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On Soft SIgδs-closed sets

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Abstract. In this paper, we introduce some new notions called soft SIgδs - closed sets, soft SIgδs - open sets. We obtain several characterizations and some of their properties. Also, we investigate its relationship with other types of sets. 2010 Mathematics Subject Classification. 54A10, 54A20, 54C08. **Key words and phrases:** soft sets, soft topological spaces, soft regular open, soft δ-cluster point, soft SIgδs -closed.

1 Introduction

The concept of soft sets was first introduced by Molodtsov [12] in 1999 as a general mathematical tool for dealing with uncertain objects. In [12, 13], Molodtsov [12] first suggested the idea of soft sets in 1999 as a generic mathematical technique for handling ambiguous things. In [12, 13], Molodtsov successfully implemented the soft theory in a number of areas, including probability, theory of measurement, Riemann integration, game theory, operations research, and smoothness of functions.

With the presentation of soft set operations [11], the properties and applications of soft set theory have undergone a growing amount of research [3, 8, 13]. By incorporating the concepts of fuzzy sets, numerous intriguing applications of soft set theory have recently been explored [1, 2, 4, 9, 10, 11, 13]. The operations of the soft sets are redefined to create the soft set theory, and a uni-int decision-making procedure is used was constructed by using these new operations [5].2011 saw the start of the study of soft topological spaces by Shabir and Naz [15]. On the collection of soft sets over X, they defined soft topology. As a result, they established the many properties of fundamental concepts in soft topological spaces, such as soft open and soft closed sets, soft subspace, soft interior, soft closure, soft neighbourhood of a point, soft separation axioms, soft regular spaces, and soft normal spaces. Hussain and Ahmad [6] looked into the characteristics of a point's soft interior, soft closure, soft exterior, and soft neighbourhood. Also, they defined and discussed the characteristics of soft interior, soft exterior, and soft border, which are crucial for further study of soft topology and will reinforce the theoretical underpinnings of soft topology.spaces.

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In this paper, we introduce some new notions called soft SIgδs -closed sets and soft SIgδs -open sets. We obtain several characterizations and some of their properties. Also, we investigate its relationship with other types of sets.

2 Preliminaries

In this section, we present some basic definitions and resuts which are needed in further study of this paper which may found in earlier studies. Throughout this paper, X refers to an initial universe, E is a set of parameters, ℘(X) is the power set of X, and A⊂E, Y refers to an initial universe, K is a set of parameters, B⊂K and Z refers to an initial universe, L is a set of parameters, C⊂L.

Definition 2.1. [12] A soft set FA over the universe X is defined by the set of ordered pairs

$$FA = \{(e, FA(e)) : e \in E, FA(e) \in \wp(X)\}$$

where FA: E→℘(X), such that FA (e)=∅, if e∈A⊂E and FA (e)=∅ if e∈/ A. The family of all soft sets over X is denoted by SS(X).

Definition 2.2. [11] The soft set F∅ over a common universe set X is said to be null soft set, denoted by ∅. Here F∅ (e)=∅, ∀e∈E.

Definition 2.3. [11] A soft set FA over X is called an absolute soft set, denoted by A˜, if e∈A, FA (e)=X.

Definition 2.4. [11] Let FA, GB be soft sets over a common universe set X. Then FA is a soft subset of GB, denoted FA ⊂GB if FA (e)⊂GB (e), ∀e∈E.

Definition 2.5. [11] Let FA, GB be soft sets over a common universe set X. The union of FA and GB, is a soft set HC defined by HC (e)=FA (e)∪GB (e), ∀e∈E, where C=A∪B.

That is, HC =FA ∪GB.

Definition 2.6. [11] Let FA, GB be soft sets over a common universe set X. The intersection of FA and GB, is a soft set HC defined by HC (e)=FA (e)∩GB (e), ∀e∈E, where C=A∩B.

That is, HC =FA ∩GB.

Definition 2.7. [15] The complement of the soft set FA over X, denoted by F c is defined by A (e)=X−FA (e), ∀e∈E.

Definition 2.8. [15] Let FA be a soft set over X and x∈X. We say that x∈FA if x∈FA (e), ∀e∈A.

For any x∈X, x∈/ FA if x∈/ FA (e) for some e∈A.

Definition 2.9. [15] For two soft points xe and ye′ over a common universe X, we say that the points are different points if x=y or e=e'.

Definition 2.10. [18] The soft set FA ∈SS(X) is called a soft point in SS(X) if there exist x∈X and e∈E such that F(e)={x} and F(ec)=∅ for each ec∈E−{e} and the soft point FA is denoted by xε.

Definition 2.11. [15] A soft topology τ is a family of soft sets over X satisfying the following properties.(1) ∅, X˜ belong to τ.(2) The union of any number of soft sets in τ belongs to τ. (3) The intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called a soft topological space.

Definition 2.12. [14] Let (X, τ, E) be a soft topological space over X. Then

(1) The members of τ are called soft open sets in X.

(2) A soft set FA over X is said to be a soft closed set in X if F c ∈τ.

(3) A soft set FA is said to be a soft neighborhood of a point x∈X if x∈FA and FA^ is soft open in (X, τ, E)

(4) The soft interior of a soft set FA is the union of all soft open subsets of FA. The soft interior of FA is denoted by int(FA).

(5) The soft closure of FA is the intersection of all soft closed super sets of FA. The soft closure of FA is denoted by cl(FA) or FA.

Definition 2.13. [17] A soft set FA in a soft topological space (X, τ, E) is said to be a soft regular open (resp. soft regular closed) if FA =int(cl(FA)) (resp. FA =cl(int(FA))).

The set of all soft regular open (resp. soft regular closed) sets of (X, τ, E) is denoted by SRO(X) (resp. SRC(X)).

Definition 2.14. Let I be a non-null collection of soft sets over a universe X with the same set of parameters E. Then I ⊂SS(X) is called a soft ideal on X with the same set E if

(1) FA ∈I and GA ∈I ⇒FA ∪GA ∈I. (2) FA ∈I and GA ⊂FA ⇒GA ∈I.

Definition 2.15. Let (X, τ, E) be a soft topological space and I be a soft ideal over X with thesame set of parameters E. Then F∗ =∪{xe ∈X : Oxe

∩ FA ∈/ I, for all Oxe ∈τ } is called the soft localfunction of FA with respect to I and τ, where Oxe is a τ -open set containing xe.**Theorem 2.16**. Let I and J be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E). Let FA, GA ∈SS(X). Then

(1) (∅)∗ =∅.

(2) FA ⊂GA ⇒ F∗ ⊂G∗. A A

(3) I ⊂J ⇒ F∗ (J)⊂FA∗ (I).

(4) F∗ ⊂cl(FA), where cl is the soft closure w.r.t τ.(5) F∗ is τ -closed soft set.(6) (F∗)∗ ⊂ F∗. A A

(7) (FA ∪GA)∗ =F∗ ∪GA∗

Definition 2.17. Let (X, τ, E) be a soft topological space, I be a soft ideal over X with the same set of parameters E and cl∗ : SS(X) →SS(X) be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E, finer than τ, called the ⋆-soft topology, defined by τ ∗, given by τ ∗ ={ FA ∈SS(X) : cl∗ (X−FA)=X−FA}.

Definition 2.18. [7] Let FA be a soft subset of soft topological space (X, τ, E). Then

(1) xε is called a soft δ-cluster point of FA if FA ∩int(cl(UA))=∅ for every soft open set UA containing xε.

(2) The family of all soft δ-cluster point of FA is called the soft δ-closure of FA and is denoted by clδ (FA).

(3) A soft subset FA is said to be soft δ-closed if clδ (FA)=FA. The complement of a soft δ-closed set of X is said to be soft δ-open.

Lemma 2.19. [7] Let FA be a soft subset of soft topological space (X, τ, E). Then, the following properties hold:

(1) int(cl(FA)) is soft regular open,

(2) Every soft regular open set is soft δ-open,

(3) Every soft δ-open set is the union of a family of soft regular open sets.

(4) Every soft δ-open set is soft open.

Proposition 2.20. [7] Intersection of two soft regular open sets is soft regular open.

Lemma 2.21. [7] Let FA and GA be soft subsets of soft topological space (X, τ, E). Then, the following properties hold.

(1) FA ⊂clδ (FA),

(2) If FA ⊂GA, then clδ (FA)⊂clδ (GA),

(3) clδ (FA)=∩{GA ∈SS(X) : FA ⊂GA and GA is soft δ-closed},

(4) If (FA)α is a soft δ-closed set of X for each α∈∆, then ∩{(FA)α : α∈∆} is soft δ-closed,

(5) clδ (FA) is soft δ-closed.

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Theorem 2.22. [7] Let (X, τ, E) be a soft topological space and τδ ={FA ∈SS(X) : FA is a soft δ-open set}. Then τδ is a soft topology weaker than τ.

Definition 2.23. A soft subset FA of a soft topological space (X, τ, E) is said to be

(1) soft preopen if FA ⊂int(cl(FA)), (2) soft semiopen if FA ⊂cl(int(FA)), (3) soft α-open if FA ⊂int(cl(int(FA))).

The complement of a soft preopen (resp. soft semiopen, soft α-open) set is called a soft preclosed (resp. soft semiclosed, soft α -closed) set. The set of all soft preopen (resp. soft semiopen, soft α-open, soft preclosed, soft semiclosed, soft α-closed) sets of (X, τ, E) is denoted by SPO(X) (resp. SSO(X), SαO(X), SPC(X), SSC(X), SαC(X)).

intS (FA)={∪ UA : UA ⊂ FA and UA is soft semiopen sets} and clS (FA)={∩ GA : FA ⊂ GA and GA is soft semiclosed}.

Definition 2.24. A soft subset FA of a soft topological space (X, τ, E) is said to be

(1) soft regular-I -open if FA =int(cl∗ (FA)),

(2) soft pre-I -open if FA ⊂int(cl∗ (FA)), (3) soft semi-I -open if FA ⊂cl∗ (int(FA)), (4) soft α-I -open if FA ⊂int(cl∗ (int(FA))).

The complement of a soft regular-I -open (resp. soft pre-I -open, soft semi-I -open, soft α- I -open) set is called a soft regular-I -closed (resp. soft pre-I -closed, soft semi-I -closed, soft α-I -closed) set. The set of all soft regular-I -open (resp. soft pre-I -open, soft semi-I -open, soft α-I -open, soft regular-I -closed, soft pre-I -closed, soft semi-I -closed, soft α-I -closed) sets of (X, τ, E, I) is denoted by SRI O(X) (resp. SPI O(X), SSI O(X), SαI O(X), SRI C(X), SPI C(X), SSI C(X), SαI C(X)).

Definition 2.25. SIsint (FA)={∪ UA : UA ⊂ FA and UA is soft semi-I -open sets} and SIscl (FA)={∩ GA : FA ⊂ GA and GA is soft semi-I -closed sets}.

A soft subset FA is soft semi-I -closed if and only if SIscl (FA)=FA.

Definition 2.26. Let FA be a soft subset of a soft topological space (X, τ, E). The set ∩{UA ∈ τ : FA ⊂ UA } is called the kernel of FA and is denoted by ker(FA).

3 On Soft SIgδs-closed sets

Definition 3.1. A soft set FA over soft ideal topological space X is called soft generalized δ semi- closed (briefly soft SIgδs -closed) set if SIscl (FA) ⊂ GA whenever FA ⊂ GA and GA is soft δ-open over X.

A soft set FA over X is called soft generalized δ semi-open (briefly soft SIgδs -open) set if FA c is soft SIgδs-closed.

The family of all soft SIgδs -closed subsets over X is denoted by SIgδs -C(X) and soft SIgδs -open subsets over X is denoted by SIgδs -O(X).

Example 3.2. Let X = {a, b}, E={e1, e2 }, τ = {X˜, ∅, AE, BE, CE } and I = {∅, DE } where A(e1)=∅, A(e2)={a} (hereafter represent as (∅, {a})) B(e1)=∅, B(e2)={b} (hereafter represent as (∅, {b})) C(e1)=∅, C(e2)=X (hereafter represent as (∅, X)) then soft SIgδs -closed sets are ∅, X˜, (∅, {a}), (∅, {b}), (∅, X), ({a}, ∅), ({a}, {a}), ({a}, {b}), ({a}, X), {a}), ({b}, ∅), ({b}, {a}), ({b}, {b}), ({b}, X), (X, ∅), (X, {a}), (X, {b}).

Theorem 3.3. Every soft closed set is soft SIgδs -closed.

Proof: Let FA be a soft closed set over X. Let GA be a soft δ-open set such that FA ⊆ GA. Since FA is soft closed, that is cl(FA)=FA, cl(FA) ⊆ GA. But SIscl (FA) ⊆ cl(FA) ⊆ GA. Therefore SIscl (FA) ⊆ GA. Hence FA is soft SIgδs -closed over X.

Remark 3.4. (1) Every soft semi-I -closed (resp. soft pre-I -closed, soft α-I -closed) set is soft SIgδs -closed.

(2) Every soft δ-closed set is SIgδs -closed.

Example 3.5. From Example 3.2, the soft subset (∅, {a}) is Igδs -closed but not soft pre-I -closed, soft semi-I -closed, soft α-I -closed and soft δ-closed.

Definition 3.6. A soft point xe over X is said to be a soft limit point of soft set FA if and only if GA ∩ (FA − {xe }) = ∅ for every soft open set GA containing xe. The set of all soft limit points of FA is called soft derived set of FA and is denoted by D(FA).

Definition 3.7. A soft point xe over X is said to be a soft semi-I -limit point of soft set FA if and only if GA ∩ (FA − {xe }) = ∅ for every soft semi-I -open set GA containing xe. The set of all soft semi-I -limit points of FA is called soft semi-I -derived set of FA and is denoted by DS (FA).

Lemma 3.8. For any soft subset FA over X, if D(FA) ⊂ DS (FA), then cl(FA)=SIscl (FA).

Proof: For any soft subset FA over X, DS (FA) ⊂ D(FA) is always true. By hypothesis D(FA) ⊂ DS (FA). Therefore D(FA) = DS (FA). That is FA ∪ cl(FA)=FA ∪ SIscl (FA), which implies cl(FA) = SIscl (FA).

Theorem 3.9. If FA and GA are soft SIgδs -closed sets such that D(FA) ⊂ DS (FA) and D(GA) ⊂ DS (GA). Then FA ∪ GA is SIgδs -closed set over X.

Proof: Let FA and GA be soft SIgδs -closed subsets over X such that D(FA) ⊂ DS (FA) and D(GA) ⊂ DS (GA). Therefore by lemma 3.8, SIscl (FA) − cl(FA) and SIscl (GA) = cl(GA). Let HA be a soft δ-open set such that FA ∪ GA ⊂ HA, then FA ⊂ HA and GA ⊂ HA. Since FA and GA are soft SIgδs -closed sets, SIscl (FA) ⊂ HA and SIscl (GA) ⊂ HA. Since SIscl (FA ∪ GA) ⊂ cl(FA ∪ GA) = cl(FA) ∪ cl(GA) = S Iscl (FA) ∪ S Iscl (GA) ⊂ HA ∪ HA = HA. Thus, SIscl (FA ∪ GA) ⊂ HA. This shows that, FA ∪ GA is soft SIgδs-closed set over X.

Theorem 3.10. Let FA be a soft SIgδs -closed set over X. Then SIscl (FA) − FA does not contain any non empty soft δ-closed set.

Proof: Let FA be a soft SIgδs -closed set and HA be a soft δ-closed set over X such that HA ⊂ S Iscl (FA) − FA. This implies HA ⊂ S Iscl (FA) and HA ⊂ X − FA implies FA ⊂ X − HA. Now FA is soft SIgδs -closed set and X − HA is soft δ-open set containing FA. Therefore SIscl (FA) ⊂ X − HA. That is HA ⊂ X − S Iscl (FA). This implies HA ⊂ S Iscl (FA) ∩ (X − S Iscl (FA)) = ∅. This shows HA = ∅.

Theorem 3.11. A soft SIgδs -closed set FA is soft semi-I -closed if and only if SIscl (FA) − FA is soft δ-closed set.

Proof: Suppose a soft SIgδs -closed set FA is soft semi-I -closed. Then SIscl (FA) = FA. This implies SIscl (FA) − FA = ∅, which is soft δ-closed.

Conversely, SIscl (FA)−FA is soft δ-closed and FA is soft SIgδs -closed set. Now SIscl (FA)−FA is soft δ-closed subset of itself. Therefore by Theorem 3.10, SIscl (FA) − FA = ∅. That is SIscl (FA) = FA, implies FA is soft semi-I -closed.

Theorem 3.12. If FA is soft SIgδs -closed and FA ⊂ GA ⊂ S Iscl (FA), then GA is soft SIgδs -closed set.

Proof: Let GA ⊂ HA and HA is soft δ-open. Since FA ⊂ GA implies FA ⊂ HA and FA is soft SIgδs -closed, implies SIscl (FA) ⊂ HA. By hypothesis GA ⊂ S Iscl (FA), which implies SIscl (GA) ⊂ S Iscl (FA) ⊂ HA, that implies SIscl (GA) ⊂ HA. Therefore GA is soft SIgδs -closed set.

Theorem 3.13. If FA is soft δ-open and soft SIgδs -closed then FA is soft semi-I -closed and hence soft regular open.

Proof: Suppose FA is soft δ-open and soft SIgδs -closed. Since FA ⊂ FA, implies SIscl (FA) ⊂ FA. But FA ⊂ SIscl (FA) is always true. Therefore SIscl (FA) = FA. That is FA is soft semi-I -closed and hence regular open, because every soft δ-open set is soft open and every soft open and soft semi-I -closed set is soft regular open.

Theorem 3.14. For a soft ideal topological space X the following are equivalent

(1) Every soft δ-open set over X is soft semi-I -closed.

(2) Every soft subset over X is soft SIgδs -closed.

Proof: (1) ⇒ (2), Suppose (1) holds. Let FA be any soft subset over X and GA be a soft δ-open such that FA ⊂ GA. This implies SIscl (FA) ⊂ S Iscl (GA). By hypothesis, GA is soft semi-I -closed implies SIscl (GA) = GA. Hence SIscl (FA) ⊂ GA. Therefore FA is soft SIgδs -closed set over X.

(2) ⇒ (1), Suppose (2) holds and GA is soft δ-open set over X. By (2), GA is soft SIgδs -closed. Therefore SIscl (GA) ⊂ GA. But GA ⊂ S Iscl (GA) is always true. Therefore GA = S Iscl (GA) and hence GA is soft semi-I -closed.

Theorem 3.15. For any soft point xe over X, the set X − {xe } is soft SIgδs -closed set or soft δ-open.

Proof: Suppose X − {xe } is not soft δ-open. Then X˜ is the only soft δ-open set containingX − {xe }. This implies SIscl (X − {xe }) ⊂ X˜. Hence X − {xe } is soft SIgδs closed set.

Theorem 3.16. If a soft subset FA over X is soft SIgδs -closed set, then clδ ({xe }) ∩ FA = ∅ for each xe ∈ S Iscl (FA).

Proof: Suppose FA is a soft SIgδs -closed set and xe ∈ S Iscl (FA). If possible clδ ({xe}) ∩ FA = ∅. Then FA ⊂ X − clδ {xe } and X − clδ {xe } is soft δ-open set containing FA. Since FA is soft SIgδs -closed set, implies SIscl (FA) ⊂ X − clδ ({xe }), which is contradiction to xe ∈ S Iscl (FA). Therefore, clδ ({xe }) ∩ FA = ∅.

Theorem 3.17. If every soft semi-open set is soft semi-I -closed, then every soft subset over X is soft SIgδs -closed.

Proof: Suppose FA ⊂ HA where HA is soft δ-open over X. Since every soft δ-open set is soft semi-open, HA is soft semi-open. By hypothesis HA is soft semi-I -closed. Hence SIscl (FA) ⊂ HA.

Therefore FA is soft SIgδs -closed set. Since FA is arbitrary, every soft subset of X is soft SIgδs- closed set.

Theorem 3.18. Let FA be a soft SIgδs -closed set. Then FA = S Iscl (S Isint (FA)) if and only if SIscl (S Isint (FA)) − FA is soft δ-closed.

Proof: If FA = S Iscl (S Isint (FA)), then SIscl (S Isint (FA)) − FA = ∅, which is soft δ-closed. Hence SIscl (S Isint (FA)) − FA is soft δ-closed. Conversely, let SIscl (S Isint (FA)) − FA be soft δ- closed. Since SIscl (S Isint (FA)) − FA ⊂ S Iscl (FA) − FA, implies SIscl (FA) − FA contains the soft δ-closed set SIscl (S Isint (FA)) − FA. By Theorem 3.11, SIscl (S Isint (FA)) − FA = ∅, which implies SIscl (S Isint (FA)) = FA.

Theorem 3.19. A soft subset FA is soft SIgδs -closed set if and only if SIscl (FA) ⊂ ker(FA).

Proof: Suppose FA is soft SIgδs -closed set, then SIscl (FA) ⊂ GA whenever FA ⊂ GA and GA is soft δ-open. Let xe ∈ S Iscl (FA). If xe ∈/ ker(FA), then there exist a soft δ-open set GA containing FA such that xe ∈/ X˜. Since GA is soft δ-open set containing FA, implies xe ∈/ S Iscl (FA), which is a contradiction. Therefore xe ∈ ker(FA). Conversely, let SIscl (FA) ⊂ ker(FA). If GA is soft δ-open set containing FA, then ker(FA) ⊂ GA, which implies SIscl (FA) ⊂ GA. Therefore FA is soft SIgδs -closed set.

Theorem 3.20. A soft set FA is soft SIgδs -open if and only if HA ⊂ S Isint (FA), whenever HA is soft δ-closed and HA ⊂ FA.

Proof: Let FA be a soft SIgδs -open set. Suppose HA ⊂ FA, where HA is soft δ-closed. Then, X − FA is soft SIgδs -closed set contained in soft δ-open set X − HA. This implies SIscl (X − FA) ⊂ X − HA. Therefore X − S Isint (FA) ⊂ X − HA, which implies HA ⊂ S Isint (FA).

Conversely, suppose HA ⊂ S Isint (FA), whenever HA ⊂ FA and HA is soft δ-closed set. Then X − S Isint (FA) ⊂ X − HA, whenever X − FA ⊂ X − HA and X − HA is soft δ-open. This implies SIscl (X − FA) ⊂ X − HA whenever X − FA ⊂ X − HA and X − HA is soft δ-open. This shows that X − FA is soft SIgδs -closed over X, hence FA is soft SIgδs -open set over X.

Theorem 3.21. If FA is a soft SIgδs -open set over X, then GA = X˜ and soft SIsint (FA) ∪ (X − FA) ⊂ GA. Whenever GA is soft δ-open.

Proof: Let FA be a soft SIgδs -open set and GA be soft δ-open over X such that SIsint (FA) ∪ X − FA ⊂ GA. Then X − GA ⊂ X − (S Isint (FA) ∪ X − FA) ⊂ (X − S Isint (FA)) ∩ FA. That is X − GA ⊂ (S Iscl (X − FA) − (X − FA). Since X − FA is soft SIgδs -closed, by Theorem 3.11SIscl (X − FA) − (X − FA) does not contain any non empty soft δ-closed set, which implies X – GA = ∅. Hence GA =X˜.

Theorem 3.22. If SIsint (FA) ⊂ GA ⊂ FA and FA is soft SIgδs -open set, then GA is soft SIgδs - open.

Proof: Let FA be a soft SIgδs -open set and SIsint (FA) ⊂ GA ⊂ FA implies X − FA ⊂ X − GA ⊂ X − S Isint (FA), that is X − FA ⊂ X − GA ⊂ S Iscl (X − FA). Now X − FA is soft SIgδs -closed set. Hence by Theorem 3.12, X − GA is soft SIgδs -closed and hence GA is soft Igδs -open set.

Theorem 3.23. Let X be soft ideal topological space and FA, GA be soft subsets over X. If GA is soft SIgδs -open and if SIsint (GA) ⊂ FA, then FA ∩ GA is soft SIgδs -open.

Proof: Let FA and GA be arbitrary soft subsets overe X and GA is soft SIgδs -open such that SIsint (GA) ⊂ FA, This implies SIsint (GA) ∩ GA ⊂ FA ∩ GA ⊂ GA, since SIsint (GA) ⊂ GA, implies SIsint (GA) ⊂ FA ∩ GA ⊂ GA. Since GA is soft SIgδs-open, FA ∩ GA is soft SIgδs -open.

Theorem 3.24. Let SSO(X) be closed under finite intersections. If FA and GA are soft SIgδs - open, then FA ∩ GA is soft SIgδs -open.

Proof: Assume SSO(X) is closed under finite intersection. Let FA and GA be soft SIgδs -open sets such that X − (FA ∩ GA) = (X − FA) ∪ (X − GA) ⊂ OA, where OA is soft δ-open. Then X − FA ⊂ OA and X − GA ⊂ OA. Since FA and GA are soft SIgδs -open, X − FA and X − GA are soft SIgδs -closed sets. Therefore SIscl (X − FA) ⊂ OA and SIscl (X − GA) ⊂ OA. This implies SIscl (X − FA) ∪ S Iscl (X − GA) ⊂ OA. Now SIscl (X − (FA ∩ GA)) = S Iscl ((X − FA) ∪ (X − GA)) ⊂ S Iscl (X − FA) ∪ S Iscl (X − GA) ⊂ OA. This implies X − (FA ∩ GA) is soft SIgδs -closed and hence FA ∩ GA is soft SIgδs -open set.

Theorem 3.25. If FA is soft SIgδs -closed over X then SIscl (FA) − FA is soft SIgδs - open.

Proof: If FA is soft SIgδs -closed set and GA be soft δ-closed set such that GA ⊂ S Iscl (FA) − FA, This implies GA = ∅ by Theorem 3.10. Hence GA ⊂ S Isint (S Iscl (FA) − FA). Therefore SIscl (FA) − FA is soft SIgδs -open set.

Theorem 3.26. Let X be a soft ideal topological space and FA be soft set over X. Then SIscl (FA)− FA is soft SIgδs -closed if and only if FA ∪ (X − S Iscl (FA)) is soft SIgδs - open.

Proof: Let GA = S Iscl (FA) − FA. Then X − GA = (X − S Iscl (FA)) ∪ FA. Since GA is soft SIgδs -closed, X − GA is soft SIgδs -open and hence (X − S Iscl (FA)) ∪ FA is soft SIgδs -open.

Conversely, let HA = (X − S Iscl (FA)) ∪ FA be soft SIgδs -open. Hence X − HA = S Iscl (FA) − FA, which is soft SIgδs -closed, by hypothesis. This implies X − S Iscl (FA) is soft SIgδs -closed.

Theorem 3.27. Let X be a soft ideal topological space and FA be soft set over X. If SIscl (FA) – FA is soft SIgδs -closed, then FA = GA ∩ S Iscl (A), for some soft SIgδs - open set GA.

Proof: Given SIscl (FA) − FA is soft SIgδs -closed set. Therefore GA = X − (S Iscl (FA) − FA) is soft SIgδs -open. Then GA ∩ S Iscl (FA) = (X − (S Iscl (FA) − FA)) ∩ S Iscl (FA) = ((X − S Iscl (FA)) ∪ FA) ∩ S Iscl (FA) = ((X − S Iscl (FA)) ∩ S Iscl (FA)) ∪ (FA ∩ S Iscl (FA)) = ∅ ∪ FA = FA.

Definition 3.28. A soft subset FA over soft topological space X is called a soft neighbourhood (briefly, soft nhd) of a soft point xe over X, if there exists a soft open set

GA such that xe ∈ GA ⊂ FA. The collection of all soft nhd's of xe ∈ FA is called soft nhd system of xe and is denoted by N (xe).

Definition 3.29. A soft subset FA over soft ideal topological space X is called a soft SIgδs - neighbourhood (briefly, soft SIgδs -nhd) of a soft point xe over X, if there exists a soft SIgδs -open set GA such that xe ∈ GA ⊂ FA. The collection of all soft SIgδs -nhd's of xe ∈ FA is called soft SIgδs -nhd system of xe and is denoted by SIgδs -N (xe).

Remark 3.30. Every soft nhd of xe over X is a soft SIgδs-nhd of xe, because every soft open set is a soft SIgδs -open set. But converse need not be true as seen from following example.

Example 3.31. In example 3.2, it is clear that X˜, ({a}, ∅), ({a}, {a}), ({a}, b), ({a}, X), (X,∅), (X, {a}), (X, {b}) are Igδs -nhd of ({a}, ∅) but X˜
is only soft nhd of ({a}, ∅).

Lemma 3.32. An arbitrary union of soft SIgδs -nhd's of a soft point xe is again a soft SIgδs -nhd of that soft point.

Proof: Let {FAi : i ∈ I } be an arbitrary collection of soft SIgδs -nhd's of xe over X. Since for each i ∈ I, FAi is soft SIgδs -nhd of xe, there exists soft SIgδs -open set GAi such that xe ∈ GAi ⊂ FAi. But for each i ∈ I, FAi ⊂ ∪FAi, therefore xe ∈ GAi ⊂ ∪FAi, which implies ∪FAi is again soft SIgδs -nhd of xe.

Theorem 3.33. Let xe be any arbitrary soft point of a soft ideal topological space X. Then SIgδs -N (xe) satisfies the following properties.

(1) SIgδs -N (xe) = ∅.

(2) if NA ∈ SIgδs -N (xe) then xe ∈ NA.

(3) if NA ∈ SIgδs -N (xe) and NA ⊂ MA then MA ∈ SIgδs -N (xe).

Proof: (1) Since for each soft point xe over X, X˜ is a soft SIgδs -open set. Therefore xe ∈ X˜ ⊂X˜, implies X˜ is soft SIgδs -nhd of xe, hence X˜ ∈ S Igδs -N (xe). Therefore SIgδs -N (xe) = ∅.(2) Given NA ∈ S Igδs -N (xe), implies NA is a SIgδs -nhd of xe, which implies there exists a soft SIgδs -open set GA such that xe ∈ GA ⊂ NA. This implies, xe ∈ NA.

(3) Given NA ∈ S Igδs -N (xe) implies there exists soft SIgδs -open set GA such that xe ∈ GA ⊂ NA and NA ⊂ MA, which implies xe ∈ GA ⊂ MA. This shows that MA ∈ S Igδs -N (xe).

Theorem 3.34. Let FA be a soft subset over X. Then FA is soft SIgδs -open set if and only if FA is soft SIgδs -nhd of each of its soft points.

Proof: Let FA be any soft SIgδs -open set over X. Then for each soft point xe over X, xe ∈ FA ⊂ FA, implies FA is soft SIgδs -nhd of xe. Since xe is arbitrary soft point of FA, implies FA is soft SIg δs-nhd of each of its soft points. On the other hand, FA is soft SIgδs -nhd of each of its soft points, which implies for each xe ∈ FA, there exists a soft SIgδs -open set Gxe such that xe ∈ Gxe ⊂ FA...(l) Now claim that FA = ∪xe ∈FA Gxe. Suppose if xe ∈ FA, there exists at least one soft SIgδs-open set Gxe such that xe ∈ Gxe ⊂ ∪xe ∈FA Gxe. Therefore FA ⊂ ∪xe ∈FA Gxe....(2) Again if ye ∈ ∪xe ∈FA Gxe implies ye ∈ Gxe for some xe ∈ FA. From (1), ye ∈ FA. Therefore ∪xe ∈FA Gxe ⊂ FA.... (3). From (2) and (3) it follows that, FA = ∪xe ∈FA Gxe. Thus each Gxe is soft SIgδs -open set and arbitrary union of soft SIgδs -open sets is again soft SIgδs -open set. Therefore FA is soft SIgδs -open set.

Corollary 3.35. If FA is a soft SIgδs -closed subset over X and xe ∈ X − FA, then there exists a soft Igδs -nhd NA of xe such that NA ∩FA = ∅.

Proof: Given FA is soft SIgδs -closed set, implies X − FA is soft SIgδs -open set by Theorem

3.34, X − FA is soft SIgδs -nhd of each of its soft points. Let xe ∈ X − FA, implies there exists a soft SIgδs -open set NA such that xe ∈ NA ⊂ X − FA, which implies NA ∩ FA = ∅.

Definition 3.36. A soft point xe over X is said to be a soft SIgδs -limit point of soft set FA if and only if GA ∩ (FA − {xe }) = ∅ for every soft SIgδs -open set GA containing xe. The set of all soft SIgδs -limit points of FA is called soft SIgδs -derived set of FA and is denoted by SIgδs -D(FA).

Remark 3.37. Since every soft open set is soft SIgδs -open set, it follows from the definition 3.36 that, every soft SIgδs -limit point of FA is a limit point of FA. Therefore, SIgδs -D(FA) ⊂ D(FA) where D(FA) is derived set of FA.

Theorem 3.38. Let FA, GA be two soft sets of soft ideal topological space. Then the following properties hold.

(1) SIgδs-D(∅) = ∅.

(2) If FA ⊂ GA then SIgδs -D(FA) ⊂ S Igδs -D(GA).

(3) If xe ∈ S Igδs -D(FA) then xe ∈ S Igδs -D(FA − {xe }). (4) SIgδs-D(FA) ∪ S Igδs - D(GA) ⊂ S Igδs -D(FA ∪ GA). (5) SIgδs -D(FA ∩ GA) ⊂ S Igδs -D(FA) ∩ S Igδs - D(GA).

Proof: (1) Let xe be soft point over X and HA be a soft SIgδs -open set containing xe. Then

HA ∩ (∅ − {xe }) = ∅. Therefore for any soft point xe over X, xe is not a soft SIgδs-limit point of

∅. Hence SIgδs -D(∅) = ∅.

(2) Let xe ∈ S Igδs -D(FA). Then HA ∩ (FA − {xe }) = ∅, for every soft SIgδs -open set HA containing xe. Since FA ⊂ GA, implies HA ∩ (GA − {xe }) = ∅. This implies xe ∈ S Igδs -D(GA). Thus, xe ∈ S Igδs -D(FA) implies xe ∈ S Igδs -D(GA). Therefore, SIgδs - D(FA) ⊂ S Igδs -D(GA).

(3) Let xe ∈ S Igδs -D(FA). Then HA ∩ (FA − {xe }) = ∅, for every soft SIgδs -open set HA containing xe. This implies every soft SIgδs -open set HA containing xe, contains at least one soft point other than xe of FA − {xe }. Therefore xe ∈ S Igδs -D(FA − {xe }).

(4) Since FA ⊂ FA ∪ GA and GA ⊂ FA ∪ GA and by (2), SIgδs -D(FA) ⊂ S Igδs-D(FA ∪ GA) and

SIgδs -D(GA) ⊂ S Igδs -D(FA ∪ GA). Hence, SIgδs -D(FA) ∪ S Igδs -D(GA) ⊂ S Igδs -D(FA ∪ GA).

(5) Since FA ∩ GA ⊂ FA and FA ∩ GA ⊂ GA and by (2), soft SIgδs -D(FA ∩ GA) ⊂ S Igδs -D(FA) and SIgδs -D(FA ∩ GA) ⊂ S Igδs -D(GA). Therefore SIgδs -D(FA ∩ GA) ⊂ S Igδs -D(FA) ∩ S Igδs - D(GA).

Theorem 3.39. If FA is a soft subset of soft ideal topological space X, then FA ∪ S Igδs -D(FA) is soft SIgδs -closed set.

Proof: To prove FA ∪ S Igδs -D(FA) is soft SIgδs -closed set, it is sufficient to prove X − (FA ∪ S Igδs -D(FA)) is soft SIgδs -open. If X − (FA ∪ S Igδs -D(FA)) = ∅, then it is clearly soft SIgδs -open set. Let X − (FA ∪ S Igδs -D(FA)) = ∅ and xe ∈ X − (FA ∪ S Igδs -D(FA)), implies xe ∈/ FA ∪ S Igδs - D(FA). This implies xe ∈/ FA and xe ∈/ S Igδs - D(FA). Now xe ∈/ S Igδs -D(FA), implies xe is not soft SIgδs -limit point of FA. Therefore there exists a soft SIgδs -open set HA containing xe such that HA ∩ (FA − {xe }) = ∅. Since xe ∈/ FA, implies HA ∩ FA = ∅. This implies xe ∈ HA ⊂ X − FA...(l). Again HA is soft SIgδs -open set and HA ∩ FA = ∅ implies no soft point of HA can be soft SIgδs - limit point of FA. This follows HA ∩ S Igδs -D(FA) = ∅, implies xe ∈ HA ⊂ X − S Igδs - D(FA).... (2). This shows from (1) and (2), xe ∈ HA ⊂ (X − FA)∩ (X − S Igδs-D(FA)) = X − (FA ∪ S Igδs - D(FA)). That is xe ∈ HA ⊂ X − (FA ∪ S Igδs -D(FA)). This implies X − (FA ∪ S Igδs -D(FA)) is soft SIgδs -nhd of each of its soft points. By Theorem 3.34, X − (FA ∪ S Igδs -D(FA)) is soft SIgδs -open set and hence FA ∪ S Igδs -D(FA) is sotf SIgδs - closed set.

Theorem 3.40. Let X be a soft ideal topological space and FA be soft set over X. Then FA is soft SIgδs -closed set if and only if SIgδs -D(FA) ⊂ FA.

Proof: Suppose FA is soft SIgδs -closed set. If SIgδs -D(FA) = ∅, then the result is obvious. If SIgδs -D(FA) = ∅ then xe ∈ S Igδs-D(FA), implies HA ∩ (FA − {xe }) = ∅ for every soft SIgδs -open set HA containing xe. If xe ∈/

FA then xe ∈ X − FA. Since FA is soft SIgδs -closed setand X − FA is soft SIgδs -open set containing xe and not containing any other soft point of FA, which is contradiction to xe ∈ S Igδs -D(FA), therefore xe ∈ FA. Thus, xe ∈ S Igδs -D(FA) implies xe ∈ FA. Hence SIgδs -D(FA) ⊂ FA. On the other hand, SIgδs -D(FA) ⊂ FA. To prove FA is soft SIgδs -closed set; it is equivalent to prove X − FA is soft SIgδs -open set. Let xe ∈ X − FA implies xe ∈/ FA. Since SIgδs -D(FA) ⊂ FA, implies xe ∈/ S Igδs -D(FA), which implies there exists a soft SIgδs -open set HA containing xe such that HA ∩ (FA − {xe }) = ∅. That is HA ∩ FA = ∅ as xe ∈/ FA, implies, xe ∈ HA ⊂ X − FA. Therefore X − FA is soft SIgδs -nhd of xe. Since xe is arbitrary, X − FA is soft SIgδs -nhd of each of its points. By Theorem 3.34, X − FA is soft SIgδs -open set. Hence FA is soft SIgδs -closed set.

4. Soft SIgδs-closure and soft SIgδs -interior

Definition 4.1. Let X be a soft ideal topological space and FA be a soft subset over X. Then soft SIgδs -closure of FA denoted by SIgδs -cl(FA) and defined as the intersection of all soft SIgδs -closed sets over X containing FA.

Theorem 4.2. Let FA be any soft subset of soft ideal topological space X. Then

(1) SIgδs -cl(FA) is the smallest soft SIgδs -closed superset of FA.

(2) FA is soft SIgδs -closed if and only SIgδs -cl(FA) = FA.

(3) SIgδs -cl(FA) = FA ∪ S Igδs -D(FA).

Proof: (1) Let {GAi : i ∈ I } be the collection of all soft SIgδs -closed subsets over X containing FA. Therefore SIgδs -cl(FA) = ∩{GAi : i ∈ I }, by the definition of the SIgδs -cl(FA). Since the intersection of an arbitrary collection of soft SIgδs -closed sets is a soft SIgδs -closed set, implies

∩{GAi : i ∈ I } is soft SIgδs -closed set. Therefore SIgδs -cl(FA) is a soft SIgδs -closed set. Also since FA ⊂ GAi for each i ∈ I, implies FA ⊂ ∩{GAi : i ∈ I } = S Igδs -cl(FA). Thus SIgδs -cl(FA) is soft SIgδs -closed set containing FA. Also since SIgδs -cl(FA) = ∩{GAi : i ∈ I }, implies SIgδs - cl(FA) ⊂ GAi for each i ∈ I. Consequently, SIgδs -cl(FA) is the smallest soft SIgδs -closed superset of FA.

(2) If FA is soft SIgδs -closed set, then obviously it is the smallest soft SIgδs -closed superset of FA, therefore it must coincide with SIgδs -cl(FA). Hence FA is soft SIgδs -closed, implies SIgδs - cl(FA) = FA. Again if SIgδs -cl(FA) = FA, then SIgδs -cl(FA) is soft SIgδs -closed set, so FA is soft SIgδs -closed set. Hence FA is soft SIgδs -closed if and only SIgδs - cl(FA) = FA.

(3) By Theorem 3.39, FA ∪ S Igδs -D(FA) is a soft SIgδs -closed set. Also FA ⊂ FA ∪ S Igδs - D(FA), implies FA ∪ S Igδs -D(FA) is a soft SIgδs -closed set containing FA. Therefore SIgδs - cl(FA) ⊂ FA ∪ S Igδs -D(FA)......(1). Again FA ⊂ S Igδs -cl(FA), implies SIgδs -D(FA) ⊂ S Igδs - D(S Igδs -cl(FA)) ⊂ S Igδs -cl(FA), because SIgδs -cl(FA) is soft SIgδs -closed set. Hence FA ∪ S Igδs - D(FA) ⊂ S Igδs -cl(FA)......(2). From (1) and (2), SIgδs -cl(FA) = FA ∪ S Igδs -D(FA).

Remark 4.3. From the Theorem 4.2 it is clear that FA ⊂ S Igδs -cl(FA) and also since every soft closed set is soft SIgδs -closed, implies FA ⊂ S Igδs -cl(FA) ⊂ cl(FA). But the equality does not holds as seen from the following example.

Example 4.4. In Example 3.2, let FA =(∅, {a}), it is clear that cl(FA)=(X, {a})⊄SIgδs -cl(FA)=(∅,

{a}).

Theorem 4.5. For any soft subsets FA and GA of a space X the following properties hold(1) SIgδs -cl(∅) = ∅, SIgδs -cl(X˜) = X˜

and SIgδs -cl(SIgδs -cl(FA)) = SIgδs -cl(FA).(2) If FA ⊂ GA, then SIgδs -cl(FA) ⊂ S Igδs -cl(GA).

(3) SIgδs -cl(FA) ∪ S Igδs -cl(GA) ⊂ S Igδs -cl(FA ∪ GA).

(4) SIgδs -cl(FA ∩ GA) ⊂ S Igδs -cl(FA) ∩ S Igδs -cl(GA).Proof: (1) Since each one of the sets ∅, X˜ and SIgδs -cl(FA) being soft SIgδs -closed, implies byTheorem 4.2, SIgδs -cl(∅) = ∅, SIgδs -cl(X˜) = X˜

and SIgδs -cl(SIgδs -cl(FA)) = SIgδs -cl(FA).(2) Let FA ⊂ GA then FA ⊂ GA ⊂ S Igδs -cl(GA). This implies SIgδs -cl(GA) is soft SIgδs-closed uperset of FA. But SIgδs -cl(FA) is the smallest soft SIgδs -closed superset of FA. Therefore, SIgδs -cl(FA) ⊂ S Igδs -cl(GA).

(3) Since FA ⊂ FA ∪ GA and GA ⊂ FA ∪ GA. From (2), SIgδs -cl(FA) ⊂ S Igδs -cl(FA ∪ GA) and

SIgδs -cl(GA) ⊂ S Igδs -cl(FA ∪ GA). Therefore, SIgδs -cl(FA) ∪ S Igδs -cl(GA) ⊂ S Igδs -cl(FA ∪ GA). (4) Since FA ∩ GA ⊂ FA and FA ∩ GA ⊂ GA. From (2), SIgδs -cl(FA ∩ GA) ⊂ S Igδs -cl(FA) and

SIgδs -cl(FA ∩ GA) ⊂ S Igδs -cl(GA). Therefore, SIgδs-cl(FA ∩ GA) ⊂ S Igδs -cl(FA) ∩ S Igδs -cl(GA).

Theorem 4.6. Let FA be a soft subset of a space X. Then xe ∈ S Igδs -cl(FA) if and only if

HA ∩ FA = ∅ for every soft SIgδs -open set HA containing xe.

Proof: Let xe ∈ S Igδs -cl(FA). Suppose there exists soft SIgδs -open set HA containing xe such that FA ∩ HA = ∅. Then FA ⊂ X − HA. Now X − HA is soft SIgδs -closed set containing FA implies SIgδs -cl(FA) ⊂ X − HA and xe ∈/ X − HA implies xe ∈/ S Igδs -cl(FA). This is contradiction to hypothesis. Hence FA ∩ HA = ∅.

Conversely, let HA ∩ FA = ∅ for every soft SIgδs -open set HA containing xe. Suppose xe ∈/ S Igδs -cl(FA). There exists a soft SIgδs -closed set OA containing FA such that xe ∈/

OA. This impliesFA ∩ (X − OA) = ∅ and X − OA is soft SIgδs -open set containing xe. This is contradiction to the hypothesis. Therefore xe ∈ S Igδs-cl(FA).

Definition 4.7. A soft point xe over X is called soft SIgδs -interior of FA if there exists a soft SIgδs -open set HA over X such that xe ∈ HA ⊂ FA.

In other words, xe is soft SIgδs -interior point of FA if FA is soft SIgδs -nhd of xe. The set of all soft SIgδs -interior points of FA is denoted by SIgδs -int(FA).

Remark 4.8. Since every soft open set is soft SIgδs -open set, implies every interior point of FA is soft SIgδs -interior point of FA. Therefore, int(FA) ⊂ S Igδs-int(FA) for any soft set FA over X. But the equality does not holds as seen from the following example.

Example 4.9. In the example 3.2, Let FA =({a}, ∅) it is clear that int(FA) is ∅ but SIgδs -int(FA) is ({a}, ∅) is not a soft subset of ∅.

Theorem 4.10. Let FA be any soft subset of a soft ideal topological space X. Then SIgδs -int(FA) is the union of all soft SIgδs-open subsets over X.

Proof: Let HA be the collection of all soft SIgδs -open subsets over X. To prove that SIgδs - int(FA) = ∪{HA ∈ S Igδs -O(X) : HA ⊂ FA }; If xe ∈ S Igδs -int(FA), then xe is soft SIgδs - interior point of FA, so there exists a soft SIgδs -open subset HA over X containing xe such that xe ∈ HA ⊂ FA. Consequently, xe ∈ ∪{HA ∈ S Igδs -O(X) : HA ⊂ FA }. This shows that, SIgδs -int(FA) ⊂ ∪{HA ∈ S Igδs-O(X) : HA ⊂ FA }....(1) Again if xe ∈ ∪{HA ∈ S Igδs - O(X) : HA ⊂ FA }, then xe is contained in some soft SIgδs -open subset HA of FA, that is xe ∈ HA ⊂ FA. Therefore xe is soft SIgδs -interior point of FA and so xe ∈ S Igδs -int(FA). Thus, ∪{HA ∈ S Igδs -O(X) : HA ⊂ FA } ⊂ S Igδs -int(FA)..... (2). Hence from (1) and (2), SIgδs -int(FA) = ∪{HA ∈ S Igδs -O(X) : HA ⊂ FA}.

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Theorem 4.11. Let X be a soft ideal topological space and FA and GA be soft subsets over X. Then the following properties hold.

(1) SIgδs -int(FA) is a soft SIgδs -open set.

(2) SIgδs -int(FA) is largest soft SIgδs -open set contained in FA.

(3) FA is soft SIgδs -open if and only if FA = S Igδs -int(FA).

(4) SIgδs-int(∅) = ∅ and SIgδs -int(X˜) = X˜.

(5) If FA ⊂ GA then SIgδs -int(FA) ⊂ S Igδs -int(GA).

(6) SIgδs-int(FA) ∪ S Igδs -int(GA) ⊂ S Igδs -int(FA ∪ GA). (7) SIgδs -int(FA ∩ GA) ⊂ S Igδs -int(FA) ∩ S Igδs -int(GA). (8) SIgδs -int(S Igδs -int(FA)) = S Igδs -int(FA).

Proof: (1) Since union of soft SIgδs -open sets is again soft SIgδs -open set and SIgδs -int(FA) is union of soft SIgδs -open sets contained in FA. Therefore SIgδs -int(FA) is soft SIgδs -open set.

(2) Let HA be any soft SIgδs -open subset of FA and if xe ∈ HA, then xe ∈ HA ⊂ FA. Since HA being soft SIgδs -open set, implies FA is soft SIgδs -nhd of xe. Therefore xe is soft SIgδs -interior point of FA. Thus xe ∈ HA implies xe ∈ S Igδs -int(FA). This implies every soft SIgδs -open subset of FA is contained in SIgδs -int(FA). Therefore SIgδs -int(FA) is the largest soft SIgδs -open set contained in FA.

(3) Let FA be a soft SIgδs -open set. Since FA ⊂ FA, implies FA is identical with largest soft SIgδs-open subset of FA. By (2), SIgδs -int(FA) is the largest soft SIgδs -open subset of FA. Therefore FA = S Igδs -int(FA).(4) Since X˜

and ∅ are soft SIgδs -open sets, by (3) SIgδs -int(X˜) = X˜

and SIgδs -int(∅) = ∅.5) Let FA ⊂ GA and xe ∈ S Igδs -int(FA), implies there exists a soft SIgδs -open set HA such that xe ∈ HA ⊂ FA, which implies xe ∈ HA ⊂ FA ⊂ GA. That is, xe ∈ HA ⊂ GA. Therefore xe is soft SIgδs -interior of GA. That is xe ∈ S Igδs -int(GA). Thus xe ∈ S Igδs -int(FA) implies xe ∈ S Igδs -int(GA). Therefore SIgδs -int(FA) ⊂ S Igδs -int(GA).

(6) Since FA ⊂ FA ∪GA and GA ⊂ FA ∪GA, then by (5), SIgδs -int(FA) ⊂ S Igδs -int(FA ∪GA) and SIgδs -int(GA) ⊂ S Igδs-int(FA ∪ GA), implies SIgδs -int(FA) ∪ S Igδs -int(GA) ⊂ S Igδs -int(FA ∪ GA). (7) Since FA ∩ GA ⊂ FA and FA ∩ GA ⊂ GA, then from (5), SIgδs -int(FA ∩ GA) ⊂ S Igδs -int(FA) and SIgδs -int(FA ∩ GA) ⊂ S Igδs -int(GA). Thus, SIgδs -int(FA ∩ GA) ⊂ S Igδs -int(FA) ∩ S Igδs -

int(GA).

(8) By (3), FA is soft SIgδs -open if and only if FA = S Igδs -int(FA) and by (1), SIgδs -int(FA) is soft SIgδs -open set. Therefore, SIgδs -int(S Igδs -int(FA)) = S Igδs -int(FA).

Theorem 4.12. If FA is a soft subset over soft ideal topological space X, then

(1) SIgδs -cl(X − FA) = X − S Igδs -int(FA). (2) SIgδs -int(X − FA) = X − S Igδs -cl(FA). (3) SIgδs -int(FA) = X − S Igδs -cl(X − FA). (4) SIgδs -cl(FA) = X − S Igδs -int(X − FA).

Proof: (1) Consider, SIgδs -cl(X − FA) = ∩{GAi : GAi is soft SIgδs -closed set over X and X − FA ⊂ GAi } = ∩{GAi : X − GAi is soft SIgδs -open subset over X and X − GAi ⊂ FA } = X − ∪i (X − GAi), where X − GAi is soft SIgδs -open set over X and X − GAi ⊂ FA, therefore X − (union of all soft SIgδs -open sets contained in FA) = X − S Igδs -int(FA).

(2) Consider, X − S Igδs -cl(FA) = X − ∩{GAi : GAi is soft SIgδs -closed subset over X and FA ⊂ GAi } = ∪{X − GAi : X − GAi is soft SIgδs -open subset over X and X − GAi ⊂ X − FA } = union of all soft SIgδs -open sets contained in X − FA = S Igδs -int(X − FA).

(3) Obtained by replacing FA by X − FA in result (2). (4) Obtained by replacing FA by X − FA in result (1).

Theorem 4.13. If FA is soft subset of soft ideal topological space X, then SIgδs -int(FA) equals to the set of all those soft points FA which are not soft SIgδs -limit points of (X − FA). That is, SIgδs -int(FA) = FA − S Igδs -D(X − FA).

Proof: Let xe ∈ FA − S Igδs -D(X − FA), implies xe ∈ FA and xe ∈/ S Igδs -D(X − FA). This implies xe is not soft SIgδs -limit point of (X − FA), therefore there exists a soft SIgδs -open set HA containing xe but not contains the soft points of (X − FA). That is HA ∩ (X − FA) = ∅. This implies HA ⊂ FA. Thus xe ∈ HA ⊂ FA implies xe ∈ S Igδs -int(FA). Therefore FA − S Igδs - D(X − FA) ⊂ S Igδs -int(FA)...(1). On the other hand, if xe ∈ S Igδs -int(FA) then xe ∈ FA as SIgδs -int(FA) ⊂ FA and also SIgδs -int(FA) is soft SIgδs -open set containing xe and not containing any other points of (X − FA), implies xe is not soft SIgδs-limit point of (X − FA). Since xe is arbitrary implies, every soft point of SIgδs -int(FA) is soft limit point of FA but not a soft limit point of (X − FA). This shows that xe ∈ FA − S Igδs -D(X − FA). Therefore SIgδs -int(FA) ⊂ FA − S Igδs - D(X − FA)...(2). From (1) and (2) SIgδs -int(FA) = FA − S Igδs -D(X − FA). Hence SIgδs -int(FA) equals to the set of all those soft points FA which are not soft SIgδs -limit points of (X − FA).

Theorem 4.14. For the soft subsets FA and GA of soft ideal topological space X, following statements are true

(1) SIgδs -int(X − FA) ⊂ X − S Igδs -int(FA).

(2) SIgδs -int(FA − GA) ⊂ S Igδs -int(FA) − S Igδs-int(GA).

Proof: (1) Let xe ∈ S Igδs -int(X −FA). Since SIgδs -int(X −FA) ⊂ X −FA, implies xe ∈/ FA and hence xe ∈/ S Igδs -int(FA). This implies xe ∈ X − S Igδs -int(FA). Therefore, SIgδs -int(X − FA) ⊂ X − S Igδs -int(FA).

(2) SIgδs-int(FA −GA) = S Igδs -int(FA ∩X −GA)) ⊂ S Igδs -int(FA)∩S Igδs -int(X −GA) ⊂ S Igδs -
int(FA) ∩ (X − S Igδs -int(GA)) = S Igδs -int(FA) − S Igδs -int(G).

Definition 4.15. A soft point xe over X is called exterior point of FA if xe is interior of (X − FA). The set of exterior points of FA is denoted by Ext(FA). That is, Ext(FA) = int(X − FA).

Definition 4.16. A soft point xe over X is called soft SIgδs -exterior point of FA if xe is soft SIgδs - interior of (X − FA). The set of soft SIgδs -exterior points of FA is denoted by SIgδs -E xt(FA). That is, SIgδs -E xt(FA) = S Igδs -int(X − FA).

Theorem 4.17. For soft subsets FA and GA of a soft ideal topological space X the following statements hold.

(1) E xt(FA) ⊂ S Igδs-E xt(FA).

(2) SIgδs -E xt(FA) is soft SIgδs -open set.

(3) SIgδs -E xt(X˜) = ∅ and SIgδs -E xt(∅) = X˜.

(4) SIgδs -E xt(FA) ⊂ X − FA.

(5) SIgδs -E xt(X − S Igδs -E xt(FA)) = S Igδs -E xt(FA). (6) If FA ⊂ GA then SIgδs -E xt(GA) ⊂ S Igδs -E xt(FA). (7) SIgδs -E xt(FA) = X − S Igδs -cl(FA).

(8) SIgδs -E xt(FA ∪ GA) ⊂ S Igδs -E xt(FA) ∪ S Igδs -E xt(GA). (9) SIgδs -E xt(FA) ∩ S Igδs -E xt(GA) ⊂ S Igδs -E xt(FA ∩ GA). (10) SIgδs -Ext(SIgδs -Ext(FA)) = SIgδs -int(SIgδs -cl(FA)).

(11) SIgδs -int(FA) ⊂ S Igδs -E xt(S Igδs -E xt(FA)).

(12) FA ∩ S Igδs -E xt(FA) = ∅.

Proof: (1) For any soft subset GA over X, int(GA) ⊂ S Igδs -int(GA). Put GA = X − FA, then
int(X − FA) ⊂ S Igδs -int(X − FA). This implies, E xt(FA) ⊂ S Igδs -E xt(FA).

(2) By definition, SIgδs -E xt(FA) = S Igδs -int(X − FA) and SIgδs -int(X − FA) is soft SIgδs -open set. Therefore, SIgδs -E xt(FA) is soft SIgδs -open set.

(3) By definition, SIgδs -E xt(X˜) = S Igδs -int(X − X˜) = S Igδs -int(∅) = ∅ and SIgδs -E xt(∅) =
S Igδs -int(X − ∅) = Igδs -int(X˜) = X˜.

(4) By definition, SIgδs -E xt(FA) = S Igδs -int(X − FA) ⊂ X − FA.

(5) Consider SIgδs -Ext(X − S Igδs -Ext(FA)) = SIgδs -int(X − (X − S Igδs -Ext(FA)) = SIgδs - int(SIgδs -Ext(FA)) = SIgδs -int(SIgδs -int(X − FA)) = SIgδs -int(X − FA) = SIgδs - Ext(FA).

(6) If FA ⊂ GA then X − GA ⊂ X − FA. This implies SIgδs -int(X − GA) ⊂ S Igδs - int(X − FA). Therefore, SIgδs -E xt(GA) ⊂ S Igδs -E xt(FA).

(7) By definition, SIgδs -E xt(FA) = S Igδs -int(X − FA) = X − S Igδs -cl(FA).

(8) Since, FA ⊂ FA ∪ GA and GA ⊂ FA ∪ GA. By (6), SIgδs -E xt(FA ∪ GA) ⊂ S Igδs -E xt(FA) and SIgδs -E xt(FA ∪ GA) ⊂ S Igδs -E xt(GA). Therefore, SIgδs -E xt(FA ∪ GA) ⊂ S Igδs -E xt(FA) ∪ S Igδs - E xt(GA).

(9) Since, FA ∩ GA ⊂ FA and FA ∩ GA ⊂ GA. By (6), SIgδs -E xt(FA) ⊂ S Igδs -E xt(FA ∩ GA) and SIgδs -E xt(GA) ⊂ S Igδs -E xt(FA ∩ GA). Hence SIgδs -E xt(FA) ∩ S Igδs -E xt(GA) ⊂ S Igδs - E xt(FA ∩ GA).

(10) Consider, SIgδs -E xt(S Igδs -E xt(FA)) = S Igδs -E xt(S Igδs -int(X − FA)) = S Igδs -E xt(X −
S Igδs -cl(FA)) = S Igδs -int(X − (X − S Igδs -cl(FA))) = S Igδs -int(S Igδs -cl(FA)).

(11) Since FA ⊂ S Igδs -cl(FA), implies SIgδs -int(FA) ⊂ S Igδs -int(S Igδs -cl(FA)) = S Igδs -int(X − S Igδs -int(X − FA)) = S Igδs -E xt(S Igδs -int(X − FA)) = S Igδs -E xt(S Igδs -E xt(FA)). Thus, SIgδs - int(FA) ⊂ S Igδs -E xt(S Igδs -E xt(FA)).

(12) FA ∩ S Igδs -E xt(FA) = FA ∩ S Igδs -int(X − FA) ⊂ FA ∩ (X − FA) = ∅. Therefore FA ∩ S Igδs -
E xt(FA) = ∅.

Definition 4.18. For any soft subset FA of soft ideal space X, FA − int(FA) is defined as soft border of FA and is denoted by nbd(FA).

Definition 4.19. For any soft subset FA of soft ideal space X, FA − S Igδs -int(FA) is defined as soft SIgδs -border of FA and is denoted by SIgδs -nbd(FA).

Theorem 4.20. For any soft set FA over X, the following statements hold

(1) SIgδs -nbd(FA) ⊂ nbd(FA).
(2) SIgδs -int(FA) ∩ S Igδs -nbd(FA) = ∅.
(3) FA is soft SIgδs -open if and only if SIgδs -nbd(FA) = ∅.
(4) SIgδs -int(S Igδs -nbd(FA)) = ∅.
(5) SIgδs -nbd(S Igδs -int(FA)) = ∅.
(6) SIgδs -nbd(S Igδs -nbd(FA)) = S Igδs -nbd(FA).
(7) SIgδs -nbd(FA) = FA − S Igδs -int(FA) = FA ∩ S Igδs -cl(X − FA).
(8) If FA ⊂ GA then SIgδs -nbd(GA) ⊂ S Igδs -nbd(FA).
(9) SIgδs -nbd(FA ∪ GA) ⊂ S Igδs -nbd(FA) ∪ S Igδs -nbd(GA).
(10) SIgδs -nbd(FA) ∩ S Igδs -nbd(GA) ⊂ S Igδs -nbd(FA ∩ GA).
(11) SIgδs -nbd(FA) = S Igδs -D(X − FA) and SIgδs -D(FA) = S Igδs -nbd(X − FA).
(12) FA = S Igδs -int(FA) ∪ S Igδs -nbd(FA).

Proof: (1) Since int(FA) ⊂ S Igδs -int(FA) ⇒ X − S Igδs -int(FA) ⊂ X − int(FA) ⇒ FA ∩ (X − S Igδs -int(FA)) ⊂ FA ∩ (X − int(FA)) ⇒ FA − S Igδs -int(FA) ⊂ FA − int(FA). Therefore, SIgδs -nbd(FA) ⊂ nbd(FA).

(2) Consider SIgδs -int(FA) ∩ S Igδs -nbd(FA) = S Igδs -int(FA) ∩ (FA − S Igδs - int(FA)) = Igδs - int(FA) ∩ (FA ∩ (X − S Igδs -int(FA))) = S Igδs -int(FA) ∩ (X − S Igδs - int(FA)) ∩ FA = ∅ ∩ FA = ∅. (3) Any soft subset FA over soft ideal topological space X is soft SIgδs -open if and only if
FA = S Igδs -int(FA) ⇔ FA − S Igδs -int(FA) = ∅ ⇔ S Igδs -nbd(FA) = ∅.

(4) Consider, SIgδs -int(S Igδs -nbd(FA)) = S Igδs -int(FA − S Igδs -int(FA)) = S Igδs - int(FA ∩ (X − S Igδs -int(FA))) ⊂ S Igδs -int(FA) ∩ S Igδs-int(X − S Igδs -int(FA)) ⊂ S Igδs -int(FA) ∩ (X − S Igδs - int(FA)) = ∅, as SIgδs -int(FA) ⊂ FA. Therefore, SIgδs -int(S Igδs -nbd(FA)) = ∅.

(5) Consider, SIgδs -nbd(S Igδs -int(FA)) = S Igδs -int(FA) − S Igδs -int(S Igδs -int(FA)) = S Igδs -
int(FA) − S Igδs -int(FA) = ∅.

(6) Consider SIgδs -nbd(S Igδs -nbd(FA)) = S Igδs -nbd(FA) − S Igδs -int(S Igδs -nbd(FA)) = S Igδs −
nbd(FA) (because by (4)) SIgδs -int(S Igδs -nbd(FA)) = ∅.

(7) SIgδs -nbd(FA) = FA − S Igδs -int(FA) = FA ∩ (X − S Igδs -int(FA)) = FA ∩ S Igδs -cl(X − FA). (8) If FA ⊂ GA then SIgδs -int(FA) ⊂ S Igδs -int(GA) ⇒ X − S Igδs -int(GA) ⊂ X − S Igδs - int(FA) ⇒ FA ∩ (X − S Igδs -int(GA)) ⊂ FA ∩ (X − S Igδs -int(FA)) ⇒ FA − S Igδs -int(GA) ⊂
FA − S Igδs -int(FA) ⇒ S Igδs -nbd(GA) ⊂ S Igδs -nbd(FA).

(9) Since, FA ⊂ FA ∪ GA and GA ⊂ FA ∪ GA. By (8), SIgδs -nbd(FA ∪ GA) ⊂ S Igδs -nbd(FA) and SIgδs -nbd(FA ∪ GA) ⊂ S Igδs -nbd(GA). Therefore, SIgδs-nbd(FA ∪ GA) ⊂ S Igδs-nbd(FA) ∪ S Igδs - nbd(GA).

(10) Since FA ∩ GA ⊂ FA and FA ∩ GA ⊂ GA. By (8), SIgδs -nbd(FA) ⊂ S Igδs -nbd(FA ∩ GA) and SIgδs -nbd(GA) ⊂ S Igδs -nbd(FA ∩ GA). Therefore, SIgδs -nbd(FA) ∩ S Igδs -nbd(GA) ⊂ S Igδs - nbd(FA ∩ GA).

(11) SIgδs -nbd(FA) = FA − S Igδs -int(FA) = FA − (FA − S Igδs -D(X − FA)) = S Igδs -D(X − FA)
and SIgδs -nbd(X − FA) = S Igδs -D(FA) is obtained by replacing FA by X − FA.

(12) SIgδs -int(FA) ∪ S Igδs -nbd(FA) = S Igδs -int(FA) ∪ (FA − S Igδs -int(FA)) = S Igδs -int(FA) ∪ (FA ∩ (X − S Igδs -int(FA))) = (Igδs -int(FA) ∪ FA) ∩ (S Igδs -int(FA) ∪ (X − S Igδs -int(FA))) = FA ∩ X = FA. Therefore, FA = S Igδs-int(FA) ∪ S Igδs -nbd(FA).

Definition 4.21. For any soft subset FA over soft ideal topological space X, cl(FA) − int(FA) is defined as soft frontier of soft set FA and is denoted by Fr(FA).

Definition 4.22. For any soft subset FA over soft ideal topological space X, SIgδs -cl(FA) − S Igδs -
int(FA) is defined as soft SIgδs -frontier of set FA and is denoted by SIgδs -F r(FA).

Theorem 4.23. For a soft subset FA over soft ideal topological space X the following results hold

(1) SIgδs-F r(FA) ⊂ F r(FA).

(2) SIgδs -nbd(FA) ⊂ S Igδs -F r(FA).

(3) SIgδs -cl(FA) = S Igδs -int(FA) ∪ S Igδs -F r(FA).

(4) SIgδs -int(FA) ∩ S Igδs -F r(FA) = ∅.

(5) SIgδs -F r(FA) = S Igδs -nbd(FA) ∪ S Igδs -D(FA).

(6) FA is soft SIgδs -open if and only if SIgδs -F r(FA) = S Igδs -D(FA).

(7) SIgδs -F r(FA) = S Igδs -cl(FA) ∩ S Igδs -cl(X − FA).

(8) SIgδs -F r(FA) = S Igδs -F r(X − FA). (9) SIgδs -F r(FA) is soft SIgδs -closed set. (10) SIgδs -int(FA) = FA − S Igδs -F r(FA).

(11) SIgδs-F r(FA) = ∅ if and only if FA is soft SIgδs -open as well as soft SIgδs -closed.

(12) SIgδs -F r(S Igδs -int(FA)) ⊂ S Igδs -F r(FA).

(13) SIgδs -int(FA) ∪ S Igδs -int(X − FA) = X − S Igδs -F r(FA).

(14) SIgδs-F r(S Igδs -cl(FA)) ⊂ S Igδs -F r(FA).

(15) SIgδs -cl(FA) = FA ∪ S Igδs -F r(FA).

Proof: (1) Since int(FA) ⊂ S Igδs -int(FA), implies X − S Igδs -int(FA) ⊂ X − int(FA). Also SIgδs -cl(FA) ⊂ cl(FA). Therefore, SIgδs -cl(FA) ∩ (X − S Igδs -int(FA)) ⊂ cl(FA) ∩ (X − int(FA)). This implies SIgδs -cl(FA) − S Igδs -int(FA) ⊂ cl(FA) − int(FA). Hence SIgδs -F r(FA) ⊂ F r(FA).

(2) Since $FA \subset S\ Ig\delta s\text{-cl}(FA)$, implies $FA \cap (X - S\ Ig\delta s\text{-int}(FA)) \subset S\ Ig\delta s\text{-cl}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA))$. This implies $FA - S\ Ig\delta s\text{-int}(FA) \subset S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA)$. This shows, $SIg\delta s\text{-nbd}(FA) \subset S\ Ig\delta s\text{-}Fr(FA)$.

(3) $SIg\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-int}(FA) \cup (S\ Ig\delta s\text{-cl}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA))) =$

$(S\ Ig\delta s\text{-int}(FA) \cup (S\ Ig\delta s\text{-cl}(FA))) \cap (S\ Ig\delta s\text{-int}(FA) \cup (X - S\ Ig\delta s\text{-int}(FA))) = SIg\delta s\text{-cl}(FA) \cap \tilde{X} =$

$SIg\delta s\text{-cl}(FA)$.

(4) $SIg\delta s\text{-int}(FA) \cap S\ Ig\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-int}(FA) \cap (S\ Ig\delta s\text{-cl}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA))) =$

$S\ Ig\delta s\text{-cl}(FA) \cap (S\ Ig\delta s\text{-int}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA))) = S\ Ig\delta s\text{-cl}(FA) \cap \emptyset = \emptyset$.

(5) From (3), $SIg\delta s\text{-cl}(FA) = SIg\delta s\text{-int}(FA) \cup SIg\delta s\text{-}Fr(FA)$. This implies, $FA \cup S\ Ig\delta s\text{-}D(FA) =$

$S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}Fr(FA)$. But $FA = S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-nbd}(FA)$... (by(12) of Theorem

4.20. Therefore $SIg\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-nbd}(FA) \cup S\ Ig\delta s\text{-}D(FA) = S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}Fr(FA)$. Hence, $SIg\delta s\text{-nbd}(FA) \cup S\ Ig\delta s\text{-}D(FA) = S\ Ig\delta s\text{-}Fr(FA)$.

(6) Suppose FA is soft $SIg\delta s$-open, by Theorem 4.20 $SIg\delta s\text{-nbd}(FA) = \emptyset$. From (5), $SIg\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-nbd}(FA) \cup S\ Ig\delta s\text{-}D(FA) = S\ Ig\delta s\text{-}D(FA)$. Therefore if FA is soft $SIg\delta s$-open, $SIg\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-}D(FA)$.

On the other hand, suppose $SIg\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-}D(FA)$. From (3), $SIg\delta s\text{-cl}(FA) = S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}Fr(FA)$. That is, $FA \cup S\ Ig\delta s\text{-}D(FA) = S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}Fr(FA)$. That is $FA \cup S\ Ig\delta s\text{-}D(FA) = S\ Ig\delta s\text{-int}(FA) \cup S\ Ig\delta s\text{-}D(FA)$, by hypothesis. Therefore, $FA = S\ Ig\delta s\text{-int}(FA)$ and hence FA is soft $SIg\delta s$-open.

(7) $SIg\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA) = S\ Ig\delta s\text{-cl}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA)) = S\ Ig\delta s\text{-}$

$\text{cl}(FA) \cap S\ Ig\delta s\text{-cl}(X - FA)$.

(8) Now $SIg\delta s\text{-}Fr(X - FA) = S\ Ig\delta s\text{-cl}(X - FA) - S\ Ig\delta s\text{-int}(X - FA) = (X - S\ Ig\delta s\text{-int}(FA)) -$

$(X - S\ Ig\delta s\text{-cl}(FA)) = S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA) = S\ Ig\delta s\text{-}Fr(FA)$.

(9) A soft subset FA of X is soft $SIg\delta s$-closed if and only if $SIg\delta s\text{-cl}(FA) = FA$. Consider $SIg\delta s\text{-cl}(S\ Ig\delta s\text{-}Fr(FA)) = S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA)) = S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-cl}(FA) \cap (X - S\ Ig\delta s\text{-int}(FA))) = S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-cl}(FA) \cap S\ Ig\delta s\text{-cl}(X - FA)) \subset S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-cl}(FA)) \cap S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-cl}(X - FA)) = S\ Ig\delta s\text{-cl}(FA) \cap S\ Ig\delta s\text{-cl}(X - FA) = S\ Ig\delta s\text{-}Fr(FA)$.....by (7).

Thus, $SIg\delta s\text{-cl}(S\ Ig\delta s\text{-}Fr(FA)) \subset S\ Ig\delta s\text{-}Fr(FA)$. But $SIg\delta s\text{-}Fr(FA) \subset S\ Ig\delta s\text{-cl}(S\ Ig\delta s\text{-}Fr(FA))$ is always true. Therefore $SIg\delta s\text{-cl}(g\delta s\text{-}Fr(FA)) = S\ Ig\delta s\text{-}Fr(FA)$ and hence $SIg\delta s\text{-}Fr(FA)$ is soft $SIg\delta s$-closed set.

(10) $FA - S\ Ig\delta s\text{-}Fr(FA) = FA \cap (X - S\ Ig\delta s\text{-}Fr(FA)) = FA \cap (S\ Ig\delta s\text{-cl}(FA) \cap S\ Ig\delta s\text{-cl}(X - FA))^c = FA \cap ((S\ Ig\delta s - \text{cl}(FA))^c \cup (S\ Ig\delta s\text{-cl}(X - FA))^c) = (FA \cap (S\ Ig\delta s\text{-cl}(FA))^c) \cup (FA \cap (S\ Ig\delta s\text{-cl}(X - FA))^c) = \emptyset \cup (FA \cap S\ Ig\delta s\text{-int}(FA)) = S\ Ig\delta s\text{-int}(FA)$.

(11) If FA is both soft $SIg\delta s$-open and soft $SIg\delta s$-closed set, then $SIg\delta s\text{-int}(FA) = FA$ and $SIg\delta s\text{-}$

$\text{cl}(FA) = FA$ respectively. Now $SIg\delta s\text{-}Fr(FA) = S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA) = FA - FA = \emptyset$.

Conversely, $SIg\delta s\text{-}Fr(FA) = \emptyset \Rightarrow S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA) = \emptyset \Rightarrow S\ Ig\delta s\text{-cl}(FA) \subset S\ Ig\delta s\text{-int}(FA) \subset FA$. That is $SIg\delta s\text{-cl}(FA) \subset FA$. But $FA \subset S\ Ig\delta s\text{-cl}(FA)$ is always true. Therefore $SIg\delta s\text{-cl}(FA) = FA$. Hence FA is soft $SIg\delta s$-closed set. Again $SIg\delta s\text{-}Fr(FA) = \emptyset \Rightarrow S\ Ig\delta s\text{-cl}(FA) - S\ Ig\delta s\text{-int}(FA) = \emptyset$, $SIg\delta s\text{-cl}(FA) \subset S\ Ig\delta s\text{-int}(FA) \Rightarrow FA \cup S\ Ig\delta s\text{-}D(FA) \subset S\ Ig\delta s\text{-int}(FA) \Rightarrow FA \subset S\ Ig\delta s\text{-int}(FA)$. But $SIg\delta s$ -

int(FA) ⊂ FA is always true. Therefore FA = S Igδs -int(FA). Hence FA is soft SIgδs -open set.

(12) SIgδs -F r(S Igδs -int(FA)) = S Igδs -cl(S Igδs -int(FA)) − S Igδs -int(S Igδs-int(FA)) ⊂ S Igδs - cl(FA) − S Igδs -int(FA) as SIgδs -int(FA) ⊂ FA. This implies SIgδs -F r(S Igδs -int(FA)) ⊂ S Igδs - F r(FA).

(13) Consider X − S Igδs -F r(FA) = X − (S Igδs -cl(FA) − S Igδs -int(FA)) = (X − S Igδs -cl(FA)) ∪

S Igδs -int(FA) = S Igδs -int(X − FA) ∪ S Igδs -int(FA).

(14) SIgδs -F r(S Igδs -cl(FA)) = S Igδs -cl(S Igδs -cl(FA))−S Igδs -int(S Igδs -cl(FA)) = S Igδs -cl(S Igδs - cl(FA)) ∩ (X − S Igδs -int(S Igδs -cl(FA))) = S Igδs -cl(FA) ∩ S Igδs - cl(X − S Igδs -cl(FA))....(1). Also, FA ⊂ S Igδs -cl(FA) ⇒ X − S Igδs -cl(FA) ⊂ X − FA ⇒ S Igδs -cl(X − S Igδs -cl(FA)) ⊂ S Igδs - cl(X −FA). Substituting in (1),SIgδs -F r(S Igδs - cl(FA)) ⊂ S Igδs -cl(FA)∩S Igδs -cl(X −FA) = S Igδs -

F r(FA). Thus, SIgδs -F r(S Igδs -cl(FA)) ⊂ S Igδs -F r(FA).

(15) From (3), SIgδs -cl(FA) = S Igδs -int(FA) ∪ S Igδs -F r(FA) ⊂ FA ∪ S Igδs -F r(FA)...(l) as SIgδs -int(FA) ⊂ FA. Also from (3), SIgδs -F r(FA) ⊂ S Igδs -cl(FA) and FA ⊂ S Igδs -cl(FA) is always true. Therefore, FA ∪ S Igδs -F r(FA) ⊂ S Igδs -cl(FA)...(2). From (1) and (2) it follows that, FA ∪ S Igδs-F r(FA) = S Igδs -cl(FA).

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