Development of Methods for Calculation the Load Angle of a Synchronous Generator Based on PMU Data

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Abstract. The development of the Wide Area Measurement Systems (WAMS) in the Unified Electric Power System (UES) of Russia and using phasor measurement technology (or PMU technology) open up new prospects in the electric power system (EPS) operational dispatch and automatic control. PMU devices installation on synchronous generators (SG) allows to obtain information not only about currents and stator voltages phasors, but also information about the excitation current and excitation voltage. This set of high-precision measurements of the network state parameters, in the presence of generators mathematical models, provides the possibility of calculating such parameters, the direct measurements of which are very difficult. One of these parameters, which can be determined using PMU, is the load angle of the SG (the angle between the generalized voltage phasor and the generator EMF phasor in the *dq* axis system) in the steady states and transients. The value of the load angle can be used to analyze the static and dynamic stability, as well as to quantify the damping properties of the SG at low-frequency oscillations (LFO) of the power systems electrical parameters. This study analyzes the possibilities for calculating the load angle based on a typical set of generator PMU signals: stator current and voltage vectors, excitation current and voltage. Two methods for calculating the load angle in steady states and transients have been developed and implemented. The first method is based on solving a system of nonlinear algebraic equations, and the basis of the second method is the mathematical model of the SG. The presented methods were tested using the SimInTech software package. Further, using the developed methods, the load angles of two turbogenerators installed at the plants of the Russian UES were calculated according to the PMU data of the parameters of the electromechanical transient process.

1 Introduction

The use of the PMU technology within the Wide Area Measurement Systems (WAMS) allows solving many urgent problems, contributing to an increase in the reliability of the operation of electric power systems (EPS) [1-3]. PMU devices are able to provide information about the voltages and currents phasors in steady states and transients with high discretization and accuracy [4,5]. Also, PMUs installed on synchronous generators (SG) make it possible to obtain the values of excitation current and excitation voltage [4,5]. Such a set of high-precision data opens up broad prospects in the field of verifying models of EPS objects, identifying the actual parameters of equivalent circuits, monitoring changes in the power systems electrical parameters, as well as analyzing transient processes in EPS [1]. The availability of PMU data in conjunction with the mathematical models of the generator provides the possibility of calculating such parameters, the measurement of which is impossible, or is associated with great difficulties. One of these parameters is the SG load angle (the angle between the generalized voltage phasor and the generator EMF phasor in the *dq* axis system). Information about the actual nature of the change in the load angle in various states can help in solving problems of analyzing the dynamic behaviour of generators and equivalents of power systems during transients.

An urgent problem inherent in the EPS is low-frequency oscillations (LFO) of the parameters of the electric power state [6, 7]. Arising due to the presence of various kinds of disturbing influences in the EPS, as well as due to the lack of sufficient damping, LFO reduce the reliability of the power system and can lead to a disruption of stability [6-9]. Therefore, it is important to ensure the stability of the EPS to solve the problems of identification, monitoring of LFO, as well as minimizing the negative impact of fluctuations on the power system.

It is possible to determine whether a specific SG is a source of oscillations, as well as to characterize its quantitative participation in the damping of the LFO, by calculating the values of the synchronizing power or synchronizing energy of the generator [10–12].

To assess the synchronizing power and energy, it is important to have as accurate information as possible about the nature of the change in the load angle during the electromechanical process [10]. Direct measurements of the load angle, as well as synchronized measurements of the SG rotor speed, are almost always unavailable or difficult to implement in practice. Thus, the task of developing methods that could provide information on the nature of the change in the load angle of the SG both in real time, increasing

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the steady states and in transients with acceptable accuracy is an urgent task. The presence of synchrophasors measurements will allow determining the load angles almost in efficiency of monitoring and identification of LFO, as well as the analysis of transient processes and the stability of the EPS.

2 Methods for calculating the load angle

2.1 Method based on the solution of nonlinear algebraic equations

This method allows to determine the load angle of the SG in the steady states and transients according to the following equations, which are the dependences of the generator active and reactive powers on the load angle:

$$P_g = \frac{E'_q \cdot U_g}{X'_d} \cdot \sin\theta - U_g^2 \cdot \frac{X_q - X'_d}{X_q \cdot X'_d} \cdot \sin\theta \cdot \cos\theta; \quad (1)$$

$$Q_g = \frac{E'_q \cdot U_g}{X'_d} \cdot \cos\theta - U_g^2 \cdot \left(\frac{\sin^2\theta}{X_q} + \frac{\cos^2\theta}{X'_d} \right), \quad (2)$$

where U_g is the positive sequence phasor modulus of the SG linear voltage; P_g and Q_g are the respectively active and reactive three-phase SG powers; X_q is the synchronous reactance along the *q* axis; X'_d is the transient reactance along the *d* axis; E'_q and θ are the respectively transient EMF along the *q* axis and SG load angle.

This method requires values of phase voltages and powers, which can be obtained with PMU. The modulus of the linear voltage positive sequence phasor is calculated based on information about the voltage phasors of the stator phases. In the system of nonlinear algebraic equations (1-2), the transient EMF and the load angle SG are unknown. The solution of the presented nonlinear equations makes it possible to obtain the dependence of the change in the value of the load angle during the transient.

The presented method for calculating the load angle requires only measurements from the side of the SG stator, a minimum of passport SG parameters (only synchronous and transient resistances). Also, this method is not demanding on computing power. It allows to start the calculation from any measurement point, including from any moment during the transient, which makes it possible to use the described method to solve tasks in real time. However, the disadvantage of the method is that it does not take into account the influence of damper circuits, which can affect the accuracy of determining the load angle, and, as a result, the accuracy of calculating the synchronizing power and energy. Therefore, it is required to develop an approach that can most accurately take into account transients in all SG circuits.

2.2 Method based on the mathematical model of a synchronous generator

For a more complete account of the process in all SG circuits and, consequently, a more accurate the generator load angle determination, a method has been developed based on the “full” mathematical model of the SG based on the Gorev-Park equations [13,14]. This model takes into account transients in the stator circuit, in the excitation winding, as well as in damper circuits along the *d* and *q* axes.

$$U_d + R_a \cdot I_d + \frac{d}{dt}\psi_d + \omega \cdot \psi_q = 0; \quad (3)$$

$$U_q + R_a \cdot I_q + \frac{d}{dt}\psi_q - \omega \cdot \psi_d = 0; \quad (4)$$

$$U_f = R_f \cdot U_f + \frac{d}{dt}\psi_f; \quad (5)$$

$$0 = R_{1d} \cdot I_{1d} + \frac{d}{dt}\psi_{1d}; \quad (6)$$

$$0 = R_{1q} \cdot I_{1q} + \frac{d}{dt}\psi_{1q}; \quad (7)$$

$$\psi_d = X_d \cdot I_d + X_{ad} \cdot I_f + X_{1ad} \cdot I_{1d}; \quad (8)$$

$$\psi_q = X_q \cdot I_q + X_{1aq} \cdot I_{1q}; \quad (9)$$

$$\psi_f = X_f \cdot I_f + X_{ad} \cdot I_d + X_{1f} \cdot I_{1d}; \quad (10)$$

$$\psi_{1d} = X_{1d} \cdot I_{1d} + X_{1f} \cdot I_f + X_{1ad} \cdot I_d; \quad (11)$$

$$\psi_{1q} = X_{1q} \cdot I_{1q} + X_{1aq} \cdot I_q; \quad (12)$$

$$I_q = I_g \cdot \cos(\varphi + \theta); \quad (13)$$

$$I_d = I_g \cdot \sin(\varphi + \theta); \quad (14)$$

$$U_q = U_g \cdot \cos(\theta); \quad (15)$$

$$U_d = U_g \cdot \sin(\theta); \quad (16)$$

where U_d, U_q, I_d and I_q are the respectively voltage and current phasors *d*- and *q*- components; ψ_d and ψ_q are the respectively stator flux linkage *d*- and *q*- components; R_a is the resistance of the stator circuit; I_{1d} and I_{1q} are the respectively damper circuit currents *d*- and *q*- components; U_f is the field voltage; R_f is the resistance of the field winding; R_{1d} and R_{1q} are the respectively damper circuits resistances *d*- and *q*- components; ψ_{1d} and ψ_{1q} are the respectively damper circuits flux linkages *d*- and *q*- components; X_{1d} and X_{1q} are the respectively damper circuits reactances *d*- and *q*- components; X_{1f} is the mutual reactance between the field winding and the damper circuit along the *d*-axis; X_{1ad} is the mutual reactance between the stator circuit and the damper circuit along the *d*-axis; X_{1aq} is the mutual reactance between the stator circuit and the damper circuit along the *q*-axis; φ is the angle between positive sequence voltage and current phasors; I_g is the positive sequence current phasor modulus.

Calculation using this method requires values of phase voltages and currents, excitation voltage and excitation current, which can be obtained with PMU. In equations (3-16), the derivatives are written in terms of finite differences, and at each step, an optimization problem is solved, where the unknowns determined in the calculation process are the flux linkages of the generator circuits, the currents of the damper circuits, and the load angle of the SG. When using the described method, the calculation starts from the previous steady state.

The algorithm for calculating the load angle using the system of equations (3-16) is as follows:

1. Downloading the required data from the PMU. If necessary, pre-processing and filtering of data is carried out.
2. Calculation of the positive sequence of stator current and voltage. Recalculation of electrical parameters to relative unit values.
3. Recalculation of generator parameters. In technical data sheets, passports and other documentation, synchronous, transient, subtransient reactances, as well as transient and subtransient time constants are given. In turn, the mathematical model based on the Gorev-Park equations uses the resistances and reactances of the generator circuits. Therefore, at this stage, it is required to recalculate the parameters in accordance with one of the existing approaches [15].
4. Calculation of the initial conditions according to the steady state data preceding the transient.
5. Solution of the optimization problem, which results in the dependence of the change in the load angle during the process.

Thus, this method requires a large number of measurements, including the parameters of the rotor circuit (excitation currents and excitation voltages), as well as a full set of generator parameters (calculated reactances, time constants, no-load excitation current and voltage). The calculation in accordance with this method must be started strictly from the steady state. However, the described approach most fully reflects the transients in the rotor circuits (excitation winding and damper circuits), as a result, providing a more accurate result of calculating the SG load angle.

3 Case studies

3.1 Experiments with model data

Testing the performance of the developed methods, as well as assessing the accuracy of calculating the load angle during electromechanical transients, was carried out using dynamic simulation environment of the SimInTech software package.

Using the model of a two-machine power system, data were obtained for currents, stator voltages, excitation currents and voltages during an electromechanical transient, which is a swing with increasing amplitude and a frequency of 0,86 Hz. Also, the actual (reference) values of the load angle were obtained from the model. Then the calculated values by the proposed methods were compared with the reference values. Figure 1 illustrates the block-schema of test dynamic model.

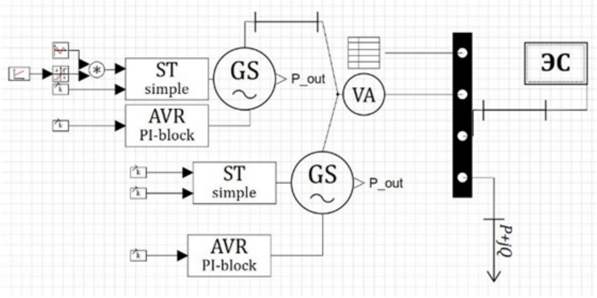

Fig.1. Block-schema of test dynamic model

Figure 2 shows a graph of the change in the active power of the modal generator and the results of calculating the load angle in comparison with the reference value during LFO simulation.

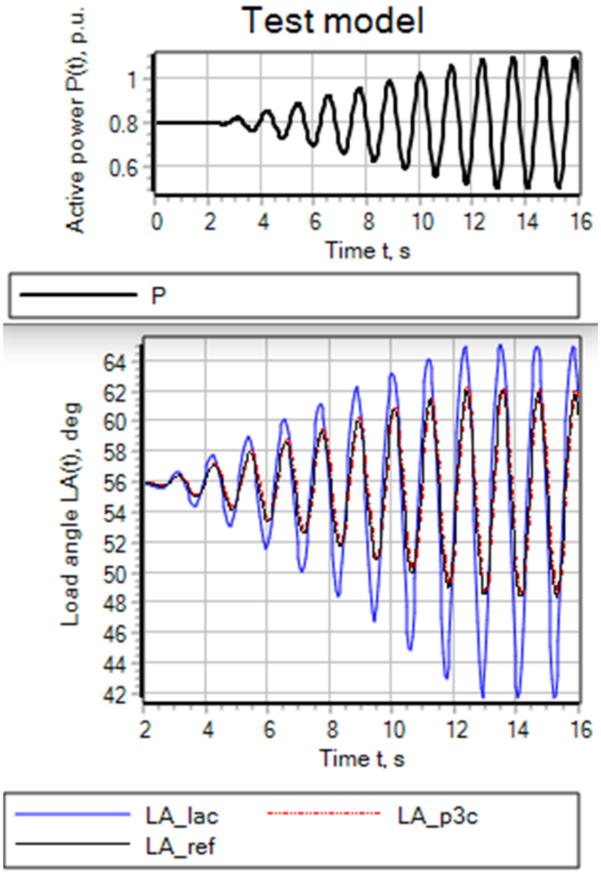

Fig. 2. Active power and load angle during test LFO simulation

In this figure and further, P is active power, LA_lac is load angle calculating by means of (1) and (2) equations; LA_p3c is load angle calculating by means of (3) - (16) equations; LA_ref is reference load angle from model.

Experiments on model data have shown that, in steady state, the methods accurately determine the load angle. In the transient, although both methods give an acceptable result, the accuracy of calculating the load angle using the SG «full» mathematical model is higher. Thus, the average difference between the calculation result and the standard at each measurement point was less than 0,5 degrees for the method based on the «full» mathematical model of the SG, and about four degrees for the method based on the solution of a system of algebraic equations.

3.2 Experiments with real-field data

Experiments with model data proved the efficiency of the developed methods and the correctness of the results of calculating the load angle in transients. At the next stage, these methods were used to calculate the load angle of real generators in the presence of oscillations in the EPS. Two turbogenerators were considered: one from the generators of the Power plant 1 and one from the generators of the Power plant 2. In both cases, LFO took place.

Figure 3 illustrates the graph of the change in the active power of the generator of the Power plant 1 and the part of the load angle calculating results during LFO.

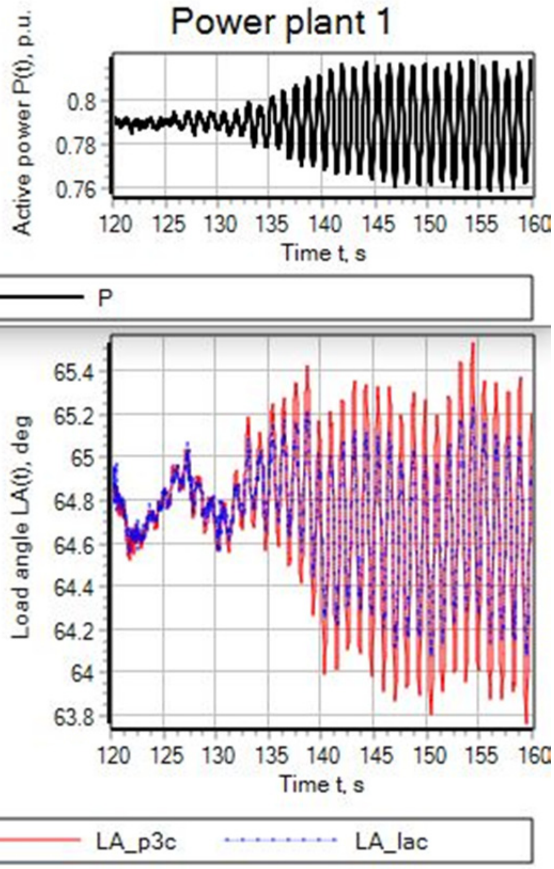

Fig. 3. Active power and load angle during LFO recorded at the Power plant 1

Figure 4 shows a graph of the change in the active power of the Power plant 2 generator and the part of the load angle calculating results during LFO.

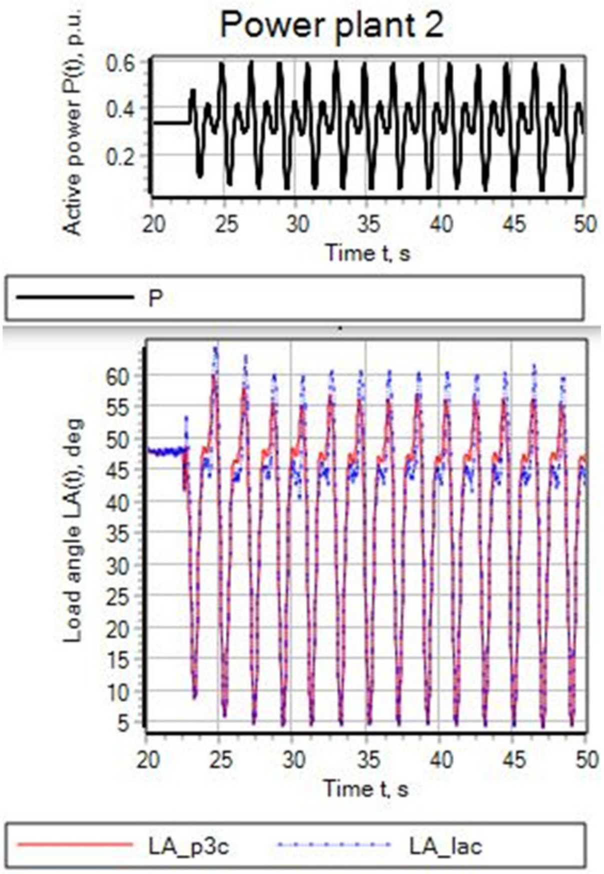

Fig. 4. Active power and load angle during LFO recorded at the Power plant 2

Table 1 shows the time for calculating the load angle of generators by different methods.

Table 1. Comparison of calculation duration.

Plant	Method	Calculation duration, s	Time of transient, s	Number of measuring points
Power plant 1	First method	87,7	220	11000
	Second method	177,7		
Power plant 2	First method	136,7	240	12000

An analysis of the results of calculating the load angle showed that the developed methods almost equally reflect the very nature of the angle change during the transient, however, there is a quantitative difference (primarily in amplitude) between the results, which is at least 10% in the transient. In steady state, the results are absolutely identical. Also, the duration of the calculation by the method based on the solution of a system of algebraic equations is almost two times less than by the method based on the use of the SG mathematical model.

Based on the results, we can conclude that if it is necessary to speed up the calculations, as well as in the presence of only stator measurements, a method based on solving a system of nonlinear algebraic equations should be used. However, in this case, not taking into account the influence of damper circuits can lead to a decrease in the accuracy of calculating the load angle. If a more complete consideration of the influence of all SG circuits is required, the load angle should be calculated using the «full» mathematical model of the generator.

4 Conclusion

The article proposes two methods for calculating the load angle. The first method is based on solving a system of nonlinear algebraic equations, requires only measurements of the stator circuit, has a short calculation time and can be used in real time. However, it does not take into account the influence of damper circuits. The second method is based on solving a system of equations representing the mathematical model of the SG, which takes into account transients in the stator circuit, excitation winding and generator damper circuits. This method requires not only measurements from the stator side, but also measurements of the excitation current and voltage. However, it gives a more accurate result.

The developed methods were successfully tested using the SimInTech software package. Further, using the presented methods, the generators load angles of the Power plant 1 and the Power plant 2 were calculated according to the transients data in the power system.

Further, the authors plan to continue research in the field of application of the calculated load angles in the tasks of developing methods for identifying the oscillations source, as well as methods for quantifying the participation of the SG in the LCO damping.

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