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Applications of some modified open sets in r-neighbourhood spaces with ideals

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Abstract. The purpose of this paper is to introduce r-α-open sets, r-semi open sets and r-pre open sets in ideal r-neighbourhood space. We also discuss their properties, characterizations and relations with existing notions with suitable examples. Then we develop this paper towards r-limit points and r-interior points in ideal r-neighbourhood space and obtain nice results. 2010 Mathematics Subject Classification. Primary 03E99; Secondary 54A05, 54A10. Key words and phrases: r-α-open sets, r-semi open sets, r-pre open sets, r-limit point.

1 Introduction

Most of the mathematical models contribute to the society to solve real life problems and general topology is the appropriate model.

The concepts of rough sets using neighbourhood system was intro- duced and studied by Lin[13] and Yao[23]. Abd El-Monsef et al [2] introduced mixed neighbourhood systems to approximate the rough sets. In 2014, Abd El-Monsef et al [1] initiated to generalize the classi- cal rough set theory by intoduced different general topologies induced from binary relationsapplied by the notion of j-neighbourhood space and also many interesting research topics in the rough set theory via topology were studied in [3, 9, 11, 12, 16, 17, 18, 19, 22, 24].

Ideal is a fundamental concept in the topological space and plays an important role in the study of topological spaces. The notion of the ideal topological spaces introduced and studied by Kuratowski [8] and Vaidyanathaswamy[21]. Some of researchers [4, 7, 20] applying the no- tion of the ideals in the rough set theory. After that Hosny [5] generate different topologies by the concept of idealization of j-approximation spaces.

Mashhour et al.[14] introduced pre-open sets and Njastad[15] intro- duced α-open sets and Levine[10] introduced semi-open sets.

In this paper, we introduce r-α-open sets, r-semi open sets and r- pre open sets in ideal r-neighbourhood space. We also discuss their properties, characterizations and relations with existing notions with suitable examples. Then we develop this paper towards r-limit points and r-interior points in ideal r-neighbourhood space and obtain nice results.

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2 Preliminaries

In this section, we present some of basic concepts of ideals, j-neighbourhood, j-neighbourhood space and j-approximations.

Definition 2.1. [6] A non-empty collection I of subsets of a set U is called an ideal on U, if it satisfies the following conditions.

(1) A∈I and B∈I ⇒ A∪B∈I .

(2) A∈I and B⊂A ⇒ B∈I .

Definition 2.2. [13] Let U be non-empty finite set and R be an ar- bitrary binary relation on U. The r-neighbourhood of x ∈ ∪(Nr (x)) is defined as r-neighbourhood: Nr (x)={y∈U : xRy}.

Definition 2.3. [13] Let U be a non-empty finite set and R be an arbitrary binary relation on U and $\sum_r$:U→P(U) be a mapping which assigns for each each x in U its r-neighbourhood in P(U). The triple (U, R, $\sum_r$) is called a r-neighbourhood space (in briefly, r-NS).

Theorem 2.4. [13] Let U be a r-NS and A⊂U. Then, the collection

τr ={A⊂U : ∀p∈A, Nr (p)⊂A} is a topology on U.

Definition 2.5. [13] Let U be a r-NS and a subset A⊂U is called r- open set if A∈τr and the complement of r-open set is called r-closed set.

Theorem 2.6. [5] Let U be a r-NS and I be an ideal on U. Then, thecollection τ_r^I = {A⊂U : ∀p∈A, Nr (p)∩A'∈I } is a topology on U.

The r-NS U with an ideal I is called ideal r-NS

Definition 2.7. [5]

(1) Let U be an ideal r-NS and a subset A⊂U is called Ir -open setif A∈τ_r^I and the complement of Ir -open set is called Ir -closed

Lemma 2.8. [5] Let U be an ideal r-NS and A, B⊂U. Then

$$\underline{R_\tau^I}(A) \subset A \subset \overline{R_r^I}(A).$$

$$A \subset B \to \overline{R_r^I}(A) \subset \overline{R_r^I}(B)$$

$$A \subset B \to \underline{R_\tau^I}(A) \subset \underline{R_\tau^I}(B)$$

$$\underline{R_\tau^I}(\underline{R_\tau^I}(A)) = \underline{R_\tau^I}(A)$$

$$\overline{R_r^I}(\overline{R_r^I}(A)) = \overline{R_r^I}(A).$$

3 Some of open sets in ideal r-NS

Definition 3.1. Let U be an ideal r-NS and A⊂U. Then A

r-pre open if $A \subset \underline{R_r^I}(\overline{R_r^I}(A))$,

r-α-open if $A \subset \underline{R_r^I}(\overline{R_r^I}(\underline{R_r^I}(A)))$, r-semi open if A⊂ $\overline{R_r^I}(\underline{R_r^I}(\text{A})))$,

The complement of r-pre open(resp. r-α-open, r-semi open) sets are called r-pre closed (resp. r-α-closed, r-semi closed) sets.

Remark 3.2. r-open→Ir -open→r-α-open→r-semi open set ↓ r-pre open

The following example shows that the reverse implications need not be true for the above diagram.

Example 3.3. Let U={a, b, c, d}, R={(a, a), (a, b), (a, c), (b, c), (b, d), (c, a), (d, d)} and I ={∅, {a}}. Then τ I ={∅, U, {c}, {d}, {c, d},

{b, c, d}}, r-semi open sets are {∅, U, {c}, {d}, {a, c}, {a, d}, {b, c},

{b, d}, {c, d}, {a, b, c}, {a, c, d}, {b, c, d}, {a, b, d}}, r-α open sets are {∅, U, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}} and r-pre open sets are {∅, U, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}}.

Theorem 3.4. Let U be an ideal r-NS. Then a subset A of U is r-semi open if and only if $\overline{R_r^I}(\text{A})= \overline{R_r^I}(\underline{R_r^I}(\text{A})))$.

Proof. Let A be r-semi open set and so A⊂ $\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ which implies $\overline{R_r^I}(\text{A})$ ⊂$\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ and obviously $\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ ⊂ A. Hence $\overline{R_r^I}(\text{A})= \overline{R_r^I}(\underline{R_\tau^I}(\text{A})))$ and the converse is obvious.

Theorem 3.5. Let U be an ideal r-NS. Then a subset A of U is r-semi open if and only if there exists an Ir -open set O such that O⊂A⊂$\underline{R_r^I}(\text{O})$.

Proof. Let A be r-semi open set and so A⊂$\overline{R_r^I}(\underline{R_r^I}(\text{A})))$. Put O=$\underline{R_r^I}(\text{A})$. Then we have O⊂A⊂$\underline{R_r^I}(\text{O})$. Conversely, let O⊂A⊂$\underline{R_r^I}(\text{O})$ for some Ir –open set O. Since O⊂A and so O⊂$\underline{R_r^I}(\text{A})$ and hence $\overline{R_r^I}(\text{O})$ ⊂$\overline{R_r^I}(\underline{R_r^I}(\text{A}))$. Hence proved.

Theorem 3.6. If A is r-semi open subset of an ideal r-NS U and A⊂B⊂$\overline{R_r^I}(\text{A})$, then B is r-semi open.

Proof. Let A be r-semi open set. By Theorem 3.5, there exists an Ir -open set O such that O⊂A⊂$\underline{R_r^I}(\text{O})$. Therefore O⊂A⊂B⊂$\overline{R_r^I}(\text{A})$ ⊂ $\overline{R_r^I}(\overline{R_r^I}(\text{O}))= \overline{R_r^I}(\text{O})$ and hence O⊂B⊂$\overline{R_r^I}(\text{O})$. By theorem 3.5, B is r-semi open.

Theorem 3.7. Let U be an ideal r-NS. Then arbitrary union of r-semi open sets are r-semi open.

Proof. Let Oα , α∈Δ be arbitrary collection of r-semi open sets and so Oα⊂ $\overline{R_r^I}(\underline{R_r^I}(\text{O}\alpha))$ for each α∈Δ and by L Lbby we have $\bigcup \text{O}\alpha \subset \bigcup \overline{R_r^I}(\underline{R_r^I}(\text{O}\alpha)) \subset \overline{R_r^I}(\bigcup \underline{R_r^I}(\text{O}\alpha)) \subset \overline{R_r^I}(\underline{R_r^I}(\bigcup \text{O}\alpha))$. Hence proved

Remark 3.8. The finite intersection of r-semi open sets need not be an r-semi open set as shown in the following example.

Example 3.9. In the Example 3.3, {b, c} ∩ {b, d}={b} is not an r-semi open set.

Theorem 3.10. For an ideal r-NS, the following are equivalent.

(1) A is r-α-open,

(2) A is both r-pre open and r-semi open.

Proof. (1)→(2) Let A be an r-α-open set and so A⊂ $\underline{R_r^I}(\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ which implies that A⊂ $\underline{R_r^I}(\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ ⊂$\underline{R_r^I}(\overline{R_r^I}(\text{A}))$ and A⊂ $\underline{R_r^I}(\overline{R_r^I}(\underline{R_r^I}(\text{A})))$ ⊂ $(\overline{R_r^I}(\underline{R_r^I}(\text{A}))$. Hence proved.

(2)→(1) Since A⊂ $\underline{R_r^I}(\overline{R_r^I}(\text{A}))$ and A⊂$\overline{R_r^I}(\underline{R_r^I}(\text{A}))$ which implies A⊂ $\underline{R_r^I}(\overline{R_r^I}(\overline{R_r^I}(\underline{R_r^I}(\text{A}))))= \underline{R_r^I}(\overline{R_r^I}\ (\underline{R_r^I}(\text{A})))$.

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Theorem 3.11. If A is r-pre open subset of an ideal r-NS U and B⊂A⊂$\underline{R_r^I}$(B) then B is an r-pre open set.

Proof. Let A be r-pre open set and B⊂A ⊂$\overline{R_r^I}$(B) Therefore B⊂A⊂$\overline{R_r^I}$(A)⊂ $\overline{R_r^I}$(B) and $\overline{R_r^I}$(A) ⊂$\overline{R_r^I}$(B) implies that $\underline{R_r^I}(\overline{R_r^I}$(A)) ⊂ $\underline{R_r^I}(\overline{R_r^I}$(B)). Hence B⊂A⊂$\underline{R_r^I}(\overline{R_r^I}$(A))) ⊂$\underline{R_r^I}(\overline{R_r^I}$(A))

Definition 3.12. Let A be a subset of an ideal r-NS space U. A point x∈A is said to be r-α-limit point of A if for each r-α-open set O con- taining x, O∩(A−{x})=∅. The set of all r-α-limit points of A is called an r-α-derived set of A and is denoted by r-Dα (A).

Theorem 3.13. If A and B are subsets of an ideal r-NS U, then

(1) r-Dα (∅)=∅,

(2) If A⊂B, then r-Dα (A)⊂r-Dα (B),

(3) r-Dα (A)∪r-Dα (B)⊂r-Dα (A∪B) and r-Dα (A∩B)=r-Dα (A)∩r-Dα (B),

(4) r-Dα (r-Dα (A))−A⊂r-Dα (A),

(5) r-Dα (A∪r-Dα (A))⊂A∪r-Dα (A).

Proof. (1) Let x∈U and H be an r-α open set containing x. Then H∩(∅−{x})=∅. Therefore, for any x∈U, x is not an r-α-limit point of ∅. Hence r-Dα (∅)=∅.

(2) Let x∈r-Dα (A) and H be an r-α open set containing x and so H∩(A−{x})=∅. Since A⊂B which implies H∩(B−{x})=∅. Thus x∈r-Dα (B). Hence r-Dα (A)⊂r-Dα (B). (3) This follows by (2).

(4) Suppose x∈r-Dα (r-Dα (A))−A and O is an r-α open set containing x, then O∩(r-Dα (A)−{x})=∅. Let y∈O∩(r-Dα (A)−{x}). Since y∈O and y∈r-Dα (A), O∩(A−{y})=∅. Let z∈O∩(A−{y}). Then z=x for z∈A and x∈/A. Therefore O∩(A−{x})=∅. Hence x∈r-Dα (A).

(5) Let x∈r-Dα (A∪r-Dα (A)). If x∈A, the proof is obvious. So, let x∈r-Dα (A∪r-Dα (A))−A, then for r-α open set O containing x, O∩(A∪(r- Dα (A)−{x}))=∅. Thus, O∩(A−{x})=∅ or O∩(r-Dα (A)−{x})=∅. Now, it follows by similar proof of (4) that O∩(A−{x})=∅. Therefore x∈r- Dα (A). Hence proved. □

Definition 3.14. A point x∈U is said to be r-α-interior point of A if there exists an r-α-open set O containing x such that O⊂A. The set of all r-α-interior points of A is said to be r-α-interior of A and its denoted by α-$\underline{R_r^I}$(A).

Theorem 3.15. If A and B are subsets of an ideal r-NS U, then

α-$\underline{R_r^I}$(A) is the largest r-α-open set contained in A,

A is r-α-open if and only if A= α-$\underline{R_r^I}$(A),

α-$\underline{R_r^I}$(α-$\underline{R_r^I}$(A))= α-$\underline{R_r^I}$(A),

α-$\underline{R_r^I}$(A)=A−α-$\underline{R_r^I}$(A),

If A⊂B, then α-$\underline{R_r^I}$(A) ⊂ α-$\underline{R_r^I}$(B),

α-$\underline{R_r^I}$(A)∪ α − $\underline{R_r^I}$(B) ⊂ α-$\underline{R_r^I}$(A∪ B),

$$\alpha\text{-}\underline{R_r^I}(A \cap B) \subset \alpha\text{-}\underline{R_r^I}(A) \cap \alpha - \underline{R_r^I}(B).$$

Proof. Proof of (1), (2), (3), (5), (6) and (7) immediate follows from the Definition 3.14.

(4)If $x \in A - \alpha - \underline{R_r^I}(A')$, then $x \nexists \alpha - \underline{R_r^I}(A')$, and so there exists an r-α-open set O containing x such that $O \cap A' = \emptyset$. Then $x \in O \subset A$ and hence $x \in \alpha\text{-}\underline{R_r^I}(A)$ which implies $A - \alpha - \underline{R_r^I}(A') \subset \alpha\text{-}\underline{R_r^I}(A)$. On the other hand, if $x \in \alpha - \underline{R_r^I}(A)$, then $x \nexists$ r − Dα (A′) since $\alpha\text{-}\underline{R_r^I}(A)$ is r-α-open and $\alpha\text{-}\underline{R_r^I}(A) \cap A' = \emptyset$. Hence $\alpha\text{-}\underline{R_r^I}(A) = A - \alpha - \underline{R_r^I}(A')$.

4 Application

In this section, we give application of the above types of open sets. We collect some of well defined objects thats are virus, fever, medicine and climate in a set. That is U={virus, fever, medicine, climate}.

Then, the relation on U defined as R={(virus, virus), (virus, fever), (virus, medicine), (fever, fever), (fever, medicine), (medicine, medicine), (climate, virus), (climate, fever), (climate, climate)} and so r-Neighbourhood of virus is {virus, fever, medicine}, r-Neighbourhood of fever is {fever, medicine}, r-Neighbourhood of medicine is {medicine} and r-Neighbourhood of climate is {fever, climate}.

From the above r-Neighbourhoods, Ir -open sets are ∅, U, {medicine}, {fever, medicine}, {virus, fever, medicine}, {fever, medicine, climate} and take I ={∅} and so r-semi open sets, r-α-open sets and r-pre open sets are ∅, U, {medicine}, {virus, medicine}, {fever, medicine}, {medicine, climate}, {virus, fever, medicine}, {virus, medicine, climate}, {fever, medicine, climate} and eliminate Ir -open sets from the above modified open sets. We get {virus, medicine}, {medicine, climate}, {virus, medicine, climate}.

5 Conclusion

From the above discussion, we can come to a conclusion that the common object is medicine and hence medicine has better relation to each others.

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