q-1)d}

$$g\ (u_i u_{i+1}) = 2(s + (q-1)d) - (f\ (u_{i+1}) + f(u_i\ )) \quad 1 \leq i \leq \eta - 1$$

$$g(v_i v_{i+1}) = 2(s + (q-1)d) - (f(v_{i+1}\ ) + f\ (v_i\ )) \quad 1 \leq i \leq \eta - 1$$

$$g\ (u_i v_i) = 2(s + (q-1)d) - (f\ (v_i) + f\ (u_i\ )) \quad 1 \leq i \leq \eta$$

$$g\ (u_1 v_\eta) = 2(s + (q-1)d) - (f(u_1\ ) + f\ (v_\eta\ )\ )$$

$$g\ (u_\eta v_1) = 2(s + (q-1)d) - (f\ (v_1) + f\ (u_\eta\ )\ )$$

Therefore$f\ (u_i) + g\ (u_i\, u_{i+1}) + f\ (u_{i+1})$,$f\ (v_i) + g\ (v_i\, v_{i+1}) + f(v_{i+1})$,$f(u_i) + g\ (u_i v_i) + f\ (v_i)$, $f(u_i) + g(u_1 v_\eta) + f(v_\eta)$ and $f(u_\eta) + g\ (u_\eta v_1) + f\ (v_1)$ are constant, equals to 2 $(s + (q-1)d)$ .Hence M_η admits (s, d)magic labeling .

Table 6. Labeling of vertices and edges for the graph M_η

	Labeling of vertices			Labeling of edges		
Value of i	$f(u_i)$	$f\ (v_i)$	$f\ (v_\eta)$	$g\ (u_i u_{i+1})$	$g(v_i v_{i+1})$	$g\ (u_i v_k)$
$1 \leq i = k \leq \eta$	$s + 2(i - 1)d$	-	-	-	-	$2(s + (q - 1)d) - (f(u_i\ ) + f(v_i)\ )$
$1 \leq i \leq \eta - 1$	-	$s + (2i - 1)d$	-	$2(s + (q - 1)d) - (f(u_i\ ) + f(u_{i+1})\ )$	$2(s + (q - 1)d) - (f(v_i\ ) + f(v_{i+1})\ )$	-
$i = \eta$	-	-	$s + (2\eta + 3)d$	-	-	-

$i = 1$ and $k = \eta$	-	-	--	-	-	$2(s + (q - 1)d) - (f(u_1) + f(v_\eta))$
$i = \eta$ and $k = 1$	-	-	-	-	-	$2(s + (q - 1)d) - (f(v_1) + f(u_\eta))$

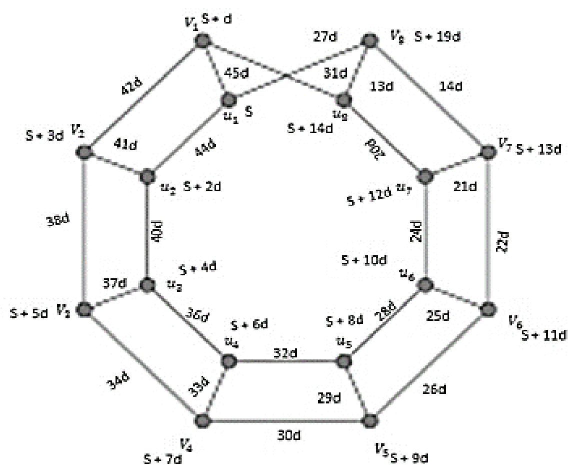

Fig 1. (S,d)Magic labeling of Mobius ladder M_8

Theorem 3.6: The Circular ladder CL_η is (s, d) magic labeling, when n is odd.

Proof: Let $V(CL_\eta) = \{u_i, v_i : 1 \le i \le \eta\}$ and $E(CL_\eta) = \{u_i u_{i+1} : 1 \le i \le \eta - 1\} \cup \{v_i v_{i+1} : 1 \le i \le \eta - 1\} \cup \{u_i v_i : 1 \le i \le \eta\} \cup \{u_1 u_\eta\} \cup \{v_1 v_\eta\}$.

By definition $f: V(G) \to \{s, s + d, s + 2d \ldots . s + (q + 1)d\}$ to label the vertices.

$$f(u_1) = s$$

$$f(u_{i+1}) = s + id \quad ; 1 \le i \le \eta - 1$$

$$f(v_1) = s + \eta d$$

$$f(v_{i+1}) = s + (\eta + i)d \quad ; 1 \le i \le \eta - 1$$

We define $g: E(G) \to \{d, 2d, 3d \ldots . . 2(q\text{-}1)d\}$

$$g(u_i u_{i+1}) = 2(s + (q - 1)d) - (f(u_{i+1}) + f(u_i)) \qquad 1 \le i \le \eta - 1$$

$$g(v_i v_{i+1}) = 2(s + (q - 1)d) - (f(v_{i+1}) + f(v_i)) \qquad 1 \le i \le \eta - 1$$

$$g(u_1 u_\eta) = 2(s + (q - 1)d) - (f(u_1) + f(u_\eta))$$

$$g\left(v_1 v_\eta\right) = 2(s + (q-1)d) - \left(f(v_1) + f\left(v_\eta\right)\right)$$

$$g\,(u_i v_i) = 2(s + (q-1)d) - (f(u_i) + f(v_i)) \qquad 1 \le i \le \eta$$

Therefore$f\,(u_i) + g\,(u_i\, u_{i+1}) + f\,(u_{i+1})$,$f\,(v_i) + g\,(v_i\, v_{i+1}) + f(v_{i+1})$,$f(u_i) + g\,(u_i v_i) + f(v_i)$,

$f\,(u_1) + g\,\left(u_1 u_\eta\right) + f\,(u_\eta)$ and $f\,(v_1) + g\,\left(v_1 v_\eta\right) + f\,(v_\eta)$ are constant, equals to $2\,(s + (q-1)d)$. .Hence CL_η admits (s, d)magic labeling.

Table 7. Labeling of vertices and edges for the graph CL_η

	Labeling of vertices		**Labeling of edges**		
Value of *i*	$\boldsymbol{f(u_{i+1})}$	$\boldsymbol{f(v_{i+1})}$	$\boldsymbol{g(u_i u_{i+1})}$	$\boldsymbol{g(v_i v_{i+1})}$	$\boldsymbol{g\,(u_i v_k)}$
$i = 0$	s	$s + \eta d$	-	-	--
$1 \le i \le \eta - 1$	$s + id$	$s + (\eta + i)d$	$2(s + (q-1)d) - (f(u_{i+1}) + f(u_i))$	$2(s + (q-1)d) - (f(v_{i+1}) + f(v_i))$	-
$1 \le i = k \le \eta$	-	-	-	-	$2(s + (q-1)d) - (f(u_i) + f(v_i))$
$i = 1$ and $k = \eta$	-	-	-	-	$2(s + (q-1)d) - (f(u_1) + f(u_\eta))$
$i = 1$ and $k = \eta$	-	-	-	-	$2(s + (q-1)d) - (f(v_1) + f(v_\eta))$

4 Conclusion

In this research, the (s, d) magic labelling number for some ladder graphs is determined. Our future work will involve calculating the (s, d) magic labelling number for more families of graphs.

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