Improved Thermal Conductivity Function for Unsaturated Soil with Physics-based Parameters

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Abstract. This paper proposes a new relationship between thermal conductivity and degree of saturation of unsaturated soils, referred to as the thermal conductivity function (TCF). The new sigmoidal relationship between thermal conductivity and degree of saturation was developed so that all parameters have a physical meaning. After calibration of the model using data from the literature and comparison with other available TCFs, the parameters of the new TCF were related to those of the soil-water retention curves for the soils investigated in the calibration process. The linkage between the parameters of the TCF and SWRC indicates that the parameters reflect the point of air entry and pore size distribution of the soil, as well as the maximum and minimum thermal conductivity values encountered at saturated and dry conditions, respectively.

1 Introduction

Thermal conductivity is a key physical property of soils that governs the rate of heat transfer due to conduction. It is well established in the literature that the thermal conductivity is a function of the degree of saturation [1-5]. The quantitative relationship between the thermal conductivity and degree of saturation is referred to as the thermal conductivity function. Several different functional relationships have been developed [6-9]. Only the most recent thermal conductivity function developed by Lu and Dong [9] was shown to be linked to the drying path of the soil-water retention curve (SWRC). The SWRC describes the relationship between degree of saturation and matric suction in soils, which is related to the pore size distribution of the soil. The most common SWRC used in practice is that of van Genuchten [10], given as follows:

$$\frac{S-S_r}{1-S_r} = \left[\frac{1}{1+(\alpha\psi)^n}\right]^{1-1/n} \quad (1)$$

where S is the degree of saturation, S_r is the residual saturation, ψ is the matric suction, and α and n are fitting parameters.

It could be expected that the pore size distribution and the amount of water stored within the soil also are related to the thermal conductivity. Having a TCF that is related to the SWRC is useful because it may simplify the testing necessary to develop the parameters [11, 12]. Specifically, the thermal conductivity could be measured for the soil in dry and saturated states, and then the shape of the TCF could be determined from the shape of the SWRC. One issue with achieving this goal using the TCF of Lu and Dong [9] is the model does not directly incorporate a measurement of the thermal conductivity at saturated conditions into its mathematical framework. Accordingly, a new TCF is proposed in this study, which is calibrated against experimental data available in the literature and compared with selected TCFs developed in previous studies.

2 Review of Available TCFs

There are several relationships available in the literature that relate the evolution in the thermal conductivity, λ, with an increase in degree of saturation from 0 (dry conditions) to 1 (saturated conditions). Based on the Johansen model [6], Côté and Konrad [7] developed an improved model by introducing an empirical relationship between the normalized thermal conductivity and degree of saturation, given as follows:

$$\frac{\lambda-\lambda_{dry}}{\lambda_{sat}-\lambda_{dry}} = \frac{\kappa S}{1+(\kappa-1)S} \quad (2)$$

where λ_{dry} and λ_{sat} are measured values of thermal conductivity for dry and saturated soil, respectively, and κ is an empirical fitting parameter. One issue with this relationship is that it may not have the same level of nonlinearity encountered in some λ versus S data and the parameter κ is not directly linked with the SWRC parameters.

The TCF developed by Lu et al. [8] is given as follows:

$$\frac{\lambda-\lambda_{dry}}{\lambda_{sat}-\lambda_{dry}} = \exp\left[\beta\left(1-S^{\beta-1.33}\right)\right] \quad (3)$$

where β is a fitting parameter. This model has similar issues to that of Côté and Konrad [7]. One advantage of the models of Côté and Konrad [7] and Lu et al. [8] is that they have been used widely in the literature and are incorporated into many analyses.

The TCF model of Lu and Dong [9] is given as follows:

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$$\frac{\lambda - \lambda_{\mathrm{dry}}}{\lambda_{\mathrm{sat}} - \lambda_{\mathrm{dry}}} = 1 - \left[1 + \left(\frac{S}{S_{\mathrm{f}}}\right)^{m}\right]^{1/m-1} \quad (4)$$

where S_{f} and m are fitting parameters. This equation has a similar form to that of the van Genuchten SWRC [10] and has two parameters that permit it to be more nonlinear than the other TCFs. Lu and Dong [9] noted that the parameters S_{f} and m represent the degree of saturation at the onset of the funicular regime, and the pore fluid network connectivity for thermal conductivity, respectively. Lu and Dong [9] showed that S_{f} and m are related to the parameters S_{r} and n of the van Genuchten (1980) SWRC given in Equation (1). An issue with Equation (4) is that when the degree of saturation is equal to 1, λ does not equal λ_{sat}. This means that λ_{sat} in Equation (4) needs to be treated as a fitting parameter and does not have a physical meaning. This is one of the main motivators for developing a new TCF in this paper.

3 Newly Proposed TCF

The proposed model from this study is a sigmoidal function that addresses reaches an upper bound value of λ_{sat} when the soil is fully saturated (S = 1), and λ_{dry} when the soil is in dry conditions (S = 0), and is given below:

$$\frac{\lambda - \lambda_{\mathrm{dry}}}{\lambda_{\mathrm{sat}} - \lambda_{\mathrm{dry}}} = 1 - (1 - S^{m})^{\frac{m}{S_{\mathrm{f}}} - 1} \quad (5)$$

where S_{f} is the degree of saturation at the onset of the funicular regime, m is a model parameter that reflects the changing rate of the thermal conductivity with the degree of saturation, λ_{dry} and λ_{sat} are measured values of thermal conductivity for dry and saturated soil, respectively. The general shape of Equation (5) is shown in Fig. 1.

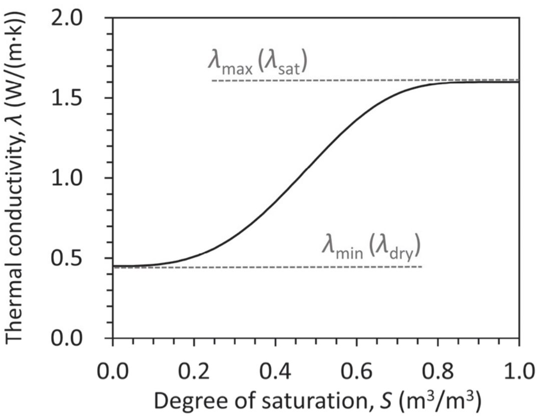

Fig. 1. Conceptual model for the thermal conductivity as a function of water retention regimes in a typical soil.

4 Model Calibration with Data from the Literature

A total of 10 soil samples were considered for the calibration of the TCF proposed in this study as well as those from previous studies. The details of the soils are summarized in Table 1. The soil data and measured thermal conductivity results were reported by Lu and Dong [9]. These soil samples consist of clayey soils, sandy soils, and silty soils. The values of λ_{dry} and λ_{sat} in Table 1 were obtained from the experimental test conducted by Dong et al. [13] or from Lu and Dong [9]. Lu and Dong [9] also report the van Genuchten SWRC [10] parameters for each of these soils.

Table 1. Soil names and physical properties used for TCF calibration.

No.	Name	Porosity	λ_{dry} (W/(m·k))	λ_{sat} (W/(m·k))
1	Ottawa F75 Sand	0.33	0.230	2.360
2	Ritzville	0.41	0.159	1.160
3	Walla-W	0.53	0.155	1.000
4	Palouse	0.53	0.220	1.000
5	Naff	0.53	0.200	1.100
6	Accusand #40/50	0.53	0.290	3.100
7	Bonny silt	0.37	0.370	1.280
8	Hoppi Clayey Sand	0.43	0.450	1.750
9	Denver Claystone	0.51	0.432	1.157
10	Georgia Kaolinite	0.51	0.239	1.556

Reference: #1: [11]; #2-5: [14]; #6-8: [15]; #9-10: [9].

5 Comparison with Other TCF models

The TCFs in Equations (2), (3), (4) and (5) were fitted to the experimental data relating the degree of saturation and thermal conductivity. The fitting parameters and coefficients of determination for the fit of each model are listed in Tables 2, 3, 4 and 5, respectively, while the experimentally-derived values of λ_{dry} and λ_{sat} for each soil were presented in Table 1. The comparisons between the models and experimental data are shown in Fig. 2.

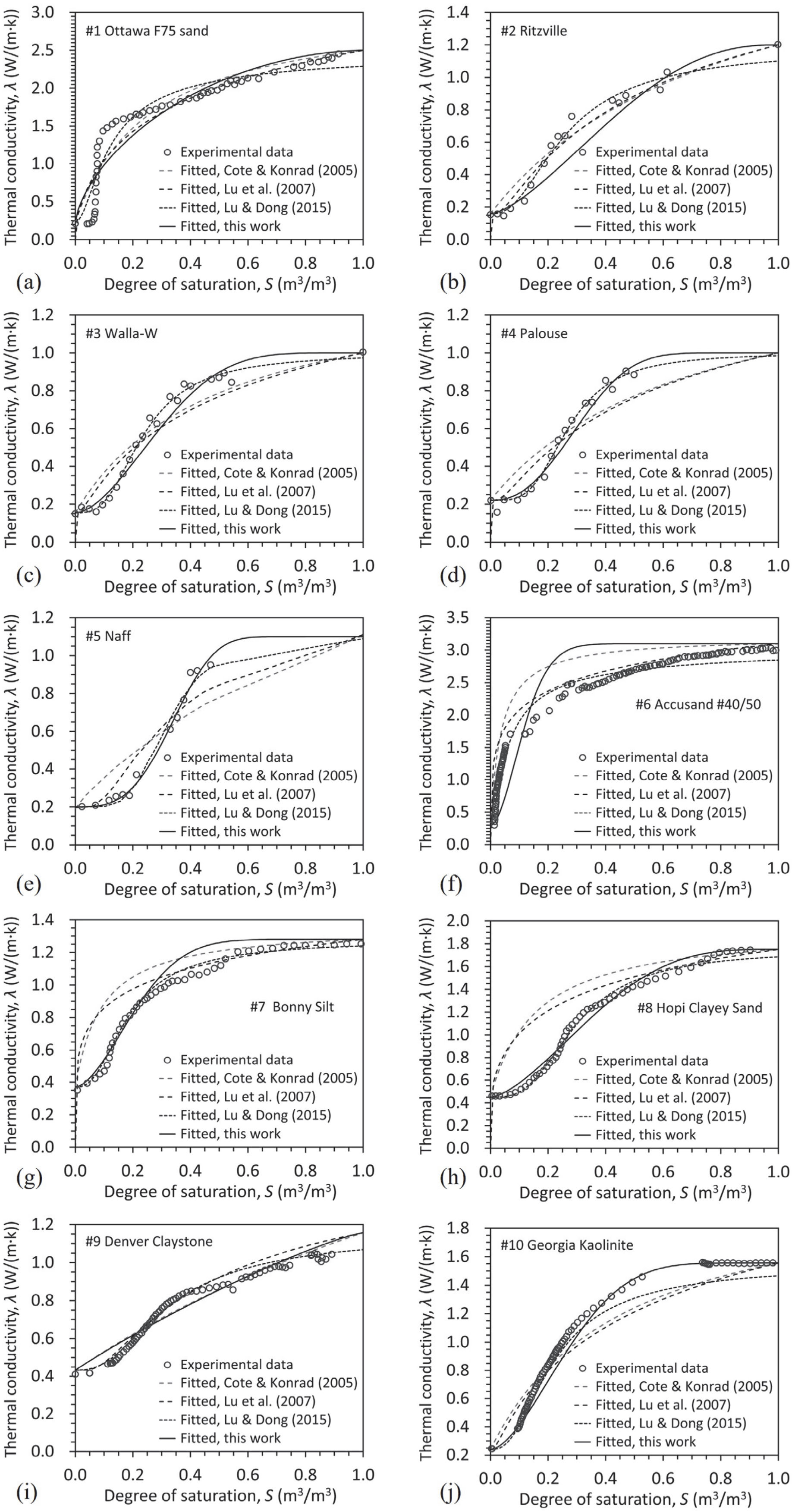

Fig. 2. Conceptual model for the thermal conductivity as a function of water retention regimes in a typical soil. (Not to scale).

Table 2. Fitting parameters and coefficient of determination for Equation (2)

Soil name	κ	R^2
Ottawa F75 Sand	4.87	0.89
Ritzville	2.21	0.96
Walla-W	3.01	0.94
Palouse	2.44	0.91
Naff	1.83	0.87
Accusand #40/50	27.84	0.98
Bonny silt	11.74	0.98
Hoppi Clayey Sand	7.28	0.95
Denver Claystone	1.43	0.96
Georgia Kaolinite	3.18	0.97

Table 3. Fitting parameters and coefficient of determination for Equation (3).

Soil name	β	R^2
Ottawa F75 Sand	1.03	0.88
Ritzville	0.79	0.98
Walla-W	0.89	0.94
Palouse	0.78	0.93
Naff	0.32	0.94
Accusand #40/50	1.19	0.98
Bonny silt	1.14	0.95
Hoppi Clayey Sand	1.08	0.93
Denver Claystone	0.51	0.99
Georgia Kaolinite	0.92	0.97

Table 4. Fitting parameters and coefficient of determination for Equation (4).

Soil name	S_f	m	R^2
Ottawa F75 Sand	0.078	1.93	0.90
Ritzville	0.201	2.46	0.98
Walla-W	0.202	3.16	0.99
Palouse	0.243	3.81	0.99
Naff	0.303	4.67	0.91
Accusand #40/50	0.032	1.70	0.98
Bonny silt	0.145	2.62	0.98
Hoppi Clayey Sand	0.253	3.17	0.99
Denver Claystone	0.255	2.521	0.98
Georgia Kaolinite	0.171	2.521	0.99

Table 5. Fitting parameters and coefficient of determination for Equation (5).

Soil name	S_f	m	R^2
Ottawa F75 Sand	0.25	0.69	0.96
Ritzville	0.42	1.49	0.99
Walla-W	0.25	1.75	0.99
Palouse	0.19	2.36	0.97
Naff	0.13	3.29	0.98
Accusand #40/50	0.05	1.80	0.95
Bonny silt	0.15	1.70	0.96
Hoppi Clayey Sand	0.34	1.56	0.97
Denver Claystone	0.43	0.95	0.98
Georgia Kaolinite	0.22	1.68	0.99

6 Equations and Discussion of Models

The SWRC relates the saturation and matric suction in a soil, while the TCF relates the degree of saturation versus the thermal conductivity of the soil. As the thermal conductivity is particularly sensitive to both the degree of saturation and the pore size distribution of the soil, it is reasonable to hypothesize that there is an intrinsic relationship between the SWRC and TCF for soils, as they both have similar sigmodal shapes that correspond to the water retention. Though not identical, experimental evidence shows some functional relationship between the saturation corresponding to adsorption capacity and the degree of saturation at the onset of the funicular regime of the SWRC, S_f. It is also possible to link the SWRC pore-size parameter n to the parameter m that captures the pore water network connectivity in the TCF.

The correlation between the parameters in SWRC and the TCF is useful to estimate thermal conductivity with a given SWRC of a soil. In other words, having an TCF that is related to the SWRC is useful as it may simplify the testing procedures to develop the parameters necessary in thermo-hydro-mechanical simulations of heat transfer and water flow. Comparisons between the parameters of the new TCF and the SWRC for each of the soils in Table 1 are shown in Fig. 3. The connection between the value of S_f from the fitted TCF and the residual degree of saturation S_r from the SWRC for seven soils (Soils 1, 2, and 6-10) is shown in Fig. 3(a). The following linear relationship was fitted to the data:

$$S_f = 1.74S_r \tag{6}$$

This equation physically implies that the inflection point of the TCF occurs at a degree of saturation that is approximately 1.74 times greater than the residual saturation, independent of the soil type. This equation has R^2 = 0.32, which indicates that there is still considerable scatter in the data. To investigate the correlation between the SWRC and the TCF, the fitted values of the parameter *n* from Lu and Dong's paper [9] were plotted versus the fitted values of the parameter *m* from this paper for the same seven soils in Fig. 3(b). The following linear relationship was fitted to the data:

$$m = 2.40 - 0.38n \tag{7}$$

where *m* and *n* are fitting parameters from the new TCF and the van Genuchten (1980) SWRC, respectively. The value of R^2 is only 0.25, however, indicating only a weak connection between these parameters. Further testing is necessary on additional soils to better establish these empirical relationships. One reason for the scatter observed in the data could be that the soil types considered in the simulation have a wide range of mineralogy and pore size distributions.

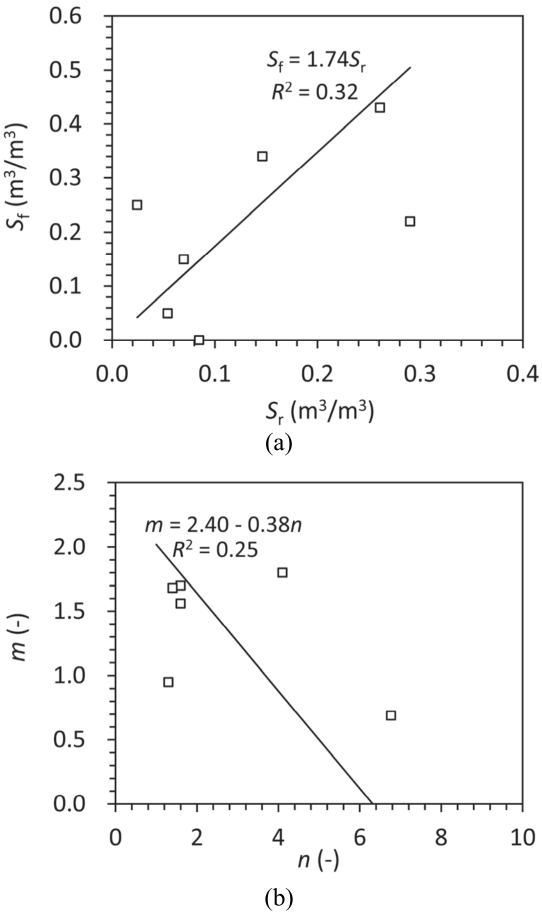

Fig. 3. Model parameter correlation between TCF and SWRC: (a) S_f vs S_r; (b) m vs n.

7 Summary

This paper presented a new thermal conductivity function for unsaturated soils that addresses issues noted in previous relationships between thermal conductivity and degree of saturation. The parameters of the thermal conductivity function were found to be approximately related to those of the soil-water retention curve, confirming the physical basis of this model. Further, the model is capable of capturing the thermal conductivities at saturated and dry conditions obtained from experiments. These features indicate that the shape of the TCF can be estimated from the SWRC and that the only measurements of thermal conductivity needed to define the TCF are those for saturated and dry conditions.

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