Determination of the parameters of the curvilinear movement of vehicles

Nikolay Petrov[1*] and *Alexander* Abramov[1]

[1]Yaroslav-the-Wise Novgorod State University, 41, str. B. St. Petersburgskaya, 173003 Veliky Novgorod, Russia

Abstract. Today, the creation of new vehicles is impossible without computer modeling. Modern vehicles have numerous electronic systems that increase its traffic safety, comfort, and route planning management. Reliable operation of all systems is achieved by using an accurate dynamic model of the vehicle to assess safety and the possibility of movement along specified trajectories. Creating an accurate dynamic vehicle model requires correct initial parameters. When modeling the curvilinear movement of motor vehicles, such parameters as the coefficients of resistance to lateral sliding of tires and the angles of sliding of axles, which are mainly determined on test benches in laboratory conditions, are important to obtain the adequacy of the object under study. However, quite often it is impossible to obtain reliable information from tire manufacturers or testing laboratories, which ultimately affects the adequacy of modeling the parameters of vehicle movement. A method is proposed for determining the design parameters of tires using primary information sensors, which are sensors of angular and linear velocities, linear accelerations, rotation angles of controlled wheels and an on-board information processing device. The advantages of this method are that it allows real-time measurement of the angles of the axles and the coefficients of resistance to the lateral withdrawal of the axles when the vehicle is moving at an arbitrary speed along a trajectory of variable curvature. This method can be used to create an accurate dynamic model in the study of the controllability and stability of the vehicle, as well as an integral part of the vehicle dynamics control system.

1 Introduction

The creation of new vehicles is impossible without computer modeling [1]. Modern vehicles have numerous electronic systems that increase its traffic safety ABS, ESP, ACC, driving comfort, route planning [2-4], image processing [5] and autonomous driving soon. All modern systems are integrated and must ensure the safety and comfort of passengers.

Reliable operation of all systems is achieved by using an accurate dynamic model of the vehicle to assess safety and the possibility of movement along specified trajectories [6].

An accurate dynamic vehicle model allows you to get many different advantages when developing an autonomous vehicle [7]. One of the advantages is that it can be used to determine whether a trajectory is feasible or not. Another advantage is the ability to include

* Corresponding author: fenrir09214576@gmail.com

a dynamic model in the simulation environment to test how the vehicle will behave under a variety of conditions [8-11]. Modeling requires qualified specialists, but it is less expensive and much safer than testing a car on real road conditions.

When modeling the curvilinear motion of motor vehicles, such parameters as tire side slip resistance coefficients K_δ and axle slip angles, which are mainly determined on test benches in laboratory conditions, are important for obtaining adequacy to the object under study. However, quite often it is not possible to obtain reliable information from tire manufacturers or testing laboratories, which ultimately affects the adequacy of the simulation of motor vehicle motion parameters.

Over the years, various tire models have been developed that provide different accuracy. These include linear tire models [12], flexible model [13] and Dugoff model [14], finite element models [15] and empirical models based on data [16]. For simple tasks, sometimes linear tire models are sufficient, for complex tasks a more accurate model is required. For complex tasks, such as controlling the trajectory of an autonomous vehicle, it is necessary to have more accurate models that ensure that the vehicle controller performs the best possible work [17]. Usually, these models are developed in specialized testing centers that can conduct controlled tests of car tires. Unfortunately, these tests are expensive and time-consuming, while leading to models that may not be accurate enough in real road conditions. One of the main limitations of testing at test facilities is that any data outside the measurement range can only be extrapolated in an approximate form. Another major limitation is that rolling tapes or drums may not correspond to real roads.

Measurements on the road can lead to better practical results but require sophisticated measuring equipment (Measurement System) Fig. 1. [9].

Fig. 1. Measurement System at the Waterloo Test Track: M. Van Gennip, Vehicle Dynamic Modelling and Parameter Identification for an Autonomous Vehicle. Master of Applied Science, University of Waterloo, December 2018 [9].

A method is proposed for determining the design parameters of tires using primary information sensors, which are sensors of angular and linear speeds, linear accelerations, angles of rotation of the steered wheels and an on-board information processing device.

The advantages of this method are that it allows one to measure in real time the slip angles of the axles and the coefficients of resistance to side slip of the axles when the motor vehicle moves at an arbitrary speed along the trajectory of variable curvature.

This method can be used not only in the study of controllability and stability of the vehicle, but also as an integral part of the vehicle dynamics control system.

 https://doi.org/10.1051/e3sconf/202337604028

2 Methods

To solve the problem associated with determining the slip angles and coefficients of resistance to side slip of the axle wheels, it is sufficient to have a flat model of the movement of the motor vehicle [18,19]. When choosing a design scheme for the movement of an automatic telephone exchange, a number of assumptions are made to obtain a simpler form of the mathematical model [20, 21]:

- motor vehicles have three degrees of freedom: linear movements in the longitudinal and transverse directions in the plane of the road and the rotation of the vehicle about a vertical axis passing through its center of gravity;
- each of the two wheels of the axle is loaded with the same forces (normal, tangential and transverse);
- all wheels of each axle have the same slip angles and the same slip resistance coefficients;
- the frame of the vehicle is assumed to be absolutely rigid;
- the position of the steered wheels is uniquely determined by the position of the steering wheel.

To draw up a design scheme for the curvilinear movement of the vehicle, it is assumed that the origin of the moving coordinate system is located at the center of gravity of the vehicle. The positive direction of the X-axis coincides with the direction of movement of the vehicle, and the Y-axis is directed towards the instantaneous center of rotation. The positive rotation of the vehicle in the plane of the road is taken to be clockwise rotation, when viewed from the origin of the Z-axis.

To compile differential equations of motion of a motor vehicle, the equations of motion of a rigid body are used. For a plane motion of a vehicle, the system of differential equations can be written in the following form

$$m \cdot j_x = \sum P_x ;$$

$$m \cdot j_y = \sum P_y ;$$

$$J_z \cdot \dot{\omega}_z = \sum M_z \tag{1}$$

where:

m is the vehicle mass;

J_z is the moment of inertia of the vehicle, relative to the vertical axis of rotation Z, passing through the center of gravity of the vehicle;

J_x, J_y are the projections of linear accelerations of the center of gravity of the vehicle on the X and Y axes;

$\dot{\omega}_z$ is the angular acceleration of the vehicle in the plane of the road;

$\sum P_x$, $\sum P_y$ are the sum of the projections on the X and Y axes of all forces acting on the vehicle;

$\sum M_z$ is the the sum of the moments about the Z axis acting on the vehicle.

The design scheme of a two-axle automatic telephone exchange is shown in Fig. 2.

 https://doi.org/10.1051/e3sconf/202337604028

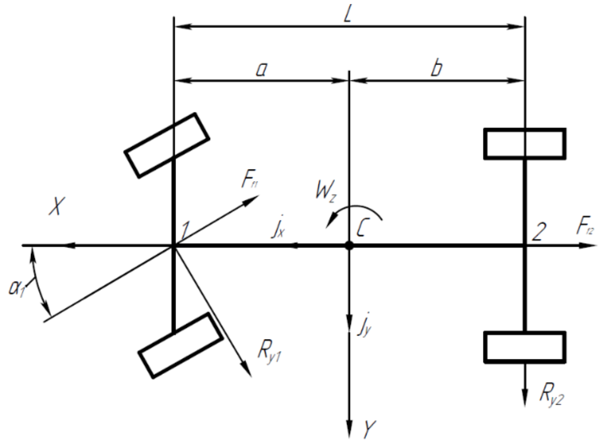

Fig. 2. Design model of the vehicle.

To simplify the differential equations of motion of the motor vehicle, it is assumed that the rolling resistance forces P_f and the stabilizing moments of the tires M_c are small compared to the lateral reactions of the wheels or can be neglected [3], i.e.

$$P_{f1} = P_{f2} = 0; \quad M_{c1} = M_{c2} = 0.$$

Then the equations for the plane motion of a two-axle motor vehicle will have the form

$$m \cdot R_y = R_{y1} \cdot cos\,\alpha + R_{y2};$$

$$J_z \cdot \omega_z = a \cdot R_{y1} \cdot cos\,\alpha - b \cdot R_{y2}. \tag{2}$$

Lateral reactions of the wheels are proportional to the slip angles

$$R_i = K_{\delta_i} \cdot \delta_i \tag{3}$$

where *i*=1,2 – wheel indexes of the front and rear axles of the vehicle.

Using equation (3), equation (2) can be written in the following form

$$m \cdot j_y = K_{\delta_1} \cdot \delta_1 \cdot cos\,\alpha + K_{\delta_2} \cdot \delta_2;$$
$$J_z \cdot \dot{\omega}_z = a \cdot K_{\delta_1} \cdot \delta_1 \cdot cos\,\alpha - b \cdot K_{\delta_2} \cdot \delta_2. \tag{4}$$

With turning radii $R \geq 15$ м, the angle of rotation of the steered wheels of the vehicle is $\alpha \leq 15^0$. In this case, we can assume that $cos\,\alpha \approx 1$. Then the equation (4) takes the following form:

$$m \cdot j_y = K_{\delta_1} \cdot \delta_1 + K_{\delta_2} \cdot \delta_2;$$
$$J_z \cdot \dot{\omega}_z = a \cdot K_{\delta_1} \cdot \delta_1 - b \cdot K_{\delta_2} \cdot \delta_2. \tag{5}$$

To determine the coefficients of resistance to side slip K_{δ_1} and $K_{\delta 2}$ in the steady state of motion, it is necessary that the motor vehicle under test move with a constant turning radius R and a constant speed V_x, i.e.

$$R = const; \qquad V_x = const.$$

With steady circular motion, i.e. when $\dot{\omega}_z = 0$, the system of equations (5) will be written in the following form:

$$\begin{aligned} m \cdot j_y &= K_{\delta_1} \cdot \delta_1 + K_{\delta_2} \cdot \delta_2; \\ O &= a \cdot K_{\delta_1} \cdot \delta_1 - b \cdot K_{\delta_2} \cdot \delta_2. \end{aligned} \tag{6}$$

Having solved the system of equations (6), it is possible to obtain formulas for calculating the coefficients of resistance to side slip of the front and rear axles of the vehicle

$$K_{\delta 1} = \frac{b \cdot m \cdot j_y}{L \cdot \delta_1}; \tag{7}$$

$$K_{\delta 2} = \frac{a \cdot m \cdot j_y}{L \cdot \delta_2}. \tag{8}$$

The slip angles of the wheels of the axles, according to the kinematic scheme of the movement of the vehicle on the turn (Fig. 3), are determined by the following expressions

$$(\alpha - \delta_1) = arctg \frac{V_{y_1}}{V_x} \approx \frac{V_{y_1}}{V_x}; \tag{9}$$

$$\delta_2 = arctg \frac{V_{y_1}}{V_x} \approx \frac{V_{y_2}}{V_x} \quad . \tag{10}$$

where V_{y_1} and V_{y_2} are the projections of the transverse speeds of the middle of the front and rear axles of the vehicle.

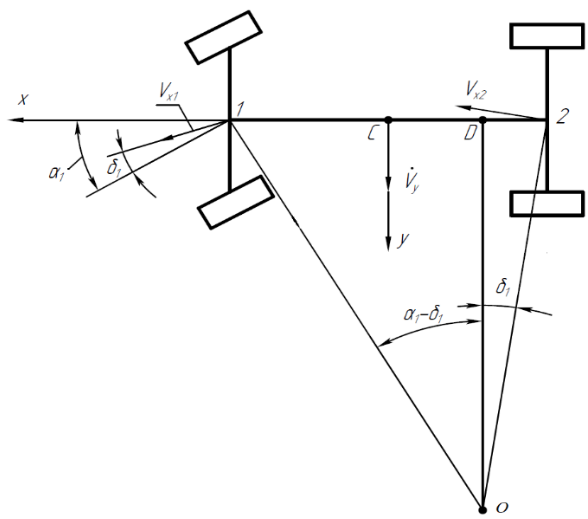

Fig. 3. Kinematic design model of a vehicle.

 https://doi.org/10.1051/e3sconf/202337604028

Projections of lateral accelerations can be determined from the equations of accelerations of these points [1].

For the front axle

$$\dot{V}_{y_1} = j_{y_1} + V_x \cdot \omega_z. \quad (11)$$

For the rear axle

$$\dot{V}_{y_2} = j_{y_2} - V_x \cdot \omega_z. \quad (12)$$

By integrating equations (11) and (12), one can obtain expressions for determining the lateral speeds of the middle of the front and rear axles of a vehicle, and by substituting equations (9) and (10) one can determine the slip angles of the wheels of the axles of a two-axle vehicle

$$\delta_1 = \frac{V_{y1}}{V_x} - \alpha; \quad (13)$$

$$\delta_2 = \frac{V_{y2}}{V_x}. \quad (14)$$

3 Results

According to mathematical expressions (7), (8), (11), (12), (13) and (14), a scheme for determining the coefficients of resistance of tires of axles to side slip K_{δ_1} and $K_{\delta 2}$, as well as a scheme for determining the slip angles of axles δ_1 and δ_2. As an element base for the implementation of the obtained mathematical expressions, digital integrated circuits are used.

Figure 4 shows a block diagram for determining the slip angles of the axes δ_1 and δ_2, compiled according to equations (13), (14), (11) and (12). This scheme allows one to determine the slip angles of the axles in any modes of motion of the vehicle, including transient modes.

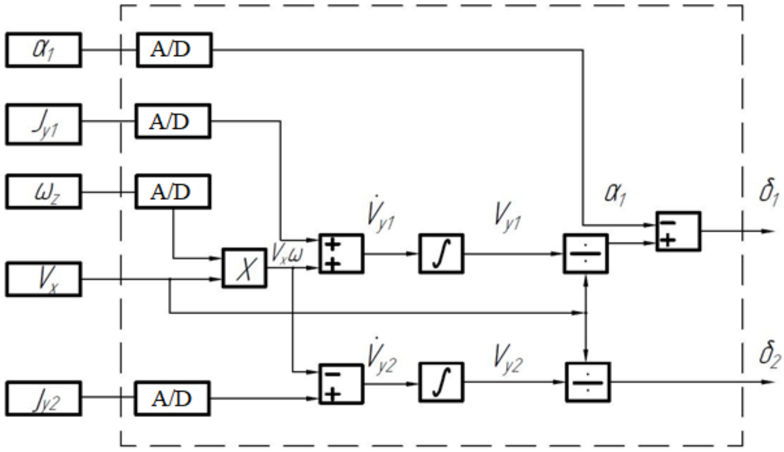

Fig. 4. Block diagram for determining the slip angles of the axes δ_1 and δ_2, A/D – analog to digital converter.

 https://doi.org/10.1051/e3sconf/202337604028

The input parameters of the circuit are the lateral accelerations of the axes j_{y_1} and j_{y_2}, vehicle angular velocity ω_z and linear velocity V_x. Accelerations j_{y_1} and j_{y_2} are determined using acceleration sensors installed on the front and rear axles of the vehicle in the transverse direction. The angular velocity ωz is determined using an angular velocity sensor installed on the longitudinal axis of the vehicle. Linear speed Vx is determined using the vehicle's electronic tachometer.

The block diagram for determining the slip angles of the axes δ_1 and δ_2, built on the basis of analog microcircuits, consists of three summing and two integrating amplifiers, a multiplication unit and two division units.

Fig. 5 shows a block diagram for determining the slip resistance coefficients of the axles K_{δ_1} and K_{δ_2}. Using this scheme, it is possible to determine K_{δ_1} and K_{δ_2} only for steady circular motion, i.e. under the condition $V_x = const,\ R = const,\ \omega_z = const.$

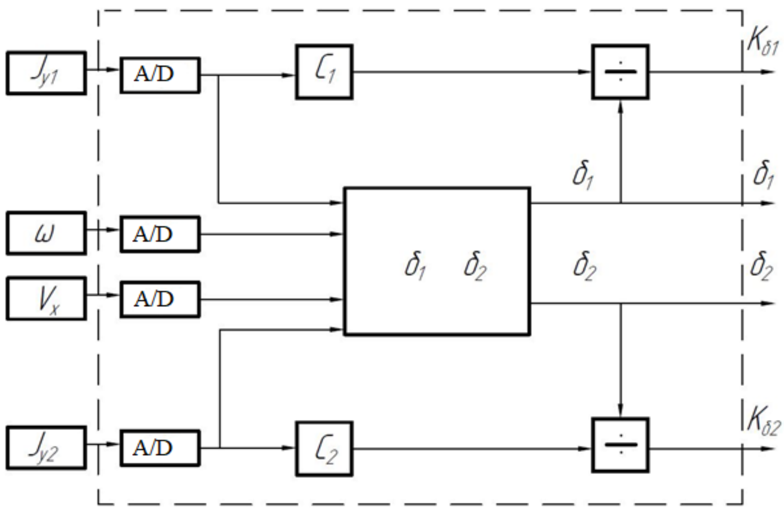

Fig. 5. Block diagram for determining the slip resistance coefficients of the axles K_{δ_1} and K_{δ_2}, A/D – analog to digital converter.

In this scheme, a block is used to determine the slip angles of the axes δ_1 and δ_2 shown in Fig. 3 and in addition, two division blocks and two multiplication blocks by constant coefficients C1 and C2 are used, equal to:

$$C_1 = \frac{\text{в} \cdot m}{L}, \tag{15}$$

$$C_2 = \frac{a \cdot m}{L}. \tag{16}$$

4 Conclusions

The proposed method makes it possible to measure in real time the side slip resistance coefficients K_δ of tires and the slip angles of the axles using primary information sensors, which are angular and linear velocity sensors, linear acceleration sensors, steered wheel rotation angle sensors and an on-board computer.

The parameters obtained experimentally can be used in a high-precision dynamic model of a vehicle for conducting computer test tests of systems controlling dynamics and trajectory movement.

The information obtained can be used both in the study of the controllability and stability of the motor vehicle, and as an integral part of microprocessor control systems for vehicle dynamics.

5 Future Work

The method of measuring tire parameters confirmed by experiment is planned to be adapted for a road train consisting of a tractor and a semi-trailer. The dynamic model of the road train will be used to work out automatic parking algorithms. During the simulation, various parking scenarios will be analyzed. Autonomous driving requires the creation of software for navigation and vehicle movement planning using numerous data from an array of simulated sensors. These inputs can then be incorporated into a complete dynamic vehicle model that will determine how the vehicle will actually move.

In the future, it will be possible to develop a specific scenario to verify the adequacy of the simulated system. A specific scenario can be implemented with a specific vehicle in order to compare the accuracy of the simulated object with real results.

References

1. Z. Cai, S. Chan, X. Tang et al. *Proceedings of the FISITA 2012 world automotive congress* (Springer, Berlin, 2012)
2. *Sicherheits- und Komfortsysteme* (Wiesbaden, Vieweg, 2004)
3. Winner Hermann, *Handbuch Fahrerassistenzsysteme : Grundlagen, Komponenten und Systeme fur aktive Sicherheit und Komfort* (Wiesbaden, Vieweg: Teubner, 2009)
4. Heissing Bernd, *Fahrwerkhandbuch: Grundlagen, Fahrdynamik, Komponenten, Systeme, Mechatronik, Perspektiven* (Wiesbaden, Vieweg: Teubner, 2008)
5. Y. Zhong, S. Wang, S. Xie et al, SAE Int. J. Passeng. Cars- Electron. Electr. Syst. **11(1)**, 48-56 (2018) https://doi.org/10.4271/07-11-01-0005
6. S. Bacha, M. Ayad, R. Saadi, et al, 5th International Conference on Electrical Engineering - Boumerdes (ICEE-B), Boumerdes, 1-12 (2017) https://doi.org/10.1109/ICEE-B.2017.8191998
7. B. Zhu, S. Liu, J. Zhao, SAE Technical Paper **2018-01-0599** (2018) https://doi.org/10.4271/2018-01-0599
8. M. Harrer, T. Stickel, P. Pfeffer, *Automatisierung fahrdynamischer Messungen. 10. Internationaler Kongress Reifen-Fahrwerk-Fahrbahn* (Hannover, 2005)
9. M. Van Gennip, *Vehicle Dynamic Modelling and Parameter Identification for an Autonomous Vehicle* (University of Waterloo, 2018)
10. A. Frings, D. Ewbank, *12th International Munich Chassis Symposium 2021 Proceedings* (Springer Vieweg, Berlin, Heidelberg, 2022) https://doi.org/10.1007/978-3-662-64550-5_4
11. S. Na, J. Jang, K. Kim, W. Yoo, Advances in Mechanical Engineering **7(11)** (2015) doi:10.1177/1687814015618126
12. R. Rajamani, *Vehicle Dynamics and Control* (Springer, 2006)

13. H. Yamashita, Y. Matsutani, H. Sugiyama, ASME. J. Comput. Nonlinear Dynam. **10(3)** (2015) https://doi.org/10.1115/1.4028335
14. L. Chen, M. Bian, Y. Luo, K. Li, International Conference on Mechanical and Automation Engineering, Jiujang, 56-61 (2013) https://doi.org/10.1109/MAEE.2013.24
15. X. Yang, O. Olatunbosun, E. Bolarinwa, SAE Int. J. Mater. Manuf. **3(1)**, 211-220 (2010) https://doi.org/10.4271/2010-01-0418
16. H.B.Pacejka, *Tyre and Vehicle Dynamics. 3nd edition* (Butterworth–Heinemann, Oxford, 2012)
17. R. Zhang, SAE Technical Paper 2016-01-1871 (2016) https://doi.org/10.4271/2016-01-187
18. P. Pfeffer, H.H. Braess, *Basics of Lateral Vehicle Dynamics. Steering Handbook* (Springer, Cham, 2017) https:// doi.org/10.1007/978-3-319-05449-0_5
19. M. Harrer, P. Pfeffer, *Fahrdynamische Grundlagen. Lenkungshandbuch. ATZ/MTZ-Fachbuch* (Springer Vieweg, Wiesbaden, 2013) https://doi.org/10.1007/978-3-658-00977-9_5
20. J. Wong, Theory of ground vehicles (Mechanical Engineering, M., 1982)
21. M. Mitschke, H. Wallentowitz, *Dynamik der Kraftfahrzeuge* (Springer Vieweg, Wiesbaden, 2014) https://doi.org/10.1007/978-3-658-05068-9_15