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A interval valued intuitionistic fuzzy multi criteria decision making

M. Arockia Ranjithkumar[1,2,4*], *J.* Boobalan[2], *L.* Godlin Atlas[3], *P.* Sudhakar[3], and *R.* Senthil Kumar[4]

[1]Department Of Mathematics, M.Kumarasamy College Of Engineering,Karur - 639 113, India
[2]Department of Mathematics,Annamalai University,Annamalai Nagar - 608002, India
[3]School of Computing Sciences, Galgotias University, Greater Noida
[4]Department of Mathematics, Mohamed Sathak Engineering College, Kilakarai, Tamil Nadu, India

Abstract. The limitations and disadvantages of the available intuitionistic fuzzy set scoring functions are investigated. Two improved methods for handling multi-criteria fuzzy decision-making problems are provided. They are based on the two theories of intuitionistic fuzzy set and cross entropy, with the adoption of cross entropy of the degree of membership from the degree of non-membership handling the effect of hesitancy degree. Score function method and weighted score function method are their names. This study presents and investigates a novel strategy for ranking interval-valued intuitionistic fuzzy sets. Examples using numbers are used to demonstrate the technique. **Keywords:** Interval-valued intuitionistic fuzzy, new Score function, Fuzzy Arithmetic aggregation operators, Fuzzy Geometric aggregation operators, Multi-criteria fuzzy decision-making.

1 Introduction

Fuzzy sets (FSs), which were suggested by Zadeh (1965), are observed as a wide-ranging tool for solving multi-criteria decision-making (MCDM) problems (Bellman & Zadeh, 1970). In order to determination the uncertainty of non-membership degrees, Atanassov (1986) presented intuitionistic fuzzy sets (IFSs), which are an extension of Zadeh's FSs. IFSs have been commonly applied in solving MCDM problems (Chen & Chang, 2015). Moreover, interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov & Gargov, 1989) were proposed, which are an extension of FSs and IFSs. In recent years, MCDM problems with IVIFSs have attracted much attention from researchers (Chen, 2014; Liu, Shen, Zhang, Chen, & Wang, 2015; Tan et al., 2014; Wan & Dong, 2014). Furthermore, the TOPSIS method, proposed by Hwang and Yoon (1981), has also been used for solving MCDM problems (Cao, Wu, & Liang, 2015; Yue, 2014; Zhang & Yu, 2012). In this paper, two optimization models are established to determine the criterion weights in multi-criteria decision-making situations where knowledge regarding the weight information.

* Corresponding author: arockiaranjithkumar@gmail.com

2 Preliminaries

Definition 2.1: Let X be an ordinary finite non-empty set. An intuitionistic fuzzy set in X is an expression given by $\Omega = \{\langle x, \mu_\Omega(x), \nu_\Omega(x)\rangle | x \in \chi\}$ Where $\mu_\Omega : \chi \to [0,1]$, $\nu_\Omega : \chi \to [0,1]$ with the condition $0 \le \sup(\mu_\Omega(x)) + \sup(\nu_\Omega(x)) \le 1$ for any $x \in \chi$.

The intervals $\mu_\Omega(x)$ and $\nu_\Omega(x)$ denote, respectively, the membership degree and the non membership degree of the element x to the set Ω.Thus, for each $x \in \chi$, $\mu_\Omega(x)$ and $\nu_\Omega(x)$ are closed intervals and their lower and upper end points are, respectively, denoted by $\mu_{\Omega L}(x)$, $\mu_{\Omega U}(x)$, $\nu_{\Omega L}(x)$, $\nu_{\Omega U}(x)$. We can denote

$$\Omega = \{\langle x, [\mu_{\Omega L}(x), \mu_{\Omega U}(x)]\ , [\nu_{\Omega L}(x), \nu_{\Omega U}(x)]\ \rangle\ | x \in \chi\},$$

where

$$0 \le \mu_{\Omega U}(x) + \nu_{\Omega U}(x) \le 1,$$

$$\mu_{\Omega L}(x) \ge 0, \nu_{\Omega L}(x) \ge 0.$$

For each element x, we can compute the unknown degree (hesitancy degree) of an intuitionistic fuzzy interval of $x \in \chi$ in Ω defined as follows: is called the intuitionistic index of the element of the element

$$\pi_\Omega(x) = 1 - \mu_\Omega(x) - \nu_\Omega(x) = \left[1 - \mu_{\Omega U}(x) - \nu_{\Omega U}(x),\ \ 1 - \mu_{\Omega L}(x) - \nu_{\Omega L}(x)\ \right].$$

The operations of IFS are defined as follows for every $C, D \in X$

1. $C \le D$ iff and $\mu_C(x) \le \mu_D(x),\ \nu_C(x) \ge \nu_D(x)\ \ \forall x \in X$
2. $C = D$ iff $C \le D$ and $D \le C$
3. $C \cap D = \{x,\ \min(\mu_C(x),\ \mu_D(x))\ ,\ \max(\nu_C(x),\ \nu_D(x)\ ,\ x \in X\}$
4. $C \cup D = \{x,\ \max(\mu_C(x),\ \mu_D(x))\ ,\ \min(\nu_C(x),\ \nu_D(x)\ ,\ x \in X\}$

Definition 2.2 (Xu 2007)

Let $\Omega = \left([a,b]\ ,\ [c,d]\right)$ be an IVIFN. Then the score function S (Ω) = $\dfrac{a+b-c-d}{2}$ and an accuracy function defined by H(Ω) = $\dfrac{a+b+c+d}{2}$

If $C = \left([0.15, 0.15]\ ,\ [0.15, 0.55]\right), D = \left([0.12, 0.23]\ ,\ [0.36, 0.39]\right)$

$S(C) = -0.2,\ S(D) = -0.2 \Rightarrow C \approx D$ .This method is also failure to ranking

Definition 2.3 (Ye 2009)

Let $\Omega = ([a,b]\,,\ [c,d])$ be an IVIFN. Then the score function

$$M(\Omega) = a + b - 1 + \left(\frac{c+d}{2}\right)$$

If $C = ([0.13, 0.26]\,,\ [0.17, 0.43]), D = ([0.17, 0.22]\,,\ [0.09, 0.51])$

$M(C) = -0.31,\ M(D) = -0.31 \Rightarrow C \approx D$ .This method is also failure to ranking

Definition 2.4 (Priyadharshini 2018)

Let $\Omega = ([a,b]\,,\ [c,d])$ be an IVIFN. Then the score function $P(\Omega) = \dfrac{a+b+c-d}{4}$

If $C = ([0.1, 0.2]\,,\ [0.3, 0.4]), D = ([0, 0.2]\,,\ [0.2, 0.2])$

$P(C) = 0.1,\ P(D) = 0.1 \Rightarrow C \approx D$ .This method is also failure to ranking

Definition 2.5 (V.L.G.Nayagam 2018)

Let $A = ([a,b]\,,\ [c,d])$ be an IVIFN. Then the score function is defined by $J(A) = \dfrac{a+b+c-d+ab+cd}{3}$

If $A = ([0, 0.1]\,,\ [0.111, 0.12]), B = ([0, 0.1]\,,\ [0.1, 0.1089])$ then

$J(A) = 0.0348,\ J(B) = 0.0339$. Here clearly

$$A \subseteq B(a_1 \le a_2, b_1 \le b_2, c_1 \ge c_2, d_1 \ge d_2).$$

But $J(A) > J(B)$ .This method is also fails to rank IVIFNs.

3 A new non hesitance score function for interval valued Intuitionistic fuzzy numbers.

Definition 3.1 : Let $A = ([a,b]\,,\ [c,d])$ be an interval valued Intuitionistic fuzzy numbers. Then the non hesitance new score of an interval valued Intuitionistic fuzzy numbers. is defined as $BV(A) = \dfrac{a+b-c-d+ad+bc}{2}$.

Theorem 3.2

Let $A=\left(\left[a_1,b_1\right],\ \left[c_1,d_1\right]\right),\ B=\left(\left[a_2,b_2\right],\ \left[c_2,d_2\right]\right)$ be any two interval valued Intuitionistic fuzzy numbers. . If $A\subseteq B\Rightarrow BV(A)\leq BV(B)$

Proof:

$$\begin{aligned}
&\mathit{If}\ A\subseteq B\ \Rightarrow\ a_1\leq a_2, b_1\leq b_2, b_1\leq b_2, d_1\geq d_2\\
&\therefore (1-a_1)\geq(1-a_2),\ (1-b_1)\geq(1-a_2)\\
&\Rightarrow d_1(1-a_1)\geq d_2(1-a_2),\ c_1(1-b_1)\geq c_2(1-a_2)\\
&BV(B)-BV(A)=(a_2-a_1)+(b_2-b1)-(c_2-c_1)-(d_2-d_1)+\\
&\qquad\qquad (d_1(1-a_1)-d_2(1-a_2))+(c_1(1-b_1)-c_2(1-a_2))
\end{aligned}$$

Therefore $BV(B)-BV(A)\geq 0\Rightarrow BV(A)\leq BV(B)$

Proposition 3.1 Let A be an interval valued Intuitionistic fuzzy numbers. Then

$$BV(A)=-1 \text{ when } A=\left([0,0],\ [1,1]\right)$$

$$BV(A)=1 \text{ when } A=\left([1,1],\ [0,0]\right)$$

4 A intuitionistic fuzzy multi-criteria decision-making method

Any decision-making problem involving following three steps:

1. Design of an information system (decision matrix) and collection of data from experts; 2. Finding of aggregated performance of each alternative with respect to all criteria; and 3. Ranking of alternatives according to its aggregated performances. In interval-valued intuitionistic fuzzy decision problem, each entry in the decision matrix is represented by IVIFNs and the aggregated performance of an alternative is also represented by interval-valued intuitionistic fuzzy numbers.

We define a decision problem mathematically as follows: Let $A=\left\{A_1, A_2, ...A_m\right\}$ be the set of alternatives and let $\overset{j=1}{C}=\left\{C_1, C_2, ...C_n\right\}$ be the set of criteria under which the performance of alternatives will be evaluated. Let w_j be the weight vector given by the decision maker for each criteria C_j ,where $w_j\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$

Definition 3.3

Let $\Omega_i\ (j=1,2,3,4......n)$ belongs to interval valued intuitionistic fuzzy set of χ , then the weighted arithmetic average operator is defined by,

$$\varnothing(\Omega_1,\Omega_2,.........\Omega_n)=\sum_{j=1}^{n} w_j\Omega_j$$

$$=\left(\begin{array}{l}\left[1-\prod_{j=1}^{n}\left(1-\mu^L{}_{\Omega_j}(x)\right)^{w_j},\left(1-\prod_{J-1}^{n}\mu^U{}_{\Omega_j}(x)\right)^{w_j}\right],\\ \times\left[\prod_{j=1}^{n}\left(\nu^L{}_{\Omega_j}(x)\right)^{w_j},\prod_{j=1}^{n}\left(\nu^U{}_{\Omega_j}(x)\right)^{w_j}\right]\end{array}\right)$$

Definition 3.4

Let $\Omega_i\ (j=1,2,3,4......n)$ belongs to interval valued intuitionistic fuzzy set of χ, then the weighted geometric average operator is defined by,

$$\Gamma(\Omega_1,\Omega_2,.........\Omega_n)=\sum_{j=1}^{n} w_j\Omega_j$$

$$=\left(\begin{array}{l}\left[1-\prod_{j=1}^{n}\left(\mu^L{}_{\Omega_j}(x)\right)^{w_j},\left(\prod_{J-1}^{n}\mu^U{}_{\Omega_j}(x)\right)^{w_j}\right],\\ \times\left[\prod_{j=1}^{n}\left(1-\nu^L{}_{\Omega_j}(x)\right)^{w_j},\prod_{j=1}^{n}\left(1-\nu^U{}_{\Omega_j}(x)\right)^{w_j}\right]\end{array}\right)$$

Example:1

Assume a panel with four possible alternative to invest the money, which are a car company A_1, Share marketing A_2, telemarketing A_3 and an footwear company A_4.The invest company wants to decide a decision according to three criteria given by risk analysis C_1, the growth analysis C_2 and the environment analysis C_3. The criterion weight is given by $W=(0.38,0.27,0.35)$

$$M=\begin{bmatrix} & C_1 & C_2 & C_3\\ A_1 & ([0.25,0.45],[0.25,0.45]) & ([0.3,0.45],[0.35,0.45]) & ([0.36,0.46],[0.36,0.45])\\ A_2 & ([0.1,0.2],[0.1,0.2]) & ([0.2,0.3],[0.2,0.3]) & ([0.5,0.5],[0.5,0.5])\\ A_3 & ([0.1,0.1],[0.1,0.1]) & ([0.2,0.2],[0.2,0.2]) & ([0.4,0.4],[0.4,0.4])\\ A_4 & ([0.1,0.3],[0.1,0.3]) & ([0.3,0.3],[0.3,0.3]) & ([0.6,0.6],[0.6,0.6])\end{bmatrix}$$

Weighted Arithmetic Average Operator Algorithm:

Under the weighted arithmetic average operator Algorithm to select the alternative

Arithmetic Average Aggregated performance of each alternative A_1, A_2, A_3, A_4 with respect to criteria C_1, C_2, C_3 can be calculated as follows

$$\Lambda_1 = [(0.3174, 0.4535), (0.2984, 0.4500)]$$

$$\Lambda_2 = [(0.2903, 0.3454), (0.2118, 0.3075)]$$

$$\Lambda_3 = [(0.2435, 0.2435), (0.1959, 0.1959)]$$

$$\Lambda_4 = [(0.3668, 0.4245) (0.2519, 0.3824)]$$

Our non hesitance new function $BV(A) = \dfrac{a+b-c-d+ad+bc}{2}$.
Applying for above Equation we get

$$BV(\Lambda_1) = 0.1503$$

$$BV(\Lambda_2) = 0.1394$$

$$BV(\Lambda_3) = 0.0953$$

$$BV(\Lambda_4) = 0.2022$$

Therefore the ranking order of the four alternatives A_1, A_2, A_3, A_4 **is** $A_4 > A_1 > A_2 > A_3$

Obviously, amongst them best alternative is A_4

Weighted Geometric Average Operator Algorithm:

Under the weighted geometric average operator Algorithm to select the alternative

Geometric Average Aggregated performance of each alternative A_1, A_2, A_3, A_4 with respect to criteria C_1, C_2, C_3 can be calculated as follows

$$\Lambda_1 = [(0.3110, 0.4535), (0.3036, 0.4500)]$$

$$\Lambda_2 = [(0.2118, 0.3075), (0.2903, 0.3454)]$$

$$\Lambda_3 = [(0.1959, 0.1959), (0.2435, 0.2435)]$$

$$\Lambda_4 = [(0.2519, 0.3824)\ (0.3668, 0.4245)]$$

Our non hesitance new function $BV(A) = \dfrac{a+b-c-d+ad+bc}{2}$.
Applying for above Equation we get

$$BV(\Lambda_1) = 0.1443$$

$$BV(\Lambda_2) = 0.0230$$

$$BV(\Lambda_3) = 0.0001$$

$$BV(\Lambda_4) = 0.0450$$

Therefore the ranking order of the four alternatives A_1, A_2, A_3, A_4 is $A_1 > A_4 > A_2 > A_3$

Obviously, amongst them best alternative is A_1
Example2:

$$M = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} C_1 & C_2 & C_3 \\ ([0.41,0.54],[0.32,0.33]) & ([0.42,0.52],[0.21,0.39]) & ([0.55,0.71],[0.14,0.18]) \\ ([0.56,0.68],[0.21,0.36]) & ([0.63,0.72],[0.23,0.26]) & ([0.44,0.56],[0.10,0.22]) \\ ([0.44,0.58],[0.32,0.41]) & ([0.53,0.66],[0.3,0.38]) & ([0.38,0.52],[0.24,0.30]) \\ ([0.68,0.75],[0.11,0.18]) & ([0.61,0.71],[0.12,0.34]) & ([0.29,0.46],[0.16,0.24]) \end{bmatrix}$$

The criterion weight is given by $W = (0.25, 0.5, 0.25)$
Weighted Arithmetic Average Operator Algorithm:
Under the weighted arithmetic average operator Algorithm to select the alternative

Arithmetic Average Aggregated performance of each alternative A_1, A_2, A_3, A_4 with respect to criteria C_1, C_2, C_3 can be calculated as follows

$$\Lambda_1 = [(0.4533, .5813), (0.2108, 0.3083)]$$

$$\Lambda_2 = [(0.5714, 0.6759)\ ,(0.1826, 0.2705)]$$

$$\Lambda_3 = [(0.4737, 0.6093), (0.2883, 0.3651]$$

$$\Lambda_4 = [(0.5688, 0.6736)\ (0.1262, 0.2658)]$$

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Our non hesitance new function $BV(A)=\dfrac{a+b-c-d+ad+bc}{2}$.

Applying for above Equation we get

$$BV(\Lambda_1)=0.3889$$

$$BV(\Lambda_2)=0.5361$$

$$BV(\Lambda_3)=0.3891$$

$$BV(\Lambda_4)=0.5493$$

Therefore the ranking order of the four alternatives A_1, A_2, A_3, A_4 is $A_4 > A_2 > A_3 > A_1$

Obviously, amongst them best alternative is A_4

Weighted Geometric Average Operator Algorithm:

Under the weighted geometric average operator Algorithm to select the alternative

Geometric Average Aggregated performance of each alternative A_1, A_2, A_3, A_4 with respect to criteria C_1, C_2, C_3 can be calculated as follows

$$\Lambda_1 = \left[(0.4466, 0.5674), (0.2227, 0.3276)\right]$$

$$\Lambda_2 = \left[(0.5592, 0.6666)\ , (0.1942, 0.2769)\right]$$

$$\Lambda_3 = \left[(0.4655, 0.6020), (0.2906, 0.3688\right]$$

$$\Lambda_4 = \left[(0.5205,\ 0.6458)\ (0.1277, 0.2782)\right]$$

Our non hesitance new function

$$BV(A)=\frac{a+b-c-d+ad+bc}{2}.$$

Applying for above Equation we get

$$BV(\Lambda_1)=0.3682$$

$$BV(\Lambda_2)=0.5195$$

$$BV(\Lambda_3) = 0.3774$$

$$BV(\Lambda_4) = 0.4938$$

Therefore the ranking order of the four alternatives A_1, A_2, A_3, A_4 is $A_2 > A_4 > A_3 > A_1$ Obviously, amongst them best alternative is A_1

5 Conclusions

This study introduces and studies a new unique accuracy function that accurately ranks all comparable interval-valued intuitionistic fuzzy sets. Our proposed method is very helpful and has numerous applications since the problem of ranking interval-valued intuitionistic fuzzy sets is very significant in real-world issues including decision-making, clustering, and artificial intelligence.

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