Enhancement of a hydro-mechanical hypoplastic model for unsaturated fine-grained soils accounting for small strain stiffness

María Pico-Duarte[1]*, *David* Mašín[1]

[1]Faculty of Science, Charles University in Prague, Czech Republic.

Abstract. Due to global climate change, larger extreme seasonal and daily moisture variations have been occurring more frequently in the last decades. This phenomenon can affect geotechnical structures by inducing cyclic coupled hydro-mechanical loads. However, reproducing this behaviour from a numerical point of view requires robust constitutive models that can predict the coupling between the hydraulic and mechanical behaviour of fine-grained soils, combined with predictions of history-dependent stiffness evolution at small strains. For this reason, in the present work the hypoplastic model for unsaturated fine-grained soils was further modified to better predict the water retention behaviour of unsaturated soils incorporating a smoothed hysteretic Water Retention Curve (WRC). In addition, the constitutive model was calibrated using experimental data available on the literature of a completely decomposed tuff (CDT) from Hong Kong. At the end, the capabilities of the extended model to predict cyclic behaviour of unsaturated soils were evaluated using cyclic constant water triaxial tests at different suctions The results indicate that the extended model is able to describe with more accuracy the cyclic hydro-mechanical behaviour of the decomposed tuff if additional suction-dependency of one of its small-strain parameters is considered. Without this, the model can be calibrated to data at a given suction but its cyclic predictions for different suctions are not reasonably accurate.

1 Introduction

Several constitutive models have been developed in the last years for predicting the behaviour of partially saturated soils e.g., [1]–[5], since there are many practical applications where unsaturated conditions should be considered, such as nuclear waste disposal facilities, pavement design and effects of cyclic variations of water conditions due to climate change [6]. Among them, Wong and Mašín [7] developed a coupled hydro-mechanical model for partially saturated fine-grained soils, which has as main features a hysteretic water retention curve that is dependent on void ratio based on the bi-linear formulation of WRC by Brooks and Corey [8], a modified intergranular strain concept for partially saturated conditions and a small strain stiffness formulation that is dependent on stress and suction history.

In this work, a hysteretic water retention curve was incorporated to the constitutive model proposed by Wong and Mašín [7] using a non-linear formulation proposed by Svoboda et al. [9] to predict the non-linear dependency of the degree of saturation and suction observed in experimental evidence [10]. Additionally, the capabilities of the extended model to predict the cyclic behaviour of unsaturated soils were evaluated using element tests simulations to predict the experimental results of constant water cyclic triaxial test from Zhou [11] at three different suctions on a completely decomposed tuff. At the end, it was observed that the extended model is able to capture the increment in stiffness, which is caused due to the increment in suction. Additionally, a linear dependency between the intergranular strain with suction was found.

2 Brief description of the model formulation

The reference model was developed by Wong and Mašín [7] through the enhancement of a previous hypoplastic model for unsaturated fine-grained soils [12]. The model incorporates a coupled hysteretic water retention model with a modified intergranular strain formulation for partially saturated soils. It was developed based on the effective stress approach for unsaturated soils and its components such as the position of the normal compression line, very small strain shear modulus, size of the small-strain stiffness elastic range and effective stress are defined in terms of the degree of saturation S_r.

The general rate form of the model is given by

$$\dot{\boldsymbol{\sigma}} = f_s(\boldsymbol{\mathcal{L}}:\dot{\boldsymbol{\epsilon}} + f_d\, \mathbf{N}\, \|\dot{\boldsymbol{\epsilon}}\|) + f_u\, \mathbf{H}_s \qquad (1)$$

Where $\dot{\boldsymbol{\sigma}}$ corresponds to the stress rate tensor, $\boldsymbol{\mathcal{L}}$ and $\mathbf{N}$ are the fourth order and second order hypoplastic

* Corresponding author: picoduam@natur.cuni.cz

tensors, respectively, $\dot{\boldsymbol{\epsilon}}$ is the Euler stretching tensor and $\mathbf{H}_s$ is a second order tensor for predicting wetting induced collapse.

The effective stress formulation was taken from Khalili and Khabbaz [13] according to

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{net} - \mathbf{1}\chi s \tag{2}$$

Where $\boldsymbol{\sigma}^{net}$ is the net stress and

$$\chi = S_r^{\left(\frac{\gamma}{\lambda_p}\right)} \tag{3}$$

Where γ is the effective stress parameter and λ_p is the slope of water retention curve (see **Fig 1).**

2.1 Modified hysteretic water retention model

The reference model adopted a hysteretic bi-linear water retention curve following the formulation by Brooks and Corey [8], see **Fig 1**. However, experimental evidence has indicated that the dependency of suction on the degree of saturation is non-linear [10]. For this reason, a smoothed WRC proposed by Svoboda et al. [9] was incorporated into the original model. The smoothed formulation also improves the numerical performance of the model, avoiding abruptly changes at the intersection of the main wetting/drying curves with the scanning curve and at the air entry/expulsion suction value due to the derivates of $\frac{\delta S_r}{\delta s}$. Moreover, the constitutive model is inherently non-linear, which is in contrast with the bi-linear formulation that resembles the response of the elasto-plastic constitutive model.

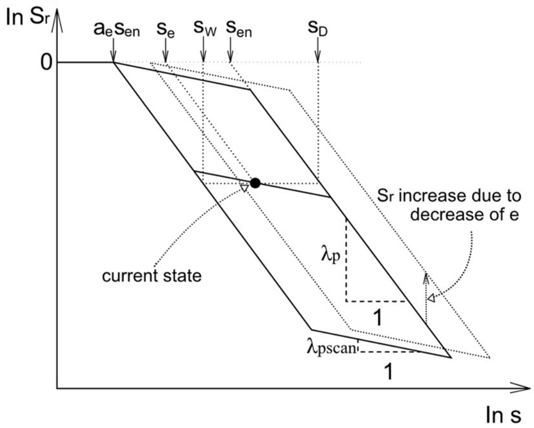

Fig 1. Original hysteretic water retention model. Taken from [7].

The water retention curve was redefined using the formulation of the factor r_λ. In this way, the complete water retention curve is given by

$$S_r = \begin{cases} 1 & for \quad s \le a_e s_{en} \\ \left(\frac{s_e}{s}\right)^{\lambda_p} & for \quad s > a_e s_{en} \end{cases} \tag{4}$$

Where λ_p corresponds to the slope of the water retention curve and s_e is calculated according to

$$s_e = s_{en}(a_e + a_{scan} + a_e a_{scan}) \tag{5}$$

Where a_{scan} corresponds to a state variable used to define the position of the current state along the scanning curves. Thus, when the soil is at the main drying curve $a_{scan} = 0$ and at the main wetting curve $a_{scan} = 1$. The rate of a_{scan} is given by

$$\dot{a}_{scan} = \frac{1-r_\lambda}{s_D(1-a_e)}\dot{s} \tag{6}$$

Where s_D is calculated according to Eq. 7. Fig. 1. illustrates the meaning of s_D and s_W.

$$s_D = \frac{s-s_W}{s-s_D} \tag{7}$$

r_λ represents to the ratio of scanning curve slope and slope of the main drying and wetting curves. In this formulation, it allows a smoothed transition between the main drying and wetting curves. For its description, three internal parameters constant values are needed: $p_{scan} = 3.0, S_{lim} = 0.75$ and $p_{wett} = 1.1$. In addition, a factor f_{scan} is defined as

$$f_{scan} = \begin{cases} a_{scan} & for \quad \dot{s} > 0 \\ 1 - a_{scan} & for \quad \dot{s} < 0 \end{cases} \tag{8}$$

Hence, the parameter r_λ is defined according to the following formulation for achieving a smoothed representation of the WRC:

$$r_\lambda = \begin{cases} 0 & for \quad s < a_e s_{en} \; and \; \dot{s} > 0 \\ \left(\frac{1-S_r}{1-S_{lim}}\right)^{p_{wett}} & for \; S_r > S_{lim} \; and \; \dot{s} < 0 \\ f_{scan}{}^{p_{scan}} & otherwise \end{cases} \tag{9}$$

2.2 Formulation for the very small strain shear modulus

For the very small strain shear modulus G_{tp0}, the formulation developed by Wong et al. [14] was adopted according to

$$G_{tp0} = p_r A_g \left(\frac{p}{p_r}\right)^{n_g} e^{(-m_g)} \left(\frac{s}{s_e}\right)^{k_g} \tag{10}$$

Where p is the mean effective stress calculated using χ from Eq. 3, p_r corresponds to a reference pressure of 1 kPa. The parameters A_g, n_g, m_g and k_g control the magnitude and dependency of G_{tp0} on the mean effective stress, void ratio, and degree of saturation, respectively.

2.3 Small strain stiffness effects

The intergranular strain by Niemunis and Herle [15] considers the strain as the product of the deformation of the intergranular interface layer and the rearrangement of the skeleton. The model uses this concept in a modified version for partially saturated condition. For this purpose, a tensorial state variable named as intergranular strain $\boldsymbol{\delta}$ is included for describing the interface deformation. The degree of mobilization of the intergranular strain is defined as

$$\rho = \frac{\|\delta\|}{R(s)} \tag{11}$$

Where $R(s)$ represents the size of the elastic range, which modified for partially saturated conditions follows

$$R(s) = R + r_m ln\frac{s}{s_e} = R - \frac{r_m}{\lambda_p} ln\, S_r \tag{12}$$

Where r_m is parameter that describes the dependency of $R(s)$ on S_r . The time derivative of $R(s)$ gives

$$\dot{R}(s) = r_m\left(r_\lambda\frac{\dot{s}}{s} + \frac{\gamma}{e\lambda_{psu}}\dot{e}\right) \tag{13}$$

The influence of suction and void ratio on size of the elastic range is then included in the rate formulation of the intergranular strain for $\dot{\boldsymbol{\delta}}$: $\boldsymbol{\epsilon} > \mathbf{0}, s > s_e$ and $R(s) < 0$ as

$$\dot{\boldsymbol{\delta}} = \left(I - \widehat{\boldsymbol{\delta}} \otimes \widehat{\boldsymbol{\delta}}\rho_r^{\beta}\right) : \boldsymbol{\epsilon} + \boldsymbol{\delta}\ \frac{R(\dot{s})}{R(s)} \tag{14}$$

The model rate formulation for unsaturated conditions can be rewritten as:

$$\dot{\boldsymbol{\sigma}}^{net} - \mathbf{1}(1 - \gamma r_\gamma)\chi\dot{\boldsymbol{s}} = \boldsymbol{\mathcal{M}}^{HM} : \dot{\boldsymbol{\epsilon}} + f_u \mathbf{H}_s \tag{15}$$

Where $\boldsymbol{\mathcal{M}}^{HM}$ is the stiffness matrix defined for partially saturated conditions as

$$\boldsymbol{\mathcal{M}}^{HM} = \boldsymbol{\mathcal{M}} - \frac{s(1-e)\gamma^2}{e\lambda_{psu}}\left(\frac{s_{en}}{s}\right)^{\lambda} \mathbf{1} \otimes \mathbf{1} \tag{16}$$

For a more detailed description of the model formulation the reader is referred to [7].

3 Soil description and experimental data

The model calibration and evaluation were performed using experimental data from the literature on a completely decomposed tuff. The soil is classified as silt (ML) according to the Unified Soil Classification System. The analysed tests included isotropic compression at four different suctions $s = \{0, 30, 100, 200\}$ kPa performed by Yung and Ng [16], two wetting and drying tests at two constant net stresses $p_{net} = \{110, 300\}$ kPa carried out by Ng et al. [17] kPa constant-water cyclic triaxial tests at three different suctions $s = \{0, 30{,}60\}$ kPa reported by Zhou [11].

4 Model calibration and element tests simulations

An extensive experimental campaign on a completely decomposed tuff in partially saturated conditions is available on the literature [11], [16]–[19]. The experimental results were used in the present study for the calibration and evaluation of the extended model. The parameters of the model can be divided into five groups: a) parameters of the basic hypoplastic model and $(\phi_c, \lambda^*, \kappa^*, N, \nu_{pp}, \alpha_G)$ b) unsaturated mechanical model (n_s, l_s, m). For its calibration, isotropic compression tests at four different suctions $s = \{0, 30, 100, 200\}$ kPa were employed. c) Parameters for the hysteretic water retention model $(s_{en0}, e_0, \lambda_{p0}, a_e)$. For this purpose, two different drying and wetting tests at two confining pressures $p_{net} = \{110{,}300\}$ kPa were used. d) parameters of the very small strain shear modulus (A_g, n_g, m_g, k_g): the calibration was performed by Wong et al. [14] using bender element tests. e) parameters for the intergranular strain model $(R, \beta_r, \chi_g, m_{rat}, r_m)$: Three different constant-water cyclic triaxial tests at three different suctions $s = \{0, 30, 60\}$ kPa were simulated. The implementation of the numerical model was done as a subroutine in the inhouse software TRIAX [20], which can be directly incorporated into open-source finite element software for the simulation of boundary value problems. The calibrated parameters for the decomposed tuff are presented in Table 1.

Table 1. Calibrated parameters for a completely decomposed tuff.

Basic model	ϕ_c 35	λ^* 0.054	κ^* 0.003	N 0.72	ν_{pp} 0.2
Unsat. Mechanical	α_G 1.0	n_s 0.038	l_s -9^{-4}	m -	
WRC model	s_{en0} -58	e_0 0.56	λ_{p0} 0.55	a_e 0.45	
Gtp0 model	A_g 4220	n_g 0.55	m_g 0.9	k_g 0.2	
Intergr. strain model	R 10^{-4}	β_r 0.5	χ_g * 3.2	m_{rat} 1.0	r_m 7^{-5}

χ_g *This value was later modified according to suction, as explained in the next sections.

4.1 Results of element tests simulations

4.1.1 Hystereric water retention behaviour

The results of the simulations for wetting and drying tests under two net stresses $p_{net} = \{110, 300\}$ kPa using the smoothed hysteretic water retention formulation are presented in Fig 2. It was observed that the smoothed formulation of the WRC produces more realistic predictions of the hysteretic hydraulic soil behaviour. In addition, the model is able to capture the dependency of the hysteretic WRC in void ratio, although the experimental evidence showed a stronger effect.

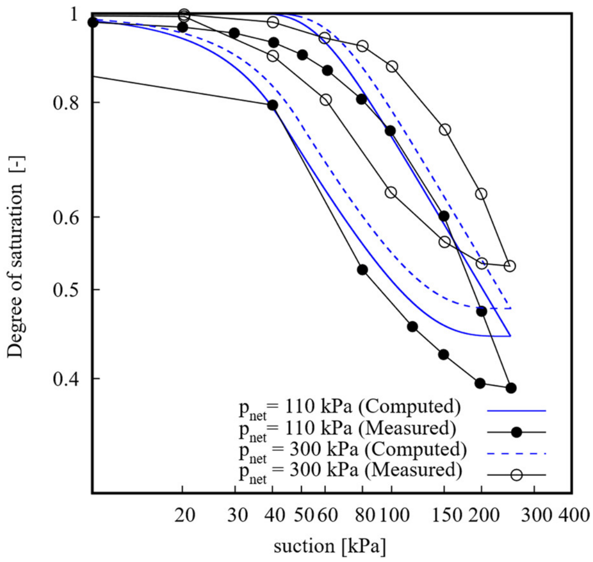

Fig 2. Smoothed hysteretic water retention curve at p_{net} = 110 kPa and p_{net} = 300 kPa. Experimental data by Ng et al. (Measured) [18] against model predictions (Computed).

4.1.2 Cyclic constant water triaxial simulations

The parameters for the intergranular strain model were calibrated using cyclic constant water triaxial test at three different suctions $s = \{0, 30, 60\}$ kPa from Zhou [11] compared the results between the model predictions and the experimental results. The model is able to predict the increasing trend in the stiffness with increasing suction that was observed on the experimental results. These results agree with the previous observations by Ng and Xu [19] who reported an increase in shear stiffness up to 35% for a suction increase from 150 to 300 kPa.

The model cyclic accumulation has been calibrated at suction = 30 kPa, and the accumulation rate of strain was for this suction well-captured by the model. However, at zero suction, the model overpredicted the accumulation in strain, meanwhile, for the highest suction, the accumulation was underpredicted (see Fig 3). For this reason, a more detailed evaluation of the effects of suction on the intergranular strain model parameters was performed. To this end, the parameters were calibrated independently for the suction values of 0 kPa and 60 kPa. After the calibration was performed, it was found that it was possible to predict the accumulation rate for suction 0 and 60 kPa by changing the value of the parameter χ_g for each suction, indicating that there is a strong dependency of χ_g on suction. The predicted accumulation curves for the updated calibration are shown in Fig 4. The obtained dependency of the parameter of the intergranular strain model χ_g with suction is illustrated in Fig 5. There is a clear trend in the dependency χ_g on suction, which will later be used in definition of an updated model considering suction variability of χ_g.

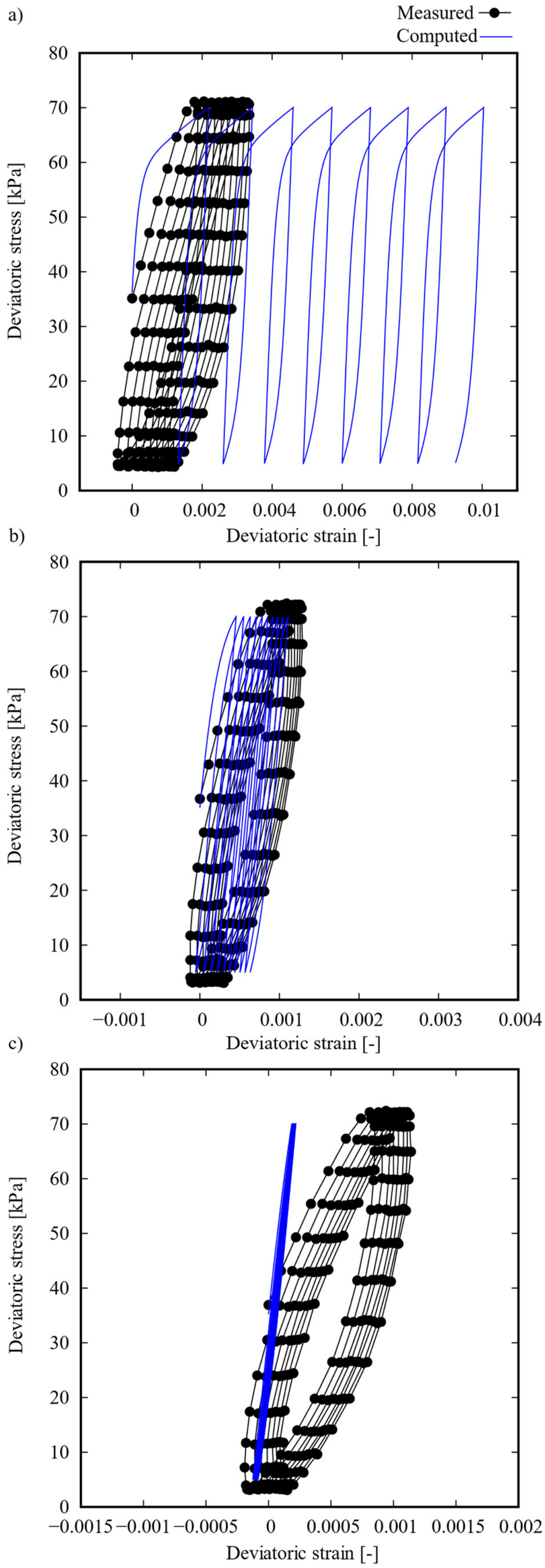

Fig 3. Cyclic constant water triaxial tests. Experimental data by Zhou [11] vs. model predictions at a) s=0 kPa, b) s=30 kPa c) 60 kPa.

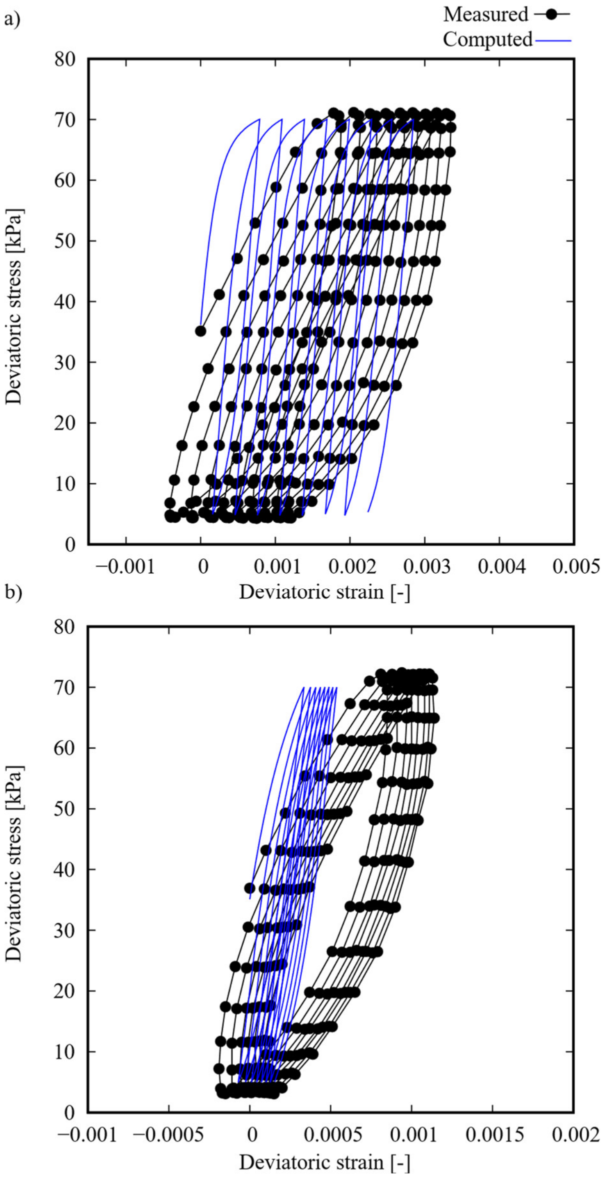

Fig 4. Recalibration results of cyclic constant water triaxial tests for suctions at a) 0 kPa and b) s=60 kPa. Experimental data by Zhou [11]vs. model predictions.

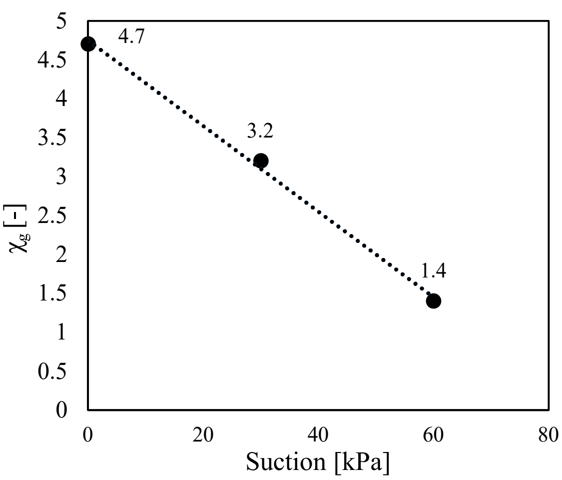

Fig 5. Dependency of the parameter of the intergranular strain model χ_g on suction.

5 Conclusions

In the present work, the coupled hypoplastic model for unsaturated fine-grained soils developed by Wong and Mašín [7] was further extended to include a smoothed hysteretic water retention curve formulation proposed by Svoboda et al.[9] The model shows better predictions of the hydraulic soil behaviour, because it is able to capture the non-linear dependency of suction on the degree of saturation.

In addition, the model was calibrated for simulating cyclic loading of partially saturated soils at different suctions. On the simulation results, it was observed that the enhanced constitutive model is able to predict the increasing stiffness effect due to suction. However, the accumulation of strain is not well capture for a single parameter set adopted for different suction levels. For this reason, a recalibration of the model was performed for each value of suction by considering different value of the parameter χ_g of the intergranular strain model for each suction. In this case, much better predictions of strain accumulation were achieved. This finding will be later used in definition of an enhanced model considering suction-dependent value of χ_g .

Aknowledgements

The authors appreciate the financial support given by the grant No. 21-35764J of the Czech Science Foundation and the Center for Geosphere Dynamics (UNCE/SCI/006). The first author acknowledges the financial support given by the Charles University Grant Agency (GAUK) with project number 260422.

References

[1] E. E. Alonso, A. Gens, and A. Josa, *Geotechnique*, **40**, 3, 405–430, (1990).
[2] D. Sheng, S. W. Sloan, and A. Gens, *Comput. Mech.*, **33**, 6, 453–465, (2004)
[3] R. Tamagnini, *Geotechnique*, **54**, 3, 223–228, (2004).
[4] D. Sheng and A. N. Zhou, *Can. Geotech. J.*, **48**, 5, 826–840, (2011)
[5] A. Zhou and D. Sheng, *Comput. Geotech.*, **63**, 46–66, (2015)
[6] G. T. Hong, C. P. Aubeny, R. Bulut, and R. L. Lytton, *Unsaturated soils 2006*, 427–438, (2006).
[7] K. S. Wong and D. Mašín, *Comput. Geotech.*, **61**, 355–369, (2014)
[8] R. H. Brooks and A. T. Corey, *HYDRAULIC PROPERTIES OF POROUS MEDIA*. 1964.
[9] J. Svoboda, D. Mašín, J. Najser, R. Vašíček, I. Hanusová, and L. Hausmannová, *Acta Geotech.*, (2022)
[10] A. S. S. Raghuram, B. M. Basha, and A. A. B. Moghal, *J. Mater. Civ. Eng.*, **32**, 4, (2020)
[11] C. Zhou, 2014.
[12] D. Mašín and N. Khalili, *Int. J. Numer. Anal. Methods Geomech.*, **30**, 13, 1303–1336, (2008)
[13] N. Khalili and M. Khabbaz, *Geotechnique*, **48**,

https://doi.org/10.1051/e3sconf/202338210005

5, 681–687, (1998).
[14] K. S. Wong, D. Mašín, and C. W. W. Ng, *Comput. Geotech.*, **56**, 28–39, (2014)
[15] A. Niemunis and I. Herle, *Mech. Cohesive-Frictional Mater.*, **2**, 4, 279–299, (1997)
[16] C. W. W. Ng and S. Y. Yung, *Geotechnique*, **58**, 1, 23–35, (2008)
[17] C. W. W. Ng, J. Xu, and S. Y. Yung, *Can. Geotech. J.*, **46**, 9, 1062–1076, (2009)
[18] C. W. W. Ng and C. Zhou, *Geotechnique*, **64**, 9, 709–720, (2014)
[19] C. W. W. Ng and J. Xu, *Can. Geotech. J.*, **49**, 2, 226–243, (2012)
[20] *https://soilmodels.com/triax/.*