Nonlinear mathematical model of an economic soliton

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Abstract. In modern conditions of economic transformation, the functioning and development of production systems is characterized by instability, nonlinearity and dynamism of the main indicators and parameters. The financial and production system is affected by a large number of market, inflationary, social and other disturbances, which leads to the losses in financial stability, a decrease in production volumes, a decrease in demand for manufactured products, etc. Therefore, it is necessary to have an effective management system and operational decision-making. The relevant direction for solving this problem is the development of a flexible intelligent control system that makes it possible to diagnose quickly the unfavorable state of the financial and production system and activate promptly the mechanisms that return the system to equilibrium. In this case, information becomes the main resource of management, which performs an integrating role and is a necessary component when using all other resources. All this predetermines the use of information technology management. With a high dynamism and instability of the external environment, it becomes necessary to apply the methods and approaches of the new science of management, concentrating its attention on the theory of complex systems and nonlinear dynamics, with the help of which complex control systems can cope effectively with uncertainty and rapid changes. The development and use of modern information technologies for enterprise management are based on the introduction of integrated information processing technology and the creation of mathematical methods and computer modeling tools. In the work of information technologies, management is implemented by creating an integrated intelligent computerized system based on the use of economic and mathematical methods, computer technology and communication means, i.e. implements a fundamentally new management platform, which is based on the integration of management information through a mechanism for generalizing the information database and knowledge. Keywords: globalization, software component, management platform, solitons, information technologies

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1 Introduction

Soliton research is carried out all over the world. Scientists consider this type of wave from the point of view of various fields of science. So, R.V. Mayer studied the formation of various solitons and their interaction by simulating the propagation of an impulse in a one-dimensional medium, its reflection from the interface between two media and its passage into a second medium, wave propagation in a dispersive medium, R.F. Malikov paid more attention to the problem of modeling solitons.

Tapert and Varma, Narayanamurti and Varma obtained the equation when studying the propagation of heat impulses in solids.

Kako and Rowlands obtained an equation for two-dimensional propagation of ion-acoustic solitons.

The universal nature of multidimensional analogs of the Korteweg-de Vries equation makes it possible to abstract from specific physical conditions and find its new multidimensional generalizations, relying on the properties of soliton equations[1].

Soliton equations have the following properties:

1 soliton equation has an infinite number of conservation laws;

2 it has a solution in the form of solitary waves - solitons;

3 if the soliton equation admits solutions of the type of solitary waves, then it should admit solutions that are a nonlinear superposition of N solitary waves for an arbitrary N;

4 it is integrable in the sense of an infinite-dimensional generalization of a fully integrable Hamiltonian system. All known soliton equations have Hamiltonian structures and an infinite set of integrals of motion in involution (that is, the Poisson's bracket vanishes);

5 there is a canonical transformation (the method of the inverse scattering problem - MISP), which transforms the soliton equation into an infinite system of separate equations with action-angle variables, each of which can be integrated in a trivial way (the existence of an infinite number of conservation laws follows from the condition of zero curvature);

6 it has the Hirota property, has bilinear forms, which makes it possible to construct N-soliton solutions (nonlinear superposition of N solitary waves);

7 Painlevé property (a test that shows that a given nonlinear equation is fully integrable).

2 Methods

Mathematically, solitons are understood as a localized nonlinear wave that behaves like a particle. It interacts with others of its kind and asymptotically restore its original shape with a possible phase shift [2].

Solitons are studied in oceans (wandering waves, tsunamis, vortex solitons), in solid crystalline bodies (dislocations, domain walls), in magnetic materials (solitons in ferromagnets, electromagnetic solitons), in optical fiber guides (optical soliton, soliton networks), in the atmosphere of the Earth and other planets (Rossby soliton, Jupiter's red spot), in galaxies (black holes), in living organisms (nerve impulses, breathers, intellectons, membranes) and in economics (economic soliton).

Sometimes a soliton is defined as a regular localized stable solution of a nonlinear differential equation.

A wide variety of systems, including socio-economic ones, can represent stable localized formations. In such organizational and economic forms as the shadow economy and urban marginal societies, the characteristics of a soliton are manifested. The collective behavior of economic entities has a nonlinear wave character. Therefore, the question arises of ensuring its structural and local stability. Both forms are resistant to external influences and have other properties of a soliton.

Note that not every solution of the (2+1)-dimensional nonlinear mathematical model A1 is a soliton. Similarly, not every collective entity has the characteristics of a soliton [3].

An economic soliton is understood as "the form of behavior of micro-subjects of the economy, a characteristic feature of which is the stable tendency of micro-subjects to certain types of activity and the continuous reproduction of the functional qualities of this collective formation." For the formation of a soliton, certain conditions are imposed on the function $\Psi = \Psi\,(x,y,t)$ that determines the behavior of the individual. The wave of probability is a nonlinear wave that characterizes the behavior of micro-subjects of the economy. Then the probability density $\rho\left(x,y,t\right) = \left|\Psi\left(x,y,t\right)\right|^2$ determines the hit of the individual at a certain point in the economic space. Therefore, "in a specific area of economic space, a stable localization of the probability density of microeconomic entities entering this area can be considered a soliton in the economy."

The direct scattering method for (2+1)-dimensional nonlinear mathematical model A1 consists of the following stages.

1. An auxiliary linear system is constructed, a Lax pair for (2+1)-dimensional nonlinear mathematical model A1.
2. The equation of compatibility of the auxiliary linear system with the (2+1)-dimensional nonlinear mathematical model A1, which is the conditions of zero curvature, is determined.
3. For the first equation of the auxiliary linear system, we consider the spectral problem.
4. Using the given function, we find the components of the scattering matrix at fixed y and t.
5. Using the 2nd equation of the auxiliary linear system, we restore the evolution of the components of the scattering matrix and discrete spectrum in variables y and t.

For a (2+1)-dimensional nonlinear mathematical model A1, consider an auxiliary linear system of the form

$$\begin{cases} \varphi_x = P\varphi \\ \varphi_t = Q_1\varphi + \lambda I \varphi_y \end{cases}$$

where $\varphi = colon\left(\varphi_1, \varphi_2\right)$; $P = P_1 + \lambda P_2$

$$P_1 = \begin{pmatrix} 0 & \Psi \\ \overline{\Psi} & 0 \end{pmatrix},\ P_2 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix},\ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q_1 = A + \lambda B,$$

$$A=\begin{pmatrix} 0 & -\Psi_{xy}-2\Psi^2-UV \\ -\overline{\Psi}_{xy}-2\overline{\Psi}^2-\overline{U}\overline{V} & 0 \end{pmatrix},$$

$$B=\begin{pmatrix} i & -2i\partial_x^{-1}\Psi-V \\ 2i\partial_x^{-1}\overline{\Psi}-\overline{V} & -i \end{pmatrix},$$

$\lambda_t=\lambda\lambda_y$, $\lambda=\lambda(y,t)$ – spectral parameter, $\lambda=\xi_1+i\xi_2$, $\operatorname{Re}\lambda=\xi_1$, $\operatorname{Im}\lambda=\xi_2$.

The compatibility conditions for the auxiliary linear system (1) and the (2+1)-dimensional mathematical model A1 (1.76) has the form:

$$P_1t-Ax+[P_1,A]=0,$$

where $[P_1,A]=P_1A-AP_1$.

Fast descending makes it possible to define own functions $u, \overline{u}, v, \overline{v}$ with the following boundary conditions at fixed y and t

$$u\sim\begin{pmatrix} e^{-iRe\lambda x} \\ 0 \end{pmatrix} \text{ if } x\to-\infty$$

$$\overline{u}\sim\begin{pmatrix} e^{-iRe\lambda x} \\ 0 \end{pmatrix} \text{ if } x\to-\infty$$

$$v\sim\begin{pmatrix} 0 \\ e^{iRe\lambda x} \end{pmatrix} \text{ if } x\to+\infty$$

$$\overline{v}\sim\begin{pmatrix} -e^{-iRe\lambda x} \\ 0 \end{pmatrix} \text{ if } x\to+\infty$$

For the spectral problem

$$\varphi_x=P_1\varphi+\lambda P_2\varphi \tag{1}$$

according to the given solutions $\varphi = \varphi(x, y, t)$ with fixed y and t the components of the scattering matrix will be found:

$$S(\lambda) = \begin{pmatrix} a(\lambda) & b(\lambda) \\ \bar{b}(\bar{\lambda}) & -\bar{a}(\bar{\lambda}) \end{pmatrix}$$

The matrix $S(\lambda)$ characterizes the function φ, $\frac{1}{a(\lambda)}$ – transmission coefficient, $\frac{b(\lambda)}{a(\lambda)}$ – reflection coefficient [4].

Further we will omit the dependence of the functions on y and t.

Let us suppose φ and $\bar{\varphi}$ are the solution to the spectral problem (1.79)

$$\varphi = \varphi(x, \mathrm{Re}\lambda) = colon\left(\varphi_1(x, \mathrm{Re}\lambda), \varphi_2(x, \mathrm{Re}\lambda)\right),$$

$$\bar{\varphi} = \bar{\varphi}(x, \mathrm{Re}\lambda) = colon\left(\bar{\varphi}_1(x, \mathrm{Re}\lambda), \bar{\varphi}_2(x, \mathrm{Re}\lambda)\right).$$

Then

$$\frac{d}{dx} W(\varphi, \bar{\varphi}) = 0,$$

where $W(\varphi, \bar{\varphi})$ – Wronskian φ and $\bar{\varphi}$:

$$W(\varphi, \bar{\varphi}) = \varphi_1 \bar{\varphi}_2 - \varphi_2 \bar{\varphi}_1 .$$

As far as $W(u, \bar{u}) = -1$ and $W(v, \bar{v}) = 1$, then the solutions v and $\bar{v}$ are linearly independent, so we can express the functions u and $\bar{u}$ in terms of v and $\bar{v}$:

$$u = a(\mathrm{Re}\lambda)\bar{v} + b(\mathrm{Re}\lambda)v$$

$$\bar{u} = -\bar{a}(\mathrm{Re}\,\lambda)v + \bar{b}(\mathrm{Re}\,\lambda)\bar{v}$$

or

$$\begin{pmatrix} u \\ \bar{u} \end{pmatrix} = \begin{pmatrix} a(\mathrm{Re}\,\lambda) & b(\mathrm{Re}\,\lambda) \\ \bar{b}(\mathrm{Re}\,\lambda) & -\bar{a}(\mathrm{Re}\,\lambda) \end{pmatrix} \begin{pmatrix} \bar{v} \\ v \end{pmatrix}$$

wherein

$$a(\mathrm{Re}\,\lambda)\bar{a}(\mathrm{Re}\,\lambda) + b(\mathrm{Re}\,\lambda)\bar{b}(\mathrm{Re}\,\lambda) = 1 .$$

Now let us set the analytical properties of the scattering matrix.

If Ψ and $\overline{\Psi}$ are absolutely integrable, i.e. $\Psi, \overline{\Psi} \in L_1$, then the functions $u(x,\lambda)e^{i\lambda x}$ and $v(x,\lambda)e^{-i\lambda x}$ are analytic in the upper half-plane, and the functions $\bar{u}(x,\bar{\lambda})e^{-i\bar{\lambda}x}$ and $\bar{v}(x,\bar{\lambda})e^{i\bar{\lambda}x}$ are analytic in the lower half-plane, where

$$u(x,\lambda) = colon(u_1(x,\lambda), u_2(x,\lambda)) ,$$

$$v(x,\lambda) = colon(v_1(x,\lambda), v_2(x,\lambda)) .$$

Then the function

$$a(\lambda) = W(u(x,\lambda), v(x,\lambda)) = u_1(x,\lambda)v_2(x,\lambda) - u_2(x,\lambda)v_1(x,\lambda)$$

is analytic in the upper half-plane, and the function

$$\bar{a}(\bar{\lambda}) = W(\bar{u}(x,\bar{\lambda}), v(x,\bar{\lambda})) = \bar{u}_1(x,\bar{\lambda})\bar{v}_2(x,\bar{\lambda}) - \bar{u}_2(x,\bar{\lambda})\bar{v}_1(x,\bar{\lambda})$$

is analytic in the lower half-plane.

The functions

$$b(\lambda) = -W(u(x,\lambda), \bar{v}(x,\bar{\lambda})) ,$$

$$\bar{b}(\bar{\lambda}) = -W(\bar{u}(x,\bar{\lambda}), v(x,\lambda))$$

generally speaking, do not possess analytic properties [5].

To establish the analyticity properties of the scattering matrix, we represent the function $u(x,\lambda) = colon(u_1(x,\lambda), u_2(x,\lambda))$ as an integral equation.

$$u_1(x,\lambda)e^{i\lambda x} = 1 + \int_{-\infty}^{x} M(x,z_1,\lambda) u_1(z_1,\lambda) e^{i\lambda z_1} dz_1 , \tag{2}$$

$$u_2(x,\lambda)e^{i\lambda x} = \int_{-\infty}^{x} e^{2i\lambda(x-z_1)}\overline{\Psi}(z_1)u_1(z_1,\lambda)e^{i\lambda z_1}dz_1, \tag{3}$$

$$M(x,z_1,\lambda) = \overline{\Psi}(z_1)\int_{z_1}^{x} e^{2i\lambda(z_2-z_1)}\Psi(z_2)dz_2. \tag{4}$$

Arise from [4] it is possible to show absolute convergence, boundedness of functions $u_1(x,\lambda)e^{i\lambda z}$, $u_2(x,\lambda)e^{i\lambda z}$ and their derivatives with respect to variables λ, what sets the analyticity of functions $u_1(x,\lambda)e^{i\lambda z}$ and $u_2(x,\lambda)e^{i\lambda z}$ in the upper half-plane [6] $\mathrm{Im}(\lambda) > 0$.

If $\Psi, \overline{\Psi} \in L_1$ and an additional condition on the function Ψ and $\overline{\Psi}$:

$$\left|\overline{\Psi}(x)\right| \le Ce^{-2K|x|},$$

$$\left|\Psi(x)\right| \le Ce^{-2K|x|},$$

where $C, K\ (K > 0)$ – constants, we get the analyticity of the vector functions $u(x,\lambda)e^{i\lambda x}$, $v(x,\lambda)e^{-i\lambda x}$ and $a(\lambda)$ with all $\mathrm{Im}(\lambda) > -K$, in this case, the vector functions $\overline{u}(x,\overline{\lambda})e^{-i\overline{\lambda}x}$ and $\overline{v}(x,\overline{\lambda})e^{i\overline{\lambda}x}$ and $\overline{a}(\overline{\lambda})$ analytic at $\mathrm{Im}(\lambda) < K$. Besides $b(\lambda)$ and $\overline{b}(\overline{\lambda})$ are analytical in the strip [7] $-K < \mathrm{Im}(\lambda) < K$.

In order to find the asymptotic expansion with large λ in the upper half-plane, we integrate (1) - (3) by parts, we obtain:

$$u_1(x,\lambda)e^{i\lambda x} = 1 - \frac{1}{2i\lambda}\int_{-\infty}^{x}\overline{\Psi}(z_1)dz_1 + O(\lambda^2),$$

$$u_2(x,\lambda)e^{i\lambda x} = -\frac{1}{2i\lambda}\overline{\Psi}(x) + O(\lambda^{-2})$$

$$v_1(x,\lambda)e^{-i\lambda x} = \frac{1}{2i\lambda}\Psi(x) + O(\lambda^{-2})$$

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$$v_2(x,\lambda)e^{-i\lambda x} = -\frac{1}{2i\lambda}\int_x^\infty \overline{\Psi}(z)\Psi(z)dz \ + O(\lambda^{-2})$$

For coordinates of vector functions $\overline{u}(x,\overline{\lambda}) = colon(\overline{u}_1(x,\overline{\lambda}), \overline{u}_2(x,\overline{\lambda}))$, $\overline{v}(x,\overline{\lambda}) = colon(\overline{v}_1(x,\overline{\lambda}), \overline{v}_2(x,\overline{\lambda}))$, for all λ, lying in the lower half-plane, we obtain the asymptotic expansion

$$\overline{u}_1(x,\overline{\lambda})e^{i\overline{\lambda}x} = 1 - \frac{1}{2i\overline{\lambda}}\Psi(x) + O(\lambda^2)$$

$$\overline{u}_2(x,\overline{\lambda})e^{-i\overline{\lambda}x} = -1 - \frac{1}{2i\overline{\lambda}}\int_{-\infty}^{x} \Psi(z_1)\overline{\Psi}(z_1)dz_1 + O(\overline{\lambda}^2)$$

$$\overline{v}_1(x,\overline{\lambda})e^{-i\overline{\lambda}x} = 1 + \frac{1}{2i\overline{\lambda}}\int_x^\infty \Psi(z_1)\overline{\Psi}(z_1)dz_1 + O(\overline{\lambda}^2)$$

$$\overline{v}_2(x,\overline{\lambda})e^{-i\overline{\lambda}x} = -\frac{1}{2i\overline{\lambda}}\overline{\Psi}(x) + O(\lambda^{-2}).$$

Then if $|\lambda| \to \infty$ in the corresponding half-planes, we have asymptotic expansions for $a(\lambda)$ and $\overline{a}(\overline{\lambda})$:

$$a(\lambda) = 1 - \frac{1}{2i\lambda}\int_{-\infty}^{\infty} \Psi(z_1)\overline{\Psi}(z_1)dz_1 + O(\lambda^2)$$

$$\overline{a}(\overline{\lambda}) = 1 - \frac{1}{2i\overline{\lambda}}\int_{-\infty}^{\infty} \Psi(z_1)\overline{\Psi}(z_1)dz_1 + O(\overline{\lambda}^{-2})$$

If Ψ and $\overline{\Psi}$ are not too small, then the spectral problem

$$\varphi_x = P_1\varphi + \lambda P_2\varphi$$

can have discrete eigenvalues, when $a(\lambda)$ has zeros in the upper half-plane ($\mathrm{Im}(\lambda) > 0$) and $\overline{a}(\overline{\lambda})$ has zeros in the lower half-plane ($\mathrm{Im}(\lambda) < 0$) [8].

Let us suppose that Ψ and $\overline{\Psi}$ decrease fast enough at $|x| \to \infty$, e.i.

$$\int_{-\infty}^{\infty} |z_1|^n |\Psi(z_1)| dz_1 < \infty,$$

$$\int_{-\infty}^{\infty} |z_1|^n |\overline{\Psi}(z_1)| dz_1 < \infty$$

for all n. Then $a(\lambda), \overline{a}(\overline{\lambda})$, $b(\lambda)$, $\overline{b}(\overline{\lambda})$ are entire functions. In this case, we can continue $b(\lambda)$, $\overline{b}(\overline{\lambda})$ and get $\gamma_k(\lambda) = b(\lambda_k)$ and $\overline{\gamma}_k(\overline{\lambda}) = \overline{b}(\overline{\lambda}_k)$.

Then the functions $a(\lambda), \overline{a}(\overline{\lambda})$ are analytical in the upper and lower half-planes, respectively, and on the real axis. Therefore $a(\lambda)$ has only a finite number of zeros if $\mathrm{Im}(\lambda) > 0$ [9].

Let λ_k, $k = \overline{1, N}$ be function zeros $a(\lambda)$ in the upper half-plane ($\mathrm{Im}(\lambda) > 0$), where N is the number of related states. If $\lambda = \lambda_k, k = \overline{1, N}$ we have a connection:

$$u(x, \lambda) = \gamma_k(\lambda) v(x, \lambda),\ \gamma_k(\lambda) = b(\lambda_k),\ k = \overline{1, N}.$$

Let $\overline{\lambda}_k$, $k = \overline{1, N}$ be function zeros $\overline{a}(\overline{\lambda})$ in the lower half-plane. Than if $\overline{\lambda} = \overline{\lambda}_k, k = \overline{1, N}$ we have bound states:

$$\overline{u}(x, \overline{\lambda}) = \overline{\gamma}_k(\overline{\lambda}) \overline{v}(x, \overline{\lambda}),\ \overline{\gamma}_k(\overline{\lambda}) = \overline{b}(\overline{\lambda}_k),\ k = \overline{1, N}.$$

Using the second equation of the auxiliary linear system (1.79)

$$\varphi_t = \lambda \varphi_y + A\varphi + B\lambda\varphi, \tag{5}$$

where the spectral parameter $\lambda = \lambda(x, y, t)$ satisfies the differential equation

$$\lambda_t = \lambda \lambda_y, \tag{6}$$

we restore the dependence of the scattering matrix $S(\lambda)$ from variables y and t.

The equation (1.84) has two solutions:

$$\lambda(y,t) = \lambda_1 = const,$$

$$\lambda(y,t) = \lambda_2(y,t) = \frac{y + c_1 + ic_2}{t_0 - t},$$

where c_1, c_2, t_0 are some real constants.

Scattering matrix

$$S(\lambda) = \begin{pmatrix} a(\lambda) & b(\lambda) \\ \bar{b}(\bar{\lambda}) & -\bar{a}(\bar{\lambda}) \end{pmatrix}$$

by virtue of (6) satisfies the evolution equation

$$S_t = \lambda S_y. \tag{7}$$

Where we get the evolution from x, y and t transition coefficients and discrete spectrum

$$a(x,y,t,\lambda) = a(x, y + \lambda t, \lambda)$$

$$b(x,y,t,\lambda) = b(x, y + \lambda t, \lambda)$$

$$\gamma_j(x,y,t,\lambda_j) = \gamma_j(x, y + \lambda_j t, \lambda_j),\ j = \overline{1,N}.$$

It can be seen from (1.85) that the components of the scattering matrix $S(\lambda)$ satisfy the evolutionary equations $b_t = \lambda b_y$, $a_t = \lambda a_y$, $(\gamma_j)_t = \lambda(\gamma_j)_y$ [10].

Thus, for the (2+1)-dimensional nonlinear mathematical model A1, we have constructed an auxiliary linear system, the condition of zero curvature, for the first equation of the auxiliary linear system we have considered the spectral problem, for a given function φ the components of the scattering matrix and the discrete spectrum were determined at fixed y and t. Using the second equation of the auxiliary linear system, we reconstructed the evolution of the data of the scattering matrix and the discrete spectrum in variables y and [11].

3 Conclusion

Based on the conducted research the following conclusions were made.

The (2+1)-dimensional nonlinear mathematical model A1, generalizing the Korteweg-de Vries equation, was considered. Its conservation laws are shown, an auxiliary linear system is considered for it, a condition of zero curvature is determined, which connects the auxiliary linear system with this model. The direct scattering problem for this model is solved. From the given function, the components of the scattering matrix are determined, and the evolution of the scattering matrix in variables y и t is reconstructed.

The model of the economic soliton describes "the behavior of micro-subjects of the economy, a characteristic feature of which is the stable tendency of micro-subjects to certain types of activity and the continuous reproduction of the functional qualities of this collective formation." The direct scattering problem method for economic problems corresponds to the implementation of a positive approach to economic policy [12].

The space-two-dimensional evolutionary equation A6, its N-soliton solutions, visualization of the 1-soliton solution of this equation are presented.

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