The influence of mathematical modeling on the physical impact in soils

Elifkhan Agakhanov[1*], *Murad* Agakhanov[2], and *Gadji* Agakhanov[1]

[1]Dagestan State Technical University, 70, Imam Shamil Avenue, 367015, Makhachkala, Russia
[2]Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, 129337, Moscow, Russia

Abstract. The behavior of soils in the foundations of buildings and structures, as well as in ground structures and massifs, is formed under the influence of force (surface and volume) and physical (temperature, humidity, etc.) influences. However, if mathematical modeling of force impacts in ground environments can be considered a completely solvable task today, then we consider such a statement premature in relation to physical impacts. The peculiarity of physical impacts is that they can be both the cause of forced deformations and the cause of changes in soil properties. The ground environment with creep properties, which is under both force and physical influences, is considered. The systems of equations of mechanics of a deformable solid for a triaxial stress state and a differential equation of thermal conductivity are used. Within the framework of the model of a linearly deformable continuous isotropic body, mathematical modeling of physical impacts in ground media with creep properties was performed. In a quasi-elastic formulation, a solution is obtained for an unlimited soil massif with a flat surface under the action of a plane -parallel heat flow. In the absence of force influences in a quasi-elastic medium for deformations from physical influences, we have the usual equations of elasticity theory. The solution obtained for an unlimited soil massif allows the calculation of stresses taking into account creep caused by temperature fluctuations on the surface in it. Keywords: mathematical modeling, physical impacts, forced deformations, non-invariant soil environment, linear-hereditary creep, stress-strain state.

1 Introduction

The behavior of soils in the foundations of buildings and structures, as well as in ground structures and massifs, is formed under the influence of force (surface and volume) and physical (temperature, humidity, etc.) influences [1-4]. However, if mathematical modeling of force impacts in ground environments can be considered a completely solvable task today, then we consider such a statement premature in relation to physical impacts [5-7]. The peculiarity of physical impacts is that they can be both the cause of forced deformations and the cause of changes in soil properties [8, 9]. An example is structurally unstable frozen, permafrost, loess and swelling soils, which are characterized by the ability to sharply reduce the strength of structural bonds between particles under physical

* Corresponding author: muradak@mail.ru

influences: when heated – for some, humidification, which may be associated with heating – for others. Soils, especially in the presence of physical influences, do not have great stability of deformations under load, and with modern requirements for the accuracy of calculation results, it is impossible to ignore the pronounced property of soils to deform over time, i.e. creep [10-12].

2 Materials and methods

We consider a ground environment with creep properties, which is under both force and physical influences. Taking the model of a linearly deformable solid isotropic body, we will write the system of equations for a triaxial stress state in the form of only the first line, bearing in mind that the other two lines can be obtained by a circular permutation of the indices x, y, z [13-16].

Let's write the differential equations of equilibrium in displacements are:

$$\left.\begin{array}{l} S_c\left(\nabla^2 u+\frac{1}{3}\cdot\frac{\partial\varepsilon_o}{\partial x}\right)+S_o\left(\frac{\partial\varepsilon_o}{\partial x}-3\frac{\partial\varepsilon_в}{\partial x}\right)+X=0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \quad (1)$$

The conditions of continuity of deformations are:

$$\left.\begin{array}{c} \frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial x\partial y}; \\ \dots\dots\dots\dots\dots\dots \\ 2\frac{\partial^2\varepsilon_x}{\partial x\partial z}=\frac{\partial}{\partial x}\left(-\frac{\partial\gamma_{yz}}{\partial x}+\frac{\partial\gamma_{zx}}{\partial y}+\frac{\partial\gamma_{xy}}{\partial z}\right); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \quad (2)$$

Non - invariant in time the linear-hereditary creep equation is:

$$\left.\begin{array}{c} \varepsilon_x=\varepsilon_в+L_c\left(\frac{2\sigma_x-\sigma_y-\sigma_z}{6}\right)+L_o\left(\frac{\sigma_x+\sigma_y+\sigma_z}{9}\right); \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \gamma_{xy}=L_c(\tau_{xy}); \\ \dots\dots\dots\dots \end{array}\right\} \quad (3)$$

In the form solved with respect to stresses, they look like the folliwing:

$$\left.\begin{array}{c}\sigma_x = S_c\left(\dfrac{4\varepsilon_x - 2\varepsilon_y - 2\varepsilon_z}{3}\right) + S_o(\varepsilon_x + \varepsilon_y + \varepsilon_z - 3\varepsilon_в);\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \tau_{xy} = S_c(\gamma_{xy});\\ \dots\dots\dots\end{array}\right\} \quad (4)$$

The following notation is used in dependencies (1) - (4):

$$\nabla^2(\ ) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2};\ \varepsilon_o = \varepsilon_x + \varepsilon_y + \varepsilon_z;$$

$\varepsilon_в$ - forced deformations;

$S_c(\ ),\ S_o(\ ),\ L_c(\ ),\ L_o(\ )$ - operators having the form:

$$L_c(p) = \frac{1}{G(t)}\left[p(t) + \int_{t_0}^{t} p(\tau)K_c(t,\tau)d\tau\right];$$

$$L_o(p) = \frac{1}{E_o(t)}\left[p(t) + \int_{t_0}^{t} p(\tau)K_o(t,\tau)d\tau\right];$$

$$S_c(p) = G(t)p(t) - \int_{t_0}^{t} p(\tau)G(\tau)R_c(t,\tau)d\tau;$$

$$S_o(p) = E_o(t)p(t) - \int_{t_0}^{t} p(\tau)E_o(\tau)R_o(t,\tau)d\tau;$$

t_o - the moment of the start of the download;

$K_c(t,\tau) = -\dfrac{\partial F_3(t,\tau)/\partial\tau}{F_3(t,t)} = \dfrac{1+\nu(t,\tau)}{1+\mu(t,t)}K(t,\tau)$ - hereditary kernel for shear deformations;

$K_o(t,\tau) = \dfrac{1-2\nu(t,\tau)}{1-2\mu(t,t)}K(t,\tau)$ - hereditary kernel for volumetric deformations;

$K(t,\tau) = -\dfrac{\partial F_1(t,\tau)/\partial\tau}{F_1(t,t)}$ - the kernel of the creep equation;

$R_c(t,\tau)$ - kernel resolvent $K_c(t,\tau)$;

$R_o(t,\tau)$ - kernel resolvent $K_o(t,\tau)$;

$E_o(t)=\dfrac{E(t)}{3-6\mu(t,t)}$ - volumetric instantaneous modulus of elasticity;

$$E(t)=\frac{1}{F_1(t,\tau)};\ \mu(t,\tau)=\frac{F_2(t,\tau)}{F_1(t,\tau)};\ G(t)=\frac{1}{F_3(t,\tau)}=\frac{E(t)}{2\left[1+\mu(t,\tau)\right]};$$

$F_1(t,\tau),\ F_2(t,\tau),\ F_3(t,\tau)$ - experimentally obtained dependencies and of the three functions, only two are independent, and the third is uniquely expressed through them by the formula

$F_3(t,\tau)=2\left[F_1(t,\tau)+F_2(t,\tau)\right]$;

$$\nu(t,\tau)=\frac{\partial F_2(t,\tau)/\partial\tau}{\partial F_1(t,\tau)/\partial\tau}.$$

Between coefficients $\nu(t,\tau)$ and $\mu(t,\tau)$ there is a dependency

$$\nu(t,\tau)=\mu(t,\tau)+\frac{F_1(t,\tau)}{\partial F_1(t,\tau)/\partial\tau}\partial\mu(t,\tau)/\partial\tau$$

3 Research results

In the case of a quasi-elastic medium, the first Poisson's ratio $\mu(t,\tau)$ does not depend on the moment of application of the load τ and $\nu=\mu=\mu(t)$, $K_c(t,\tau)=K_o(t,\tau)=K(t,\tau)$.

Then the differential equations of equilibrium (1) take the form:

$$\left.\begin{array}{l} S_c\left[\nabla^2 u+\dfrac{1}{1-2\mu(t)}\cdot\dfrac{\partial\varepsilon_o}{\partial x}-\dfrac{2+2\mu(t)}{1-2\mu(t)}\cdot\dfrac{\partial\varepsilon_в}{\partial x}\right]+X=0; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \quad (5)$$

and, going to the operator L_c, the reverse S_c:

$$\left.\begin{array}{l} \nabla^2 u+\dfrac{1}{1-2\mu(t)}\cdot\dfrac{\partial\varepsilon_o}{\partial x}-\dfrac{2+2\mu(t)}{1-2\mu(t)}\cdot\dfrac{\partial\varepsilon_в}{\partial x}-L_c(X)=0; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \quad (6)$$

Putting the force effects equal to zero, we have the usual equations of elasticity theory for deformations from physical influences in a quasi-elastic medium [17-19].

 https://doi.org/10.1051/e3sconf/202337603035

For a quasi-elastic medium, the deformation equations (3) take the form:

$$E(t)\varepsilon_x = E(t)\varepsilon_в + L(\sigma_x) - \mu(t)L(\sigma_y + \sigma_z);$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \tag{7}$$
$$G(t)\gamma_{xy} = L(\tau_{xy});$$
$$\dots\dots\dots\dots$$

where $L(\;)\}$ - an operator having the form:

$$L(\sigma) = \sigma(t) + \int_{t_0}^{t} \sigma(\tau)K(t,\tau)d\tau$$

We consider the problem for an unlimited soil mass with a flat surface, which we will consider an infinite half-space, which is under the action of forced deformations $\varepsilon_в$ caused by a plane -parallel heat flow. At the same time, the displacements u and v in the directions perpendicular to the heat flow, as well as deformations ε_x and ε_y, are equal to zero. Displacements w and deformations ε_z depend only on the variable z. Volumetric forces are considered equal to zero. In the case of a quasi-elastic soil mass, only one equation remains from equations (6)

$$\frac{\partial^2 w}{\partial z^2} = \frac{\partial \varepsilon_z}{\partial z} = \frac{1+\mu}{1-\mu} \cdot \frac{\partial \varepsilon_в}{\partial z}, \tag{8}$$

from where, integrating, we get

$$\varepsilon_z = \frac{1+\mu}{1-\mu}\varepsilon_в + C \tag{9}$$

The constant C is determined by the beginning of the reference scale of forced deformations. Assuming that by $\varepsilon_в = 0$ the deformation of the array is equal to zero, we get:

$$C = 0; \;\; \varepsilon_z = \frac{1+\mu}{1-\mu}\varepsilon_в \tag{10}$$

To determine the stresses, it is necessary to solve equations (7), in which it is necessary to put:

$\sigma_x = \sigma_y$; $\varepsilon_x = \varepsilon_y = 0$ and ε_z, according to (10), equal to $\frac{1+\mu}{1-\mu}\varepsilon_в$;

$$\left.\begin{aligned}\frac{2\mu E}{1-\mu}\varepsilon_{в} &= L(\sigma_z)-2\mu L(\sigma_x);\\ E\varepsilon_{в} &= \mu L(\sigma_z)-(1-\mu)L(\sigma_x).\end{aligned}\right\} \tag{11}$$

From here, excluding $\varepsilon_{в}$, we get:

$$\frac{2\mu^2+\mu-1}{1-\mu}L(\sigma_z)=0;\ \ L(\sigma_z)=0 \tag{12}$$

The last equality means that

$$\sigma_z=0 \tag{13}$$

This conclusion could also be reached directly by noting that the outer surface of the soil mass is free from normal stresses σ_z; from the equilibrium condition of any layer of finite thickness, these stresses should be zero everywhere.

Given (13), we get from (11):

$$L(\sigma_x)=-\frac{E}{1-\mu}\varepsilon_{в} \tag{14}$$

or in expanded form: $\sigma_x(t)+\int\limits_{t_0}^{t}\sigma_x(\tau)K(t,\tau)d\tau=-\dfrac{E}{1-\mu}\varepsilon_{в}$

In the right part, all values can be functions of time.

Considering further the soil massif invariant in time, we determine the stresses $\sigma_x=\sigma_y$ from forced deformations $\varepsilon_{в}$ from equation (15), which receives water:

$$\sigma_x(t)+\int\limits_{-\infty}^{t}\sigma_x(\tau)K(t-\tau)d\tau=-\frac{E}{1-\mu}\varepsilon_{в}(t) \tag{16}$$

By replacing the independent variable τ under the sign of the integral by θ the formulas: $\tau=t-\theta;\ \ \theta=t-\tau,$

we get the law (16) in the following form:

$$\sigma_x(t)+\int\limits_{0}^{\infty}\sigma_x(t-\theta)K(\theta)d\theta=-\frac{E}{1-\mu}\varepsilon_{в}(t) \tag{17}$$

 https://doi.org/10.1051/e3sconf/202337603035

Suppose that the effect of forced deformations $\varepsilon_в$ in a soil massif is caused by the temperature T_0 on its surface, which varies over time according to an arbitrary periodic law, represented as a series:

$$T_o = \sum A_n \sin(\omega_n t + \varphi_n) \tag{18}$$

The temperature field inside the soil mass is determined, in addition to the boundary conditions (18), by the differential equation of thermal conductivity [20]:

$$\frac{\partial T}{\partial t} = \frac{1}{Rc}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{19}$$

where R – thermal resistance of the array;
c – its specific heat capacity.

Since the temperature at the boundary of the soil mass does not depend on the coordinates x and y , then inside the array it will change only in depth z . Therefore, partial derivatives of x and y will disappear in equation (19) and it will take the form:

$$\frac{\partial T}{\partial t} = \frac{1}{Rc}\cdot\frac{\partial^2 T}{\partial z^2} \tag{20}$$

Solution of equation (20) for the n-th term of the series (18) of surface temperature $z = 0$

$$T_{z=0} = T_0 = A_n \sin(\omega_n t + \varphi_n) \tag{21}$$

has the form:

$$T = A_n e^{-\lambda_n z} \sin(\omega_n t - \lambda_n z + \varphi_n), \tag{22}$$

where

$$\lambda_n = \sqrt{\frac{Rc\omega_n}{2}} \tag{23}$$

Thus, the temperature field in the array is expressed by the formula

$$T = \sum A_n e^{-\lambda_n z} \sin(\omega_n t - \lambda_n z + \varphi_n) \tag{24}$$

Then solving the problem according to the formula (17), in which $\varepsilon_в(t) = \alpha T(t)$ (α - is the coefficient of thermal expansion), for each member of the series (18) in the case of a steady temperature-stressed oscillatory process, we obtain:

$$\sigma(t)=\sum\sigma_n(z)\sin\left[\omega_n t+\psi_n(z)\right], \tag{25}$$

where $\sigma_n(z)$ - the amplitude of voltage fluctuations, and

$$\sigma_n(z)=\frac{\alpha E}{1-\mu}\cdot\frac{A_n e^{-\lambda_n z}}{\sqrt{\left[1+B(\omega_n)\right]^2+A^2(\omega_n)}} \tag{26}$$

or

$$\sigma_n(z)=\frac{\alpha E}{1-\mu}A_n e^{-\lambda_n z}\sqrt{\left[1-D(\omega_n)\right]^2+C^2(\omega_n)}; \tag{27}$$

$\psi_n(z)$ - the phase shift angle, and

$$\psi_n(z)=\varphi_n-\lambda_n z+arctg\frac{A(\omega_n)}{1+B(\omega_n)} \tag{28}$$

or

$$\psi_n(z)=\varphi_n-\lambda_n z+arctg\frac{C(\omega_n)}{1-D(\omega_n)} \tag{29}$$

$A(\omega_n),\ B(\omega_n),\ C(\omega_n)$ и $D(\omega_n)$ they are called sine and cosine transformations of the kernel $K(\theta)$ and its resolvents $R(\theta)$, and are expressed by formulas:

$$A(\omega)=\int_0^\infty K(\theta)\sin\omega\theta d\theta;\ B(\omega)=\int_0^\infty K(\theta)\cos\omega\theta d\theta;$$

$$C(\omega)=\int_0^\infty R(\theta)\sin\omega\theta d\theta;\ D(\omega)=\int_0^\infty R(\theta)\cos\omega\theta d\theta.$$

4 Conclusions and discussion

Usually forced deformations can be considered a given function of time $\varepsilon_в(t)$, then the equations will have only one independent variable – time. It is easy to see that a time-invariant medium whose forced deformations change according to a given law in time can be considered as a medium with variable properties. Therefore, the creep calculation methods described above can be applied to it, as well as to a non-invariant medium operating under constant or variable forced deformations. For an environment with unchanging properties, all dependencies must be time invariant. In this case, the

 https://doi.org/10.1051/e3sconf/202337603035

characteristics of the medium that depend on two variables t and τ turn into functions of the difference between these two variables $t-\tau$, and the functions t turn into constants.

The solution of the problem obtained above can be used to calculate stresses taking into account creep in a soil massif from the action of forced deformations caused by temperature fluctuations on its surface.

References

1. V.A. Florin, *Fundamentals of soil Mechanics* (Stroyizdat, M., 1959)
2. N.A. Tsytovich, *Soil Mechanics* (HSE, M., 1983)
3. S.B. Ukhov, V.V. Semenov, V.V. Znamensky et al, *Soil mechanics, foundations and foundations* (ASV, M., 1994)
4. E.K. Agakhanov, M.K. Agakhanov, R.E. Agakhanova, IOP Conference Series: Materials Science and Engineering **1001** (2020) https://iopscience.iop.org/issue/1757-899X/1001/1
5. E.K. Agakhanov, M.K. Agakhanov, Scientific review **12**, 67-71 (2016)
6. G.E. Agakhanov, Bulletin of Dagestan State Technical University **1**, 8-16 (2015)
7. G.E. Agakhanov, Naukovedenie. Electronic journal **7(3)** (2015) http://naukovedenie.ru/12TVN315.pdf
8. A.R. Rzhanitsyn, *Theory of creep* (Stroyizdat, M., 1968)
9. G.E.nAgakhanov, V.B. Melekhin, Scientific Review **4**, 90-93 (2016)
10. S.R. Meschyan, Proceedings of the Academy of Sciences of the Armenian SSR, Series of Physical and Mathematical Sciences (1965)
11. V.I. Samul, *Fundamentals of the theory of elasticity and plasticity* (V.Sh., M., 1982)
12. E.K. Agakhanov, M.K. Agakhanov, Izvestiya Vuzov. The North Caucasus region. Technical sciences **1**, 25-26 (2005)
13. S.P. Timoshenko, *Goodyear J. Theory of elasticity* (Nauka, M., 1975)
14. G.S. Vardanyan, V.I. Andreev, N.M. Atarov, A.A. Gorshkov, *Resistance of materials with the basics of the theory of elasticity and plasticity* (Publishing House of the DIA, M., 1995)
15. E.K. Agakhanov, Bulletin of Dagestan State Technical University. Technical sciences **2**, 39-45 (2013)
16. V.N. Savostyanov, E.K. Agakhanov, Izv. Vuzov, Stroitelstvo, Novosibirsk **10**, 26-30 (1995)
17. E.K. Agakhanov, M.K. Agakhanov, Industrial and civil construction **11**, 40-43 (2015)
18. G.E. Agakhanov, Bulletin of Dagestan State Technical University. Technical sciences **3**, 8-15 (2015)
19. E.K. Agakhanov, M.K. Agakhanov, Bulletin of MGSU **3**, 140-143 (2010)
20. G.L. Khesin, *The method of photoelasticity* (Stroyizdat, M., 1975)