Effect of Hydro-Mechanical Hysteresis on Active Earth Pressure for Unsaturated Soils

Bestun Shwan

Department of Geotechnical Engineering, Faculty of Engineering, Koya University, Koya KOY45, Kurdistan Region-F.Iraq.
Email:bestun.shwan@koyauniversity.org or bjnshwan1@gmail.com

Abstract. Stability analysis of retaining structures carried out at unsaturated conditions mainly dismissed the effect of hydraulic and mechanical hysteresis on the exerted active thrust. This paper reports a parametric study into the influence of hydraulic and mechanical hysteresis as well as soil types on active earth pressure carried out at low and high suction profiles to envision the pattern of the exerted active thrust for various simulated soils, from sands to clays. Generally, the numerical results exhibited a non-linear relationship between active thrust and suction. The influence of hydraulic and mechanical hysteresis and soil types on the active thrust was seen very significant. A reduction in P_a of 1.62 fold at s= 137.34 kPa, owing to the influence of hydraulic and mechanical hysteresis, was obtained when comparing two simulated soils; silt and clay. A soil with a wider capillarity range showed a higher positive effect of the hydraulic and mechanical hysteresis on the exerted active thrust.

1 Introduction

Stability of the earth structures where the backfill materials are assumed dry and saturated conditions has been addressed broadly and successfully applied to the engineering practice. On the other hand, under unsaturated conditions the design requires special considerations. Backfill materials inevitably encounter various saturation conditions, attributed to the climate variability, drying and wetting cycles. Consequently, the exerting earth pressures on the retaining structures, which depend on the saturation state, change accordingly. Therefore, under such circumstances, robust approaches are necessary for a proper design. Such circumstances are mainly attributed to the influence of two stress state variables, suction (s) and degree of saturation (S_r), which impart additional strength to the soils/backfill materials, i.e. less exerted active thrust. Stability of the geotechnical problems (owing to the stress state variables), therefore, has been significantly addressed where many constitutive models have been suggested, e.g. [1-5].

A non-unique relationship between these two stress state variables has been characterised and recognised by the so-called soil water retention curve (SWRC), see Fig. 1a. The relationship identifies several important features such as air entry value and residual suction. These characteristics significantly influence the soil behaviour. Therefore, it has been common practice to correlate SWRC with the mechanical characteristics (shear strength) of unsaturated soils, [6]. Since then, studies have been reported in the literature where strength characterisation was estimated/determined based on SWRC, [7-10].

Figure 1a plots various experimental SWRCs for different types of soil, available from the literature with further details listed in Table 1. The SWRCs displayed various slopes indicating different capillarity (suction) ranges. In other words, the soils exhibited various water retaining capacity at a specific suction value. This was fundamentally attributed to different soil types, i.e. the finer the soil type, the wider the capillarity range.

Figure 1b shows two SWRCs for a soil. Suction and degree of saturation array is bounded by the drying and wetting curves. Clearly on the drying path, the amount of water retained in the soil pores at a constant suction value is higher than on the wetting path. This phenomenon is widely known as hydraulic hysteresis.

In addition, SWRC is stress-state (mechanical) and void ratio dependent, [11]. The relationship between suction and degree of saturation, therefore, is not unique (as stated previously) as it changes owing to any change in stress application or void ratio. With any change of the mechanical characteristics, the position of the curve changes/shifts. This is another SWRC characteristic known as mechanical hysteresis.

Figure 2 shows the available SWRCs on the drying path, after [11], for a soil at three initial void ratio (e) values. The curves clearly exhibited different trends, owing to the mechanical and void ratio dependency. In other words, the curve with a smaller void ratio exhibited a higher capillary (suction) range. This was attributed to mechanical hysteresis.

Hydraulic and mechanical hysteresis have significant effects on the soil behaviour and effective stress, [12]. Consequently, their influences on the geotechnical applications such as retaining structures are of great interest to the engineering practice. To account for the effects of hydraulic and mechanical hysteresis phenomena on active earth pressure, an equation originally proposed by [13] as shown below is utilised:

$$S_r = e^{-\kappa(s-AEV)} \quad (1)$$

where κ is a fitting parameter (kPa^{-1}) accounts for the hydraulic and mechanical hysteresis, discussed later, *AEV* is the air entry value (kPa).

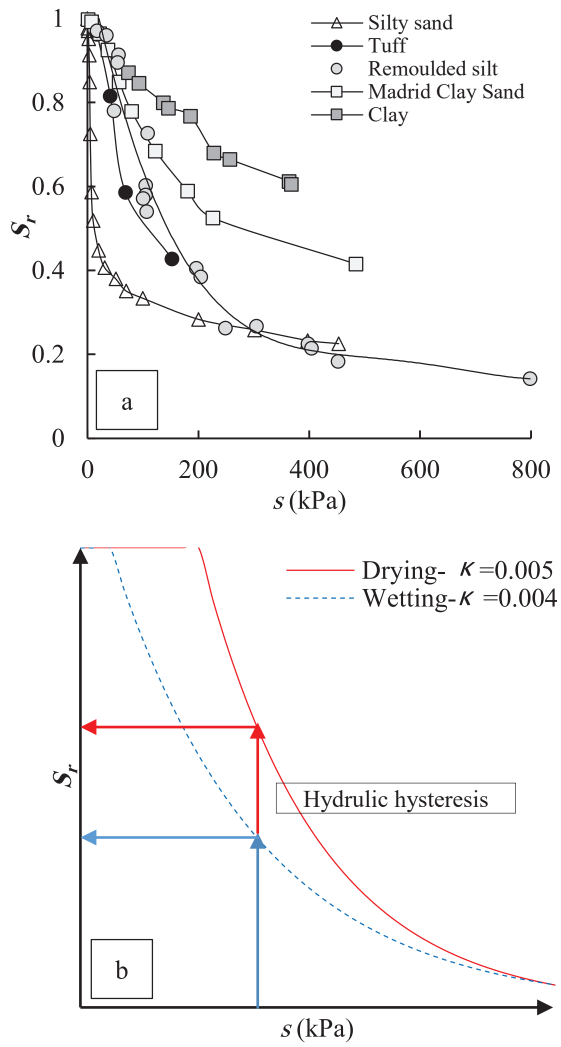

Fig. 1. (a) WRCs for various soils available from the literature and (b) Hydraulic hysteresis of the SWRC.

Table 1: Soil types referred in Fig. 1.

Authors	Soil type
Lee et al. [14]	Silty sand
Fredlund et al. [15]	Tuff
Geiser et al. [16]	Remolded silt
Krishnapillai, S.H. and Ravichandran [17]	Madrid clay sand
Vanapalli et al. [18]	Clay

Figure 3a plots a range of SWRCs, alongside with the experimental SWRCs shown previously in Fig. 1a, using different κ values and a constant *AEV*= 25 kPa.

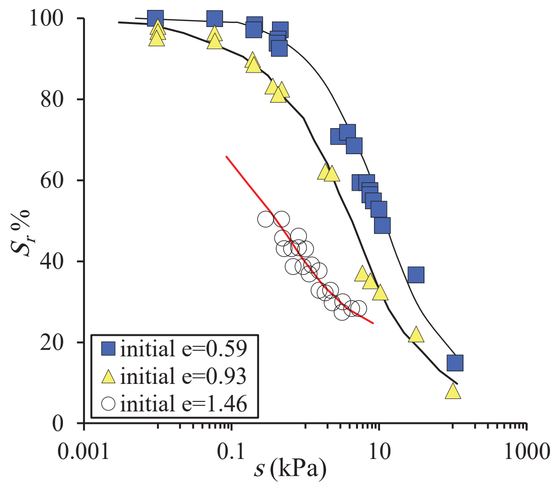

Fig. 2. Mechanical hysteresis of a soil, after [11].

The simulated SWRCs can represent different soil types at various suction ranges, covering the range of the silty sands to clays, see Fig. 3a. Clearly the smaller κ value, the wider the capillarity range. In other words, beyond the *AEV*, the particles exhibited different trends of repartition of the amount of water in the soil pores. SWRCs with higher κ values, 0.1 to 0.04, revealed a sudden decrease in degree of saturation once the *AEV* was exceeded, only microscopic water remained. These soils, therefore, could represent fine sands with/without a small amount of silt, attaining their residual saturation at a suction value less than 200 kPa, [6].

In addition, Fig. 3b plots two different SWRCs. Implicitly, the different SWRCs plotted in Fig. 3a and b using Eq. 1 were identical to what was obtained experimentally in Fig. 2. That is, repartitions of the amount of water in the soil pores were different. Thus, the parameter κ can be utilised to represent/characterise different soil types. It depends on microstructure and cycles of wetting and drying, [19]. The wetting and drying cycles, Fig. 1b, obtained at different values of κ to represent in a simple way hydraulic hysteresis. Additionally, the parameter κ can also characterise any change/shift in SWRC due to the mechanical (stress-state) behaviour. Application of the parameter κ to characterise hydraulic and mechanical hysteresis was further studied in tunnel and bearing capacity problems by Shwan, see [20-21].

Backfill materials can encounter different suction profiles. This is attributed to various soil types (generally coarse soils have narrow suction ranges as shown in Fig. 3a), or due to cycles of wetting (low suction, LS) or drying (high suction, HS) for fine soils. In this study, hydraulic and mechanical hysteresis effects were investigated at LS and HS ranges. The shaded area in Fig. 3a, represented LS and HS ranges for the fine and coarse soils. This is to ensure the transition range from full saturation up to the *AEV* as well as the range beyond the *AEV* for the fine soils and the full range for the coarse soils. The implication of the LS range in Fig. 3a is that there are many situations where backfill materials, especially fine, can be encountered within the LS range. Consequently, the exerted active thrust changes significantly.

 https://doi.org/10.1051/e3sconf/202338202006

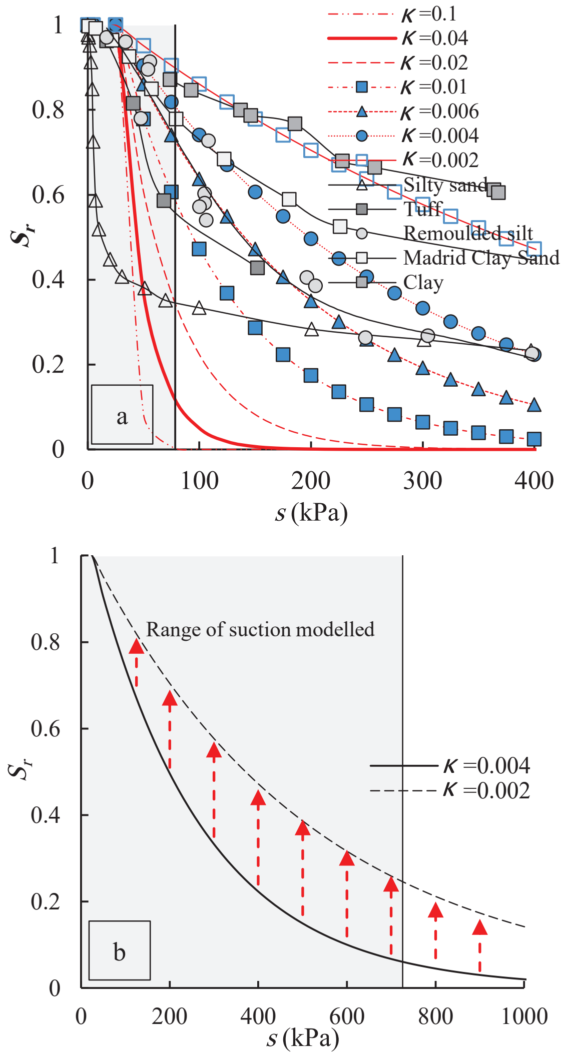

Fig. 3. (a) WRCs for various soils fitted using Eq.1 at various κ values and plotted against available SWRCs and (b) SWRC of two different soils considering hydraulic and mechanical hysteresis.

Additionally, it was also intriguing to envision the hydraulic and mechanical hysteresis effect on active earth thrust at high suction (HS) range for finer soils (fitted with κ=0.004 and 0.002 in Fig. 3b). Therefore, the numerical analysis was extended up to a suction value of about 726 kPa (shaded area in Fig. 3b). This magnitude represented almost a value beyond or at the residual suction for the two simulated soils shown in Fig. 3b. It is obvious that SWRCs with κ = 0.002 still was able to retain more than about 25% of its water, apart from SWRCs fitted with κ=0.004 (attended its residual saturation earlier). This implied various behaviour and therefore different exerted active thrust, shown later.

The aim of this paper is, therefore, to investigate the effect of hydraulic and/or mechanical hysteresis as well as soil types on the exerted active thrust using a modified upper bound theorem approach. A parametric study is carried out for a frictionless wall modelled in the analysis at two different suction profiles: low and high using different simulated soils.

2 Problem geometry

A frictionless wall of 4 m height with a soil domain of 4×4 m was selected for the analysis as shown in Fig. 4. The modelled boundary conditions were sufficient to prevent boundary restrictions, see the developed failure mechanisms in Fig. 4. Soil above the water table was assumed to be at unsaturated conditions.

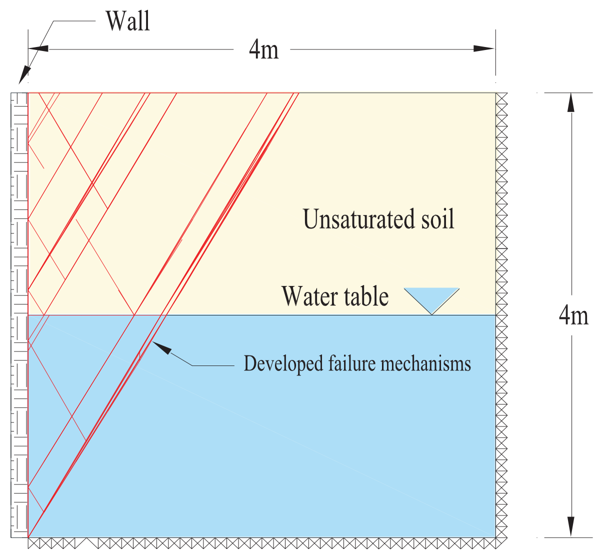

Fig. 4. Problem geometry for the modelled retaining wall.

The numerical analysis was carried out using a research version of the LimitState:GEO software, modified by the author. The modification was by inclusion of the effect of unsaturation conditions on strength into the original version of the software, see [20]. The modified version (**UNSAT-DLO**) approach was utilised to carry out a parametric study at different suction profiles for various simulated soil types as stated previously to investigate the effects of hydraulic and mechanical hysteresis on active earth pressure. The required parameters utilised in the analysis for the simulated soils are shown in Table 2.

Table 2. Parameters used in the **UNSAT-DLO** approach and Fredlund and Rahardjo equation, [22].

γ_{sat} (kN/m^3)	γ_{unsat} (kN/m^3)	c' (kPa)	ϕ (Degrees)	AEV (kPa)
20	17.5	0	30	25

3 Numerical results

First, it was necessary to validate the **UNSAT-DLO** results. Owing to a lack of experimental data for the active earth pressure problems at unsaturated conditions, the **UNSAT-DLO** approach was validated against an active earth pressure equation accounting for the suction effect. The equation, which is based on limit equilibrium method, was proposed by [22]:

$$P_a = \left[\frac{1}{2} k_a \gamma' H^2 - 2c'\sqrt{k_a} H - 2(u_a - u_w)\tan\phi^b \sqrt{k_a}\; H\right] \quad (2)$$

where P_a is the exerted effective active thrust, k_a is the active lateral earth pressure coefficient, γ' is the effective unit weight, H is the height of the wall, c' is the effective cohesion, (u_a-u_w) is the suction and ϕ^b is the angle of shearing resistance with respect to suction change . As ϕ^b is a variable parameter and considerably affects by suction, a series of ϕ^b values (0, 5, 10 and 15°) were used in the simulation. Apparently, ϕ^b may account for κ in Eq. 1. In the validation, a simulated soil, fitted with κ=0.04, represented a coarse soil (sand), see Fig. 3a.

The validation results were shown in Fig. 5. The negative values of the active earth thrust represented the case where the wall exerted no force on the wall. The **UNSAT-DLO** result exhibited a non-linear trend which was bounded between the linear lines obtained from Eq. 2 for ϕ^b= 0 kPa (fully saturated) and ϕ^b=10°. In addition, the **UNSAT-DLO** approach was capable of identifying the desaturation characteristics (the active thrust value was levelled off and then increased, e.g. capturing the increase in P_a after a specific suction value. This was attributed to the desaturation-induced strength reduction. This minimum obtained P_a was 9.60 kN at s= 58.86 kPa, before the residual suction state (see Fig. 3a- κ=0.04). Here, the residual suction was defined as a tangent of the last part of the SWRC of κ=0.04 in Fig. 3a, assumed ≈80 kPa. Obviously; Eq. 2 was unable to specify the desaturation characteristics of the simulated soil.

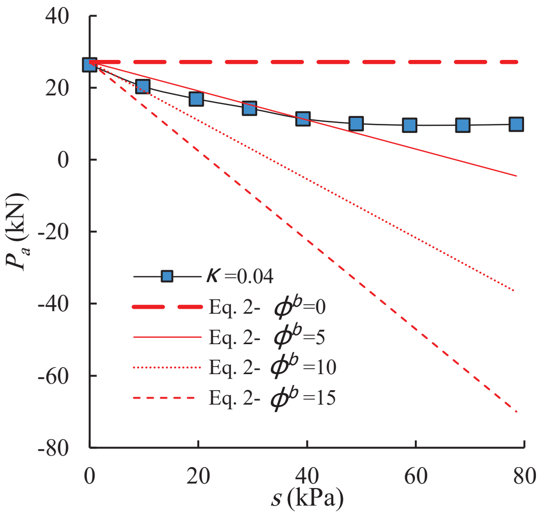

Fig. 5. Validation of the **UNSAT-DLO** approach against Eq. 2.

The results of the **UNSAT-DLO** approach obtained at two suction levels (LS and HS-explained previously by the shaded areas in Fig. 3a and b) and at various κ values were shown in Fig. 6a and b. In addition, results of Eq. 2 were also plotted. The effect of different types of soil was plotted in Fig. 6a which represented soils from sand (κ = 0.04) to clay (κ = 0.002). While, Fig. 6b represented the two simulated soils shown in Fig. 3b that accounted for the hydraulic and/or mechanical hysteresis.

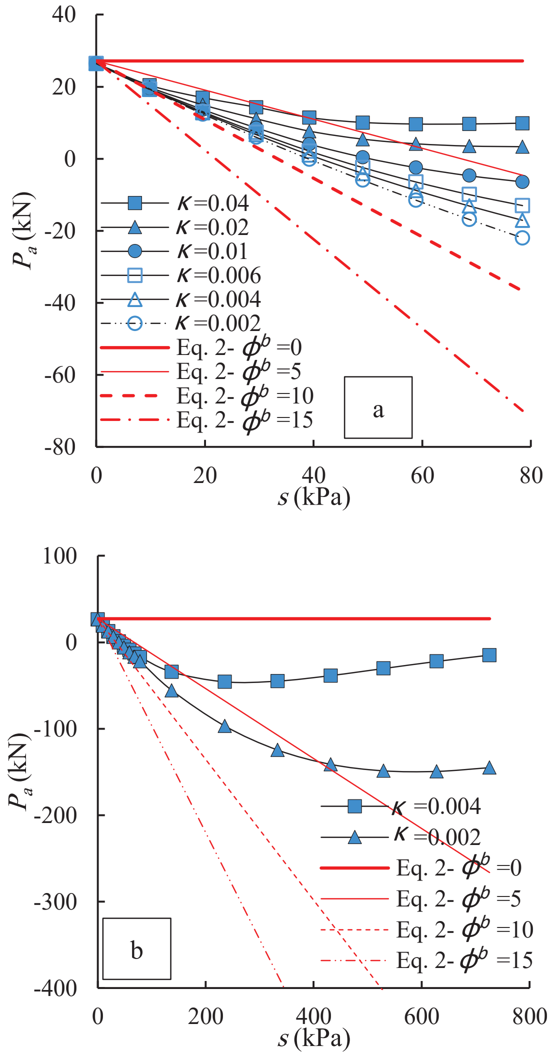

Fig. 6. The effect of: (a) soil types and (b) the hydraulic and/or mechanical hysteresis on the active thrust, P_a.

Generally, the sand soil exerted higher P_a on the wall when compared with clay soil, the latter revealed negative P_a values. A significant effect of soil types on P_a was obtained. For example, a decrease of 10.8 fold in P_a at s= 39.24 kPa was obtained when comparing κ =0.004 (almost silt) with κ=0.04 (sand). In addition, the higher κ values, represented narrower capillarity ranges and exhibited non-linear trends then levelled off beyond a specific applied suction, i.e. s ≈ 60 kPa apart from the smaller κ values. This was attributed to, as also explained previously, stability reduction induced desaturation occurred as the particles were unable to retain water and a sudden decrease in suction happened (soils lost a significant amount of their water content, see Fig. 3a- κ=0.04, 0.02 and 0.01). However, the trends were almost monotonic for the smaller κ values, e.g. κ =0.006 and smaller. This was mainly attributed to the fact that within the modelled suction range (s= 78.48 kPa), soils were able to retain a significant amount of their water, e.g. at least more than 74% of degree of saturation was retained for κ =0.004. The other κ values even showed a higher retained amount of water. Thus the desaturation state was not attended.

In addition, the influence of the hydraulic and/or mechanical hysteresis shown in Fig. 6b for the two simulated soils (almost silt and clay based on the best fit shown in Fig. 3a) was considerable. The overall trend was non-linear for the modelled HS range. A reduction in P_a of 1.62 fold at s= 137.34 kPa was obtained when comparing κ =0.002 with κ =0.004. The decrease in P_a was seen even higher at a higher suction, 4.91 fold at s= 529.74 kPa. Then, P_a was seen to increase when suction increased (when a significant amount of degree of saturation was expelled owing to the higher applied suction). Another effect of the hydraulic and/or mechanical hysteresis can be seen from Fig. 6b was that P_a for the soil with the narrower capillarity range (κ =0.004) levelled off and increased far before the other soil. For example, at suction of about 530 kPa, P_a for κ =0.002 was less by about 4.91 fold when compared with κ =0.004. Finally; the results of Eq. 2 were higher than the **UNSAT-DLO** results for ϕ^b =10 and 15°, even the result of ϕ^b =5° was not in good agreement.

4 Conclusions

A modified upper bound theorem that takes into consideration the influence of suction stress on strength, **UNSAT-DLO** approach, was utilised to carry out a parametric study into the influence of soil types (represented simulated soils from sands to clays) and hydraulic and/or mechanical hysteresis on active earth pressure (P_a). A frictionless wall of 4 m height was modelled in the analysis at two different suction profiles: low and high. The results of the approach were first validated against an active thrust equation available from the literature.

The numerical results exhibited a non-linear relationship of the obtained active thrust against suction. The effect of different types of soil was significant. Under unsaturated conditions, clays exhibited less active thrust than sands. Comparing κ =0.004 (silt) with κ =0.04 (sand), a reduction of 10.8 fold in P_a at s= 39.24 kPa was obtained.

The influence of hydraulic and/or mechanical hysteresis on P_a was also seen to be very substantial. A non-linear relationship between P_a versus suction was obtained for the range of the suction profile modelled. A decrease of 1.62 fold in P_a at s= 137.34 kPa was obtained when comparing two simulated soils, clay and silt. The **UNSAT-DLO** approach, therefore, can be utilised as a robust approach for a proper design under unsaturated conditions for retaining structures, taking into account hydraulic and mechanical hysteresis and diverse soil types.

References

1. Gens, A. and Alonso, E.E., “A framework for the behaviour of unsaturated expansive clays,” *Canadian Geotechnical Journal,* vol. 29, no. 6, pp. 1013-1032, (1991).
2. S. Wheeler, “An alternative framework for unsaturated soil behaviour,” *Géotechnique,* vol. 41, no. 2, pp. 257-261, (1991).
3. D. Toll, “A framework for unsaturated soil behaviour,” *Géotechnique,* vol. 40, no. 1, pp. 31-44, (1990).
4. A. Tarantino, “A Possible Critical State Framework for Unsaturated Compacted Soils,” *Géotechnique,* vol. 57, no. 4, pp. 385-389, (2007).
5. Alonso, E. E., Gens, A., and Josa, A., “A constitutive model for partially saturated soils,” *Géotechnique,* vol. 40, no. 3, pp. 405-430, (1990).
6. Vanapalli, S. K., Fredlund, D. G., Pufahl, D. E., & Clifton, A. W., “Model for the Prediction of Shear Strength with Respect to Soil Suction,” *Canadian geotechnical journal,* vol. 33, no. 3, pp. 379-392, (1996).
7. Zhai, Q., Rahardjo, H., Satyanaga, A. and Dai, G., “Estimation of unsaturated shear strength from soil–water characteristic curve,” *Acta Geotechnica,* vol. 14, no. 6, pp. 1977-1990, (2019).
8. B. J. Shwan, “Analysis of passive earth thrust in an unsaturated sandy soil using discontinuity layout optimization,” in *In E3S Web of Conferences: EDP Sciences.*, (2016).
9. B. J. Shwan, “The effect of soil water retention curve hysteresis on the strength of unsaturated soils,” *Journal of Duhok University,* pp. 285-292, (2017).
10. B. J. Shwan, “Numerical study of strength reduction-induced capillary rise effect for unsaturated soil,” *Geomechanics and Engineering,* vol. 31, no. 4, pp. 385-393, (2022).
11. E. E. Romero, “Characterisation and thermo-hydro-mechanical behaviour of unsaturated Boom clay: an experimental study,” (1999).
12. Nuth, M., & Laloui, L., “Advances in modelling hysteretic water retention curve in deformable soils,” *Computers and Geotechnics,* vol. 35, no. 6, pp. 835-844, (2008).
13. Shwan, B. J., and Smith, C. C., “Application of limit analysis in unsaturated soils: numerical and experimental study of bearing capacity,” in *In Unsaturated Soils: Research and Applications-Proceedings of the 6th International Conference on Unsaturated Soils*, (2014).
14. Lee, I.M., Sung, S.G. and Cho, G.C., “Effect of stress state on the unsaturated shear strength of a weathered granite,” *Canadian Geotechnical Journal,* vol. 42, no. 2, pp. 624-631, (2005).
15. Fredlund, D.G., Xing, A., Fredlund, M.D. and Barbour, S.L., “The relationship of the unsaturated

soil shear strength to the soil-water characteristic curve," *Canadian geotechnical journal, ,* vol. 33, no. 3, pp. 440-448, (1996).

16. Geiser, F., Laloui, L. and Vulliet, L., "Elasto-plasticity of unsaturated soils: laboratory test results on a remoulded silt," *Soils and Foundations,* vol. 46, no. 5, pp. 545-556, (2006).
17. Krishnapillai, S.H. and Ravichandran, N., " 2012. New soil-water characteristic curve and its performance in the finite-element simulation of unsaturated soils," *International Journal of Geomechanics,,* vol. 12, no. 3, pp. 209-219, (2012).
18. Vanapalli, S.K., Fredlund, D.G. and Pufahl, D.E., "The influence of soil structure and stress history on the soil–water characteristics of a compacted till," *Geotechnique,,* vol. 49, no. 2, pp. 143-159, (1999).
19. B. J. Shwan, "Microstructural interpretation of effective stress equations for unsaturated sands," *International Journal of Geo-Engineering,* vol. 14, no. 1, p. 4, (2023).
20. B. J. Shwan, "Upper Bound Analysis of Suction Effect on Tunnel Stability," *International Journal of Geomechanics,* vol. 22, no. 9, p. 04022158, (2022).
21. B. J. Shwan, "Hydro-Mechanical Hysteresis Effect-Induced Bearing Capacity Variations for Unsaturated Sand," *International Journal of Geomechanics,* p. In Press, (2023).
22. Fredlund, D.G. and Rahardjo, H., Soil Mechanics for Unsaturated Soils, John Wiley & Sons, (1993).