

# An evaluation of the effect of matric suction on the dynamic strain-dependent parameters of an unsaturated silt

S. Mohsen Haeri<sup>1</sup>, Behnam Sadollahzadeh<sup>1\*</sup>, and Vahid Zahed<sup>1</sup>

<sup>1</sup>Sharif University of Technology, Department of Civil Engineering, Azadi Avenue, Tehran, Iran

**Abstract.** This research investigates the influence of matric suction on the variation of shear modulus and damping ratio of a silty soil in very small to medium shear strain levels. A set of laboratory experiments including three bender elements tests in addition to three resonant column-torsional shear tests have been carried out on unsaturated Firuzkuh silt specimens. In this regard, an unsaturated triaxial cell equipped with a set of bender elements and a resonant column-torsional shear device that can apply and control matric suction have been used. All specimens had an initial void ratio of 0.7 and were tested in various matric suctions under a net mean stress of 50 kPa. For this purpose, the axis translation technique has been implemented for applying matric suction, and High Air Entry Value (HAEV) ceramic discs have been used for air-water control of unsaturated silt specimens. According to the results, a significant variation of shear modulus and damping ratio has been observed with changes in matric suction and shear strain level. The output data indicate that shear modulus increases with increasing matric suction while the damping ratio decreases with an increase in matric suction. In addition, shear modulus and damping ratio decrease and increase with increasing shear strain, respectively.

## 1 Introduction

Climate change and consequently changes in the degree of saturation imply that the soils are mostly in an unsaturated condition, especially in semi-arid areas. In addition, in earthquake-prone areas where the soils are mostly unsaturated, dynamic behavior of unsaturated soils becomes essential for the effective design of foundations and geotechnical structures. Very few researchers have conducted element tests on small strain unsaturated soils, however, their research indicated that the hydro-mechanical behavior of unsaturated soils has a significant impact on the dynamic behavior and properties of the soils [1-3].

Shear modulus ( $G$ ) and damping ratio ( $D$ ) are two key dynamic parameters of the soils. The shear modulus represents the shear stiffness of the soil and the damping ratio is representative of the dissipation of waves' energy, while waves propagate in the soil. According to the studies conducted in the last decades, several factors including mean effective stress, void ratio, degree of saturation, stress history, plasticity, soil grain characteristics, and shear strain level have remarkable effects on these two factors which affect the dynamic behavior of the soils [4-7]. Shear strain is one of the most significant parameters that influence the dynamic behavior of the soils and plays a remarkable role in the assessment and interpretation of the shear modulus and damping ratio. The small-strain, medium-strain, and large-strain shearing occur at strain levels of  $\gamma < 10^{-4}$  %,  $10^{-3}$  %  $< \gamma < 10^{-1}$  %, and  $\gamma > 10^{-1}$  %, respectively [8].

Recent research on the dynamic characteristics of unsaturated soils can be divided into two categories: 1) the small-strain level and 2) medium to large strain levels. In both categories, the impact of matric suction, net stress, degree of saturation, and hydraulic paths on the small-strain shear modulus ( $G_0$  or  $G_{max}$ ) and damping ratio ( $D_0$  or  $D_{min}$ ), and on the strain dependent shear modulus and damping ratio of the unsaturated soils are investigated. Unsaturated resonant column and bender element tests usually are implemented for small strain tests and unsaturated resonant column-torsional shear, triaxial, simple shear, and hollow cylinder tests are used for medium to large shear strain levels. On the basis of the test results, a number of empirical and semi-empirical correlations have also been proposed by researchers for the prediction of small-strain shear modulus and damping ratio [9-16]. The medium to large strain tests on unsaturated soils have also revealed the degradation of the unsaturated shear modulus and the aggradation of the damping ratio with strain level in all unsaturated tested specimens. In addition, it is shown by previous researches that an increase in matric suction would result in an increase in the shear stiffness and a decrease in the damping ratio [17-23].

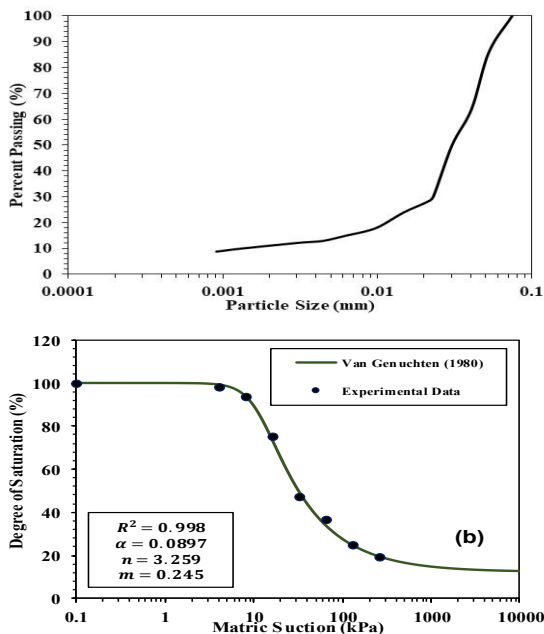
This paper aims to present the results of some unsaturated tests to assess the shear modulus and damping ratio of a specific unsaturated silt by implementing both unsaturated bender element and resonant column-torsional shear tests.

\* Corresponding author : [behnam.sadollahzadeh@sharif.edu](mailto:behnam.sadollahzadeh@sharif.edu)

## 2 Experimental Study

### 2.1 Test Material

Firuzkuh silica silt obtained from a mine in the northeast of Tehran, Iran is used in this study. The soil is a non-plastic silt with a grain size distribution curve obtained from hydrometer testing according to ASTM D7928 standard and is shown in Fig. 1a. The soil water retention curve (SWRC) was achieved by implementing the axis translation method under mean net stress of 50 kPa, and the van Genuchten model [24] was used to fit the obtained experimental data for the soil specimen in drying path, as presented in Fig. 1b. This soil is classified as ML in UCSC soil classification. The physical properties of the material are listed in Table 1. The maximum and minimum void ratios are assessed using ASTM D4254 and D4253, respectively. These methods are only applicable to soils with a maximum fines content of 15%, hence there is no standard method to acquire these parameters for soils with a higher fines content. In spite of this restriction, these methods have been used to obtain  $e_{max}$  and  $e_{min}$  in several research [e.g. 25-27].



**Fig. 1.** (a) Grain size distribution curve of Firuzkuh silt, (b) Soil Water Retention Curve (SWRC) and best fit of Van Genuchten (1980) model to the experimental data points.

### 2.2 Test Equipment

To investigate the dynamic properties of the tested soil, in small-to-medium strain levels in the unsaturated state, a triaxial cell equipped with bender elements (BE) and a resonant column-torsional shear (RC-TS) system which were developed and modified at the advanced soil mechanics laboratory of the Sharif University of Technology were implemented. These devices are suction-controlled and can control and apply the matric suction during any hydromechanical path. The axis translation technique proposed by Hilf [28] was utilized for this purpose as well.

**Table 1.** Physical properties of Firuzkuh silt.

Specifications	Value	ASTM code
<b>Classification (USCS)</b>	ML	-
$G_s$ (-)	2.65	D854
PI (%)	N.P.	D4318
$D_{50}$ (mm)	0.035	D7928
$e_{max}$ (-)	1.18	D4254
$\gamma_{min}$ (kN/m <sup>3</sup> )	12.16	D4253
$e_{min}$ (-)	0.66	D4253
$\gamma_{max}$ (kN/m <sup>3</sup> )	15.96	

The small strain shear wave velocity of the silt under various hydromechanical conditions was measured utilizing the modified triaxial cell, equipped with bender elements. The associated  $G_0$  or  $G_{max}$  was calculated using Eq. 1:

$$G_0 = \rho \times V_s^2 \quad (1)$$

Where  $V_s$  is the measured shear wave velocity and  $\rho$  is the density of the specimen. Shear modulus and damping ratio in the small to medium strain range were also measured using the resonant column-torsional shear apparatus. This device can implement both resonant column and torsional shear loadings. Resonant column test (RC) essentially involves a soil column in fixed-free end condition that is excited in a wide range of frequencies to catch the natural frequencies of the soil specimen. Once the first resonant frequency ( $f_r$ ) is obtained, the shear strain amplitude ( $\gamma$ ), shear wave velocity ( $V_s$ ), and shear modulus ( $G$ ) of the soil can be readily determined. In this test shear wave velocity was determined by measuring resonant frequency and then shear modulus was calculated by using Eq. (1). Shear strain in this test, was determined using accelerometer sensor data, and was calculated using Eq. (2):

$$\gamma = \frac{A_c \times R_{eq}}{4\pi^2 \times f_r^2 \times d \times L \times CF} \quad (2)$$

Where  $A_c$  is the output voltage of the accelerometer,  $d$  is the distance between the location of the accelerometer and the axis of the specimen,  $L$  is the length of the specimen,  $CF$  is the accelerometer calibration factor, and finally  $R_{eq}$  is the equivalent radius of the specimen (equal with  $0.707r$ ).

The damping ratio ( $D$ ) in the resonant column test can be determined from frequency response curves via the half-power bandwidth method which can be calculated using Eq. (3):

$$D(\%) = \frac{f_2 - f_1}{2f_r} \times 100 \quad (3)$$

In which  $f_1$  and  $f_2$  are the frequencies at half-power points and  $f_r$  is the resonant frequency. In the torsional shear test (TS), shear modulus and damping ratio were measured by the shear stress-shear strain hysteresis curve. The secant shear modulus was defined as the slope of the hysteresis loop and the area of the hysteresis

curve of shear stress-shear strain was used to estimate the damping ratio.

### 2.3 Specimen Preparation

The diameter and the height of the cylindrical specimens used in the bender element were 60 mm and 120 mm, respectively. In the resonant column-torsional shear test, the dimensions of the specimens were 36 mm and 72 mm, respectively. For specimen preparation, the de-aired distilled water was added to the oven-dried silt and then was blended well, and then placed in a sealed plastic bag for 24 hours. All specimens for both tests were prepared with the same initial void ratio equal to 0.7 ( $e_0 = 0,7$ ). The specimens were prepared with an initial water content of 15% and according to Fig. 1b, the value of the initial suction for the specimen was estimated at about 26 kPa and the relative density of the soil specimen was measured at approximately 93%. This high relative density indicates that the soil specimens were very dense and stiff, therefore, the volume changes in all tests were insignificant. The static compaction using the under-compaction method proposed by Ladd [29] was used to prepare the specimens by placing the soil in 10 layers for bender element specimens and 4 layers for resonant column-torsional shear specimens. No need to mention that the interfaces between the successive layers were scarified for better interpenetration of the subsequent layers.

### 2.4 Test Procedure

After the preparation of the specimen and place it on the pedestal of the cell, a vacuum of about 40 kPa as seating load was temporarily applied at the top cap to separate the mold from the noncohesive specimen and consequently to avoid deformation, disturbance, and overturning of the specimen. The focus of this research was on the measurement of the dynamic parameters of the unsaturated tested silt during the drying path in relatively high matric suctions. Therefore, all the specimens needed to be saturated and then dried. For this purpose, backpressure was implemented to saturate the specimens. In this regard, a pressure difference of 40 kPa between the back water pressure and the confining pressure was kept, and the pressures were simultaneously increased step by step to approach the saturation condition. Enough waiting time was permitted for dissolving air bubbles into the water to obtain a Skempton's  $B$ -value of larger than 0.95. When the saturation of the specimen is secured, a net mean stress ( $p_n$ ) of 50 kPa was applied and kept on the specimen to get to equilibrium. Then, the target matric suction which is the difference between the air and water pressures was applied to the specimen and was maintained for about a week or two, depending on the specimen condition, up to the time when there was no change in the water content and the volume change of the specimen, and the target hydromechanical equilibrium of the specimen was achieved. In this stage, the shear wave velocity was measured by conducting a bender element test. Since the propagation of a small

strain wave imposes no disturbance in the specimen, the next target suction could be applied on the same specimen, and after a new hydromechanical equilibrium shear wave velocity could be measured for the new target suction. This process repeated until shear wave velocities were measured for all pre-planned matric suctions.

For the implementing resonant column-torsional shear test, similar steps are taken for specimen preparation. After preparation of the specimen and saturation process, similar to that explained for the bender element test, desired matric suction was applied to the soil column. In the target hydromechanical equilibrium, the resonant column (RC) test was performed at different shear strain levels by changing the Amperage Amplitude of the current in five steps of 0.1, 0.3, 0.5, 0.7, and 1. There is a direct relationship between the value of the shear strain and the amperage amplitude in the RC device. After the RC test was completed, the torsional shear (TS) test was carried out on the same specimen, as the experienced shear strain was still enough small at the end of RC tests, to preserve the shear behavior of the tested reconstituted specimens. The torsional shear test was carried out at a medium-strain level to measure shear modulus and damping ratio corresponding to that level of strain for target hydromechanical conditions. At this part of the tests, the values of the shear modulus and the damping ratio at different strain levels, and various hydromechanical conditions were measured using the stress-strain hysteresis loop related to each test step. The RC-TS device has a restriction in the measuring of the specimen volume change, hence, the specimens were taken out of the cell, pictured, weighted, and the dimensions were measured to calculate the volume change of the specimen at the end of the test. Because the tested specimen at the end of the TS test was disturbed, a new specimen should be prepared and tested for a new set of RC-TS tests for the new target matric suctions, similar to the previously mentioned procedure. The target matric suctions of 64 kPa, 128 kPa, and 256 kPa were considered for this set of tests, under the net mean stress of 50 kPa for all tests. The measured degree of saturation for these matric suctions were equal to 37%, 25%, and 18%, respectively. The tested soils in these conditions were located in the transient and residual zones of the Soil Water Retention Curve (SWRC) (Fig. 1b). It should be mentioned that the reported experiments only encompass three suctions and one net mean stress value, and this research is being continued to be completed in the near future.

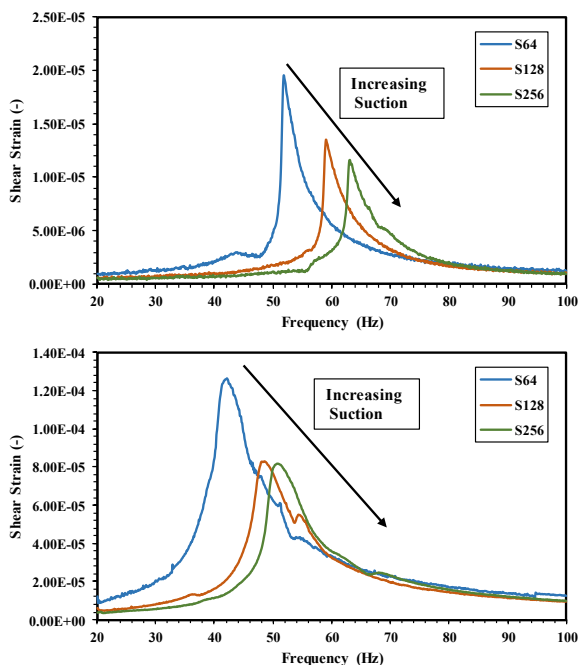
## 3 Results and Analysis

### 3.1 Effect of matric suction on the resonant frequency and frequency curves of the tested soil

To demonstrate the effect of matric suction on the resonant frequency of the tested soils, the variations of shear strain with resonant frequency is shown in Fig. 2. The figure demonstrates that the increase of matric

suction induces an increase in the soil resonance frequency and a decrease in the maximum shear strain level. It is noticeable that the matric suction has a remarkable impact on the soil behavior and the frequency response curve; although this impact reduces by the increase in matric suction. This means that the difference between the curves associated with the suctions of 128 kPa and 256 kPa is less than that for curves associated with 64 kPa and 128 kPa. Additionally, it can be observed that the shear strain level has a noticeable effect on the shape of the response curve so that the shape of the response curve for  $I = 1A$  is wider than that for  $I = 0.1A$  (Fig. 2). These shape variations affect the half-power points that are used for calculation of the soil damping ratio.

According to Fig. 2, the frequency response curves of the soil are affected by the level of the soil suction which results in an asymmetrical shape of the curves and consequently reduce the precision of the damping ratios calculated, using the half-power bandwidth method.



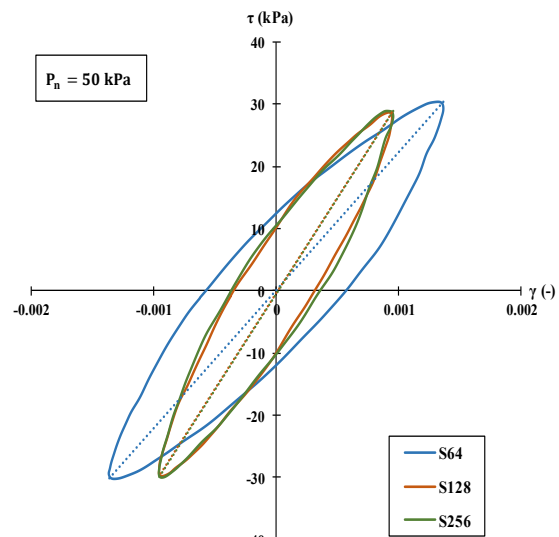
**Fig. 2.** Effects of matric suction on resonant frequency curves in Resonant Column tests: (a) at the minimum shear strain ( $I = 0.1 A$ ), (b) at the maximum shear strain ( $I = 1 A$ )

### 3.2 Effect of matric suction on the stress-strain hysteresis loop

The results of torsional shear tests are presented in Fig. 3 in the form of stress strain hysteresis loop. According to the observations from this figure, it can be concluded that increasing the matric suction from 64 to 256 kPa resulted in a hardening behavior of the soil and increase in the slope of the secant line or the shear modulus. Also, the area of the stress-strain hysteresis loop or the soil damping decreased by increase in the matric suction.

The hardening behavior is more profound for change in the matric suction from 64 kPa to 128 kPa, and after that, very small changes can be observed. This observation is due to the fact that the tested soil under both suctions of 128 kPa and 256 kPa were located in

the residual zone of the SWRC, and so the change in suction did not considerably affect the soil saturation and consequently the stiffness or damping ratio of the tested soil. In fact, the soil saturation and inter-particle forces were almost unaffected by increasing of the matric suction from 128 kPa to 264 kPa, and therefore the soil stiffness and stress-strain behavior of the tested soil were not affected as well. As demonstrated in Fig. 3, the hysteresis loop for both suctions of 128 kPa and 256 kPa were approximately identical.



**Fig. 3.** Stress-strain hysteresis loop from TS test.

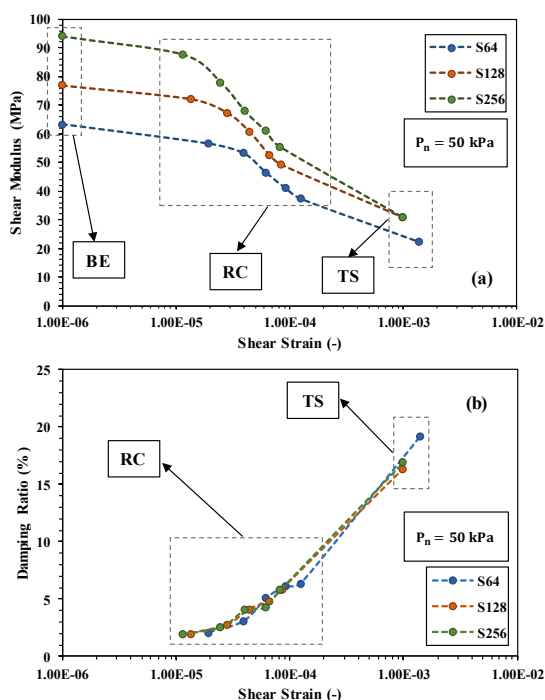
### 3.3 Effect of matric suction on the dynamic properties of the tested soil

Variations of shear modulus and damping ratio versus shear strain for tested soil under various matric suctions are presented in Fig. 4. What is obviously notable is that by increase in the strain level the shear modulus decreases and the damping ratio increases. Also, the effect of suction on the shear modulus is clearly observed. According to Fig. 4a, the results of the bender element test demonstrate that the maximum shear modulus increased significantly as the matric suction increased. Furthermore, the results of the resonant column test in the range of tested shear strain levels indicate that the effect of matric suction on the shear modulus decreased as the shear strain increased.

The data resulted from torsional shear tests shown in Fig. 4a, for the secant shear modulus also illustrate a decrease in shear modulus with an increase in strain level. However, the effect of matric suction on the stiffness degradation curve was different. The result for matric suction of 64 kPa indicates a normal decrease in shear modulus with the increase in shear strain and decrease in matric suction. However, as illustrated in Fig. 3 and discussed earlier, the results of the TS test for the suctions of 128 kPa and 264 kPa almost coincided. Namely for the tested soil, in the high matric suctions and medium range strain, and probably higher shear strain levels, the soil stiffness is independent of the

matric suction. Note that the reported result is for a net stress of 50 kPa and if the net stress changes the concluded statement may change as well.

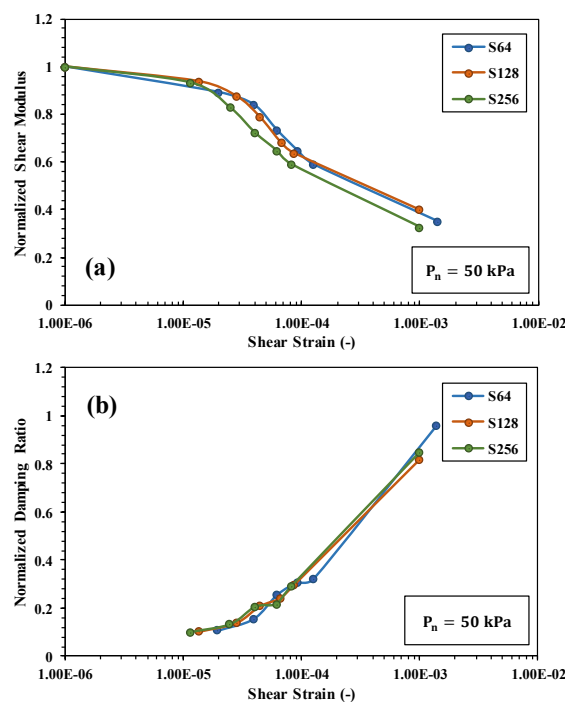
The conclusion that can be extracted from Fig. 4b is that the damping ratio of the tested soil, obtained from the bandwidth method and from the stress-strain hysteresis loop, were in the same configuration and both nonlinearly increased as the shear strain increased. The results shown in Fig. 4.b also indicate that the matric suctions had negligible effects on the damping ratio in the tested range of the matric suction and the net stress for the tested soil. However, there are some other studies that by an increase in the matric suction, the measured damping ratio decreased [17].



**Fig. 4.** Variation of (a) shear modulus and (b) damping ratio with shear strain.

Finally, the results of the tests revealed that the damping ratio determined by the bandwidth method, obtained from the resonant column were between 2% to 6% and that determined from the hysteresis loop, obtained using the torsional shear tests were between 17% to 20%.

For a better insight into the effect of matric suction on the dynamic parameters of the tested soil, the curves of changes for normalized shear modulus and damping ratio versus shear strain are prepared and are illustrated in Fig. 5. The values of shear modulus obtained from bender element test was regarded as the maximum shear modulus and the maximum damping ratio was assumed to be equal to 20% for all tests. Fig. 5a shows that the curves for different matric suctions nearly merge, however, in the higher levels of suctions higher degradation can be observed. The shape of the normalized damping ratio curves shown in Fig. 5b is very similar to the shape of the curves shown in Fig. 4b, as all variations of the latter figure (i.e Fig. 5b) are divided to a constant value to get to the former one. So probably employing Fig. 4b for damping can be more appropriate.



**Fig. 5.** Effect of matric suction on normalized (a) shear modulus and (b) damping ratio.

## 4 Conclusion

This study is mainly focused on the influence of matric suction on the variation of shear modulus and damping ratio, in the shear strain levels of very small to medium, for the tested silt. In this regard, a triaxial cell equipped with a pair of bender elements and a modified resonant column-torsional shear system were utilized to evaluate the shear modulus and damping ratio of a silt in unsaturated conditions. All the specimens had an initial void ratio of 0.7. The tests were carried out along the drying hydraulic path under a net mean stress of 50 kPa and the matric suctions of 64, 128, and 256 kPa.

The experimental measurements revealed that the shear modulus increased with an increase in the matric suction. This increase was more significant in lower shear strain levels. With the increase in shear strain level, the influence of the matric suction on the shear modulus was decreased especially in the medium strain levels. In high matric suctions and medium strain levels, an increase in suction had no remarkable impact on the stiffness of the tested soil. The degradation of the shear modulus with the increase in the shear strain level was observed for all matric suctions, however, a normalized curve indicated that the intensity of the degradation is higher for higher matric suctions.

The test results indicated that the damping ratio was not affected by the matric suction, irrespective of the methods of the measurements and calculations. However, the damping ratio of the tested soil was highly dependent on the strain level.

The tests have been performed at Advanced Soil Mechanics Laboratory of Civil Engineering Department of Sharif University of Technology which is acknowledged. Also, the authors acknowledge the financial support awarded by the research deputy of the Sharif University of Technology.

## References

1. H. Rahnema, M. Hashemi Jokar, H. Khabbaz, *Predicting the effective stress parameter of unsaturated soils using adaptive neuro-fuzzy inference system*, Scientia Iranica, **6**, (2019).
2. N. Lu, W.J. Likos, *Unsaturated soil mechanics*, Hoboken, NJ: J. Wiley, (2004).
3. D. G. Fredlund, H. Rahardjo, M. D. Fredlund, *Unsaturated soil mechanics in engineering practice*, John Willey & Sons. Inc., Hoboken, New Jersey, USA, (2012).
4. B. O. Hardin, and W. L. lack, *Vibration modulus of normally consolidated clay*, J. Soil Mech. Found. Div., **94(2)**, pp. 353–369 (1968).
5. B. O. Hardin, and V. P Drnevich, *Shear modulus and damping in soils: measurement and parameter effects*, J. Soil Mech. Found. Div., **98(sm6)** (1972).
6. B. O. Hardin, *The nature of stress-strain behavior for soils*, **Volume I** of Earthquake Engineering and Soil Dynamics Proceedings of the ASCE, , Pasadena, California, (1978).
7. X. Qian, D. H. Gray, and R. D. Woods, *Resonant column tests on partially saturated sands*, Geotech. Test. J., **14(3)**, 266-275, (1991).
8. K. Ishihara, *Soil behavior in earthquake geotechnics*, Clarendon press Oxford, (1996).
9. C. Mancuso, R. Vassallo, and A. d’Onofrio, *Small strain behavior of a silty sand in controlled-suction resonant column torsional shear tests*, Can. Geotech. J., NRC Research Press, **39(1)**, pp. 22–31, (2002).
10. L. R. Hoyos, P. Takkabutr, A. J. Puppala, M. S. Hossain, *Dynamic response of unsaturated soils using resonant column and bender element testing techniques*, in Geotech. Earthq. Eng. and Soil. Dyn. **IV**, pp. 1–8, (2008).
11. L. R. Hoyos, E. A. Suescún-florez, and A. J. Puppala, *Stiffness of intermediate unsaturated soil from simultaneous suction-controlled resonant column and bender element testing*, Eng. Geology. Elsevier B.V., **188**, pp. 10–28, (2015).
12. A. Khosravi, and J. S. McCartney, *Resonant column test for unsaturated soils with suction–saturation control*, Geotech. Test. J., **34(6)**, pp. 730–739, (2011).
13. A. Khosravi, S. Salam, J. S. McCartney, A. Dadashi, *Suction-induced hardening effects on the shear modulus of unsaturated silt*, Int. J. Geomech., **16(6)**, p. D4016007, (2016).
14. M. Jebeli, S. M. Haeri, & A. Khosravi, *Characterizing the Effect of Fines Content on the Small Strain Shear Modulus of Sand-Silt Mixtures During Hydraulic Hysteresis*, In Advances in Transport. Geotech. **IV** (pp. 837-849). Springer, Cham, (2022).
15. A. Khosravi, S. Ghadirian, & J. S. McCartney, *Effect of subsequent drying and wetting on the small strain shear modulus of unsaturated soils*, Int. J. Geolo. and Enviro. Eng., **10(2)**, 202-207, (2016).
16. Khosravi, & J. S. McCartney, *Impact of hydraulic hysteresis on the small-strain shear modulus of low plasticity soils*, J. Geotech. Geoenvironmental Eng, **138(11)**, 1326-1333, (2011).
17. M. Biglari, M. K. Jafari, et al., *Shear modulus and damping ratio of unsaturated kaolin measured by new suction-controlled cyclic triaxial device*, Geotech. Test. J., **34(5)**, pp. 525–536, (2011).
18. M. Ghayoomi, G. Suprunenko, M. Mirshekari, *Cyclic Triaxial Test to Measure Strain-Dependent Shear Modulus of Unsaturated Cyclic Triaxial Test to Measure Strain-Dependent Shear Modulus of Unsaturated Sand*, Int. J. Geomech., **1;17(9)**:04017043. (2017).
19. M. Mojezi, M. Biglari, M. K. Jafari, I. Ashayeri, *Determination of shear modulus and damping ratio of normally consolidated unsaturated kaolin*, Int. J. Geotech. Eng., Taylor & Francis, pp. 1–22, (2018).
20. S. Banar, S. M. Haeri, A. Khosravi, & M. Khosravi, *Evaluation of strain dependent shear modulus and damping ratio of unsaturated silty soil during drying path with resonant-column-torsional shear*, In Earthq. Geotech. Eng., (pp. 1281-1288). CRC Press, (2019).
21. K. N. Le, and M. Ghayoomi, *Cyclic direct simple shear test to measure strain-dependent dynamic properties of unsaturated sand*, Geotech. Test. J., **40(3)**, pp. 381–395, (2017).
22. F. Jafarzadeh, A. Ahmadinezhad, and H. Sadeghi, *Effects of initial suction and degree of saturation on dynamic properties of sand at large strain*, Sci. Iran., Sharif University of Technology, (2019).
23. F. Jafarzadeh, and H. Sadeghi, *Experimental study on dynamic properties of sand with emphasis on the degree of saturation*, Soil Dyn. Earthq. Eng. Elsevier, **32(1)**, pp. 26–41, (2012).
24. Van Genuchten, M. Th., *A closed-form equation for predicting the hydraulic conductivity of unsaturated soils*, J. Soil Sci., 44.5: **892-898**, (1980).
25. SM. Haeri, SS. Yasrebi, *Effect of amount and angularity of particles on undrained behaviour of silty sands*, Sci. Irani., **6(3):188–195**, (2016).
26. M. Goudarzy, M.M. Rahman, D. König, & T. Schanz, *Influence of non-plastic fines content on maximum shear modulus of granular materials*. Soils Found., **56(6)**, **973-983**. (2016).
27. V. T. A. Phan, D. H. Hsiao & P. T. L. Nguyen, *Effects of fines contents on engineering properties of sand-fines mixtures*, Procedia Eng., **142**, **213-220**, (2016).
28. J. W. Hilf, *An investigation of pore-water pressure in compacted cohesive soils*, University of Colorado at Boulder, (1956).
29. R. S. Ladd, *Preparing test specimens using undercompaction*, Geotec. Test. J., **1(1)**, pp. 16–23, (1978).