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Transformations of equations with approximation of characteristics of nonlinear elements by power polynomial

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Abstract. The article discusses some transformations of the equation associated with approximations in the form of a polynomial, which is quickly transferred to a convenient equation compiled with respect to the main input and output values and gives generalized dependencies for a number of nonlinear systems. As an example, a part of a magnetic circuit with longitudinal and orthogonal magnetization fields is given. Relations are presented that connect the inductions B_1, B_2 as input and output quantities, which in turn are directly related to the corresponding voltages U_1 and U_2. For all three magnetic circuits, dependencies are obtained that convert the input and output currents i_1, i_2 into input and output inductions B_1, B_2 with an approximating function in the form of a polynomial of any degree.

1 Introduction

When studying processes in electrical circuits with non-linear elements, in some cases the problem arises of transforming equations that include approximations of the characteristics of non-linear elements. These transformations are often reduced to obtaining such dependencies (usually connecting the main initial and input values) that are convenient in the study of amplitude, frequency, phase, and other characteristics. However, such transformations often cause great difficulties inherent in the operation of non-linear dependencies [1-4].

The article discusses some transformations of the equation associated with an approximation in the form of a polynomial, which is sufficient to quickly transfer to a convenient equation compiled with respect to the main input and output quantities and give generalized dependencies for a number of nonlinear systems [4-5].

2 The current state of the investigated problem

As an example, three magnetic circuits are given (Fig. 1, 2, 3), which are parts of certain electrical circuits.

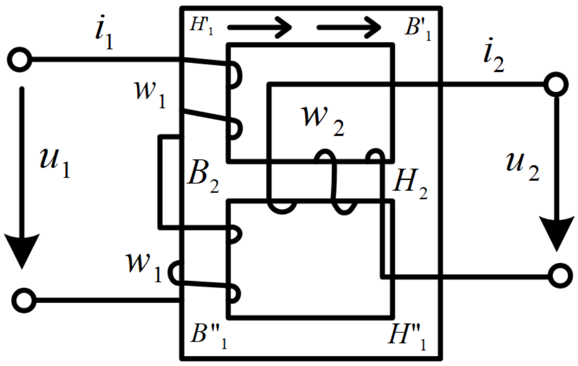

Fig. 1. Magnetic circuit with longitudinal magnetic magnetization circuits

The material of ferromagnetic cores is assumed to be isotropic and the same for all three magnetic circuits, and the dependence of the induction on the magnetic field strength (magnetization curve) is unambiguous and symmetrical relative to the origin of coordinates [6-8]. As is known, such a magnetization curve can be approximated by a polynomial:

$$H = \sum_{23.5} amB^{m} \qquad (1)$$

Where: H – magnetic field strength; m – counted integer; B – magnetic field induction; a_1, a_3, a_5 – approximation coefficients.

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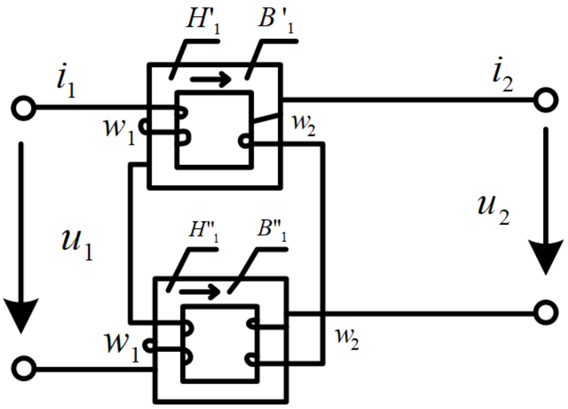

Fig. 2. Double core magnetic circuit

Figures 1, 2 show magnetic circuits with longitudinal magnetic circuits of magnetization. The end rods of a three-rod chain are assumed to be the same. The same condition (identity) is imposed on the cores of a two-core circuit. Figure 3 shows a magnetic circuit with orthogonal magnetic fields of magnetization [9-12].

Equation transformations are made for the general case when any energy sources function in primary and secondary electrical circuits (in one of the electrical circuits, instead of sources, autoparametric excitation of oscillations at any frequency can take place) [13-14].

Let us consider the functional connections of a three-rod magnetic circuit (Fig. 1):

$$i_1 = \frac{l_1}{2W_1}(H_1' + H_1''), \tag{2}$$

$$i_2 = \frac{l_1}{2W_2}(H_1' + H_1'') + \frac{l_2}{W_2}H_2 \tag{3}$$

Where: H_1', H_1'' – strength of the magnetic field of the extreme rods; l_1, l_2 – the lengths of the extreme and rods, respectively.

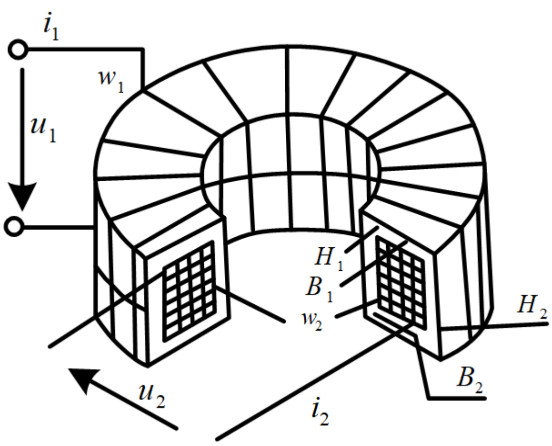

Fig. 3. Magnetic circuit with orthogonal magnetic field of magnetization

Let us introduce approximation (1) into (2) and (3).

$$i_1 = \frac{l_1}{2W_1}\sum_{m=1,3,5} am\,|(B_1')^m + (B_1'')^m| \tag{4}$$

$$i_2 = \frac{l_1}{2W_2}\sum_{m=1,3,5} am\,|(B_1')^m + (B_1'')^m| + \frac{l_2}{W_2}\sum amB_2^m \tag{5}$$

The expressions $(B_1')^m + (B_1'')^m$; $(B_1')^m - (B_1')^m$ in (4) and (5) can be converted to, respectively, linking the input and output quantities by induction [15]. For convenience, we accept $B_1' = x, B_1'' = y$.

Let's introduce new variables:

$$a = x + y,\ \ b = x - y \quad \text{or} \quad x = \frac{a+b}{2},\quad y = \frac{a-b}{2}$$

Then

$$\begin{aligned} x^m \pm y &= \frac{(a-b)^m}{2^m} \pm \frac{(a-b)^m}{2^m} \\ &= \frac{1}{2^m}[(C_m^0 a^m + C_m^1 a^{m-1}b + C_m^2 a^{m\cdot 2}b^2 + \ldots C_m^{m-1}ab^{m-1} \\ &+ C_m^0 a^3 - C_m^1 a^{m-1}b + C_m^2 a^{m-2}b^2 + \ldots + C_m^{m-1}ab^{m-1} \\ &- C_m^m b^m)] = \frac{1}{2^{m-1}}\sum_{k=m-1}^{a} C_m^k (x \pm y)^k (x \pm y)^{m-k} \end{aligned}$$

Where: $K = m-1, m-3, \ldots 2, 0;$ $\quad C_m^k$ – combinations of elements according to *K*.
So,

$$(B_1')^m \pm (B_1'')^m = \frac{1}{2^{m-1}}\sum_{k=m-1}^{0} C_m^k[(B_1') \pm (B_1'')]^k[(B_1') \pm (B_1'']^{m-k} \tag{6}$$

Obviously (Fig. 1)

$$U_1 = W_1 S_1 \frac{dB}{dt} \tag{7}$$

$$B_1 = B_1' + B_1'' \tag{8}$$

$$S_1 B_1' = S_1 B_1'' + S_2 B_2$$

Or

$$B_2 = n(B_1' - B_1'') \tag{9}$$
$$n = S_1/S_2$$

And

$$U_2 = -W_2 S_2 \frac{dB_2}{dt} \tag{10}$$

Taking into account (8) and (9), identity (6) takes the form:

$$(B_1')^m + (B_1'')^m - \frac{1}{2^{m-1}}\sum_{k=m-1}^{0} \frac{C_m^k}{n^k} B_2^k B_1^{m-k}, \tag{11}$$

$$(B_1')^m - \frac{1}{2^{m-1}} \sum_{k=m-1}^{0} \frac{C_m^k}{n^{m-k}} B_1^k B_2^{m-k}. \quad (12)$$

Thus, the expression $(B_1')^m \pm (B_1'')^m$ is converted into relations (11) and (12), which relate the inductions B1, B2 presented here as input and output quantities and, in turn, are directly related to the corresponding voltages U_1 and U_2. [16-18]

Let us introduce identities (11), (12) into (4), (5):

$$i_1 = \sum_{m=1,3,8} \sum_{k=m-1}^{0} \frac{a_m b_1 C_m^k}{2^m n^k W_1} B_2^k B_1^{m-k} \quad (13)$$

$$i_z = \sum_{m=1.3.5...} \sum_{k=m-1}^{0} \frac{a_m l_1 C_m^k}{2^m n^{m-k} \sqrt{2}} B_1^k B_2^{m-k} + \sum_{m=1.3.5...} \frac{a_m l_2}{\sqrt{2}} B_2^m \quad (14)$$

For a two-core magnetic circuit (Fig. 2), functional relationships are valid:

$$i_1 = \frac{C(H_1' + H_1'')}{2W_1} \quad (15)$$

$$i_2 = \frac{C(H_1' - H_1'')}{2W_2} \quad (16)$$

Relations (15) and (16) take the form:

$$i_1 = \sum_{m=1,3,5...} \sum_{k=m-1} \frac{a_m l C_m^k}{2^m W_1} B_2^k B_1^{m-k} \quad (17)$$

$$i_2 = \sum_{m=1,3,5...} \sum_{k=m-1}^{0} \frac{a_m l C_m^k}{2^m W_2} B_1^k B_2^{m-k} \quad (18)$$

For a magnetic circuit with orthogonal magnetic fields (Fig. 3), relations (1, 2) hold:

$$\frac{B_1}{H_1} = \frac{B_2}{H_2} = \frac{B}{H} \quad (19)$$

$$B = \sqrt{B_1^2 + B_2^2}$$

$$H = \sqrt{H_1^2 + H_2^2} \quad (20)$$

Where: B_1, B_2, B - magnetic inductions of the longitudinal, transverse and resulting magnetic fields, respectively; H_1, H_2, H - intensities of the longitudinal, transverse, resulting fields, respectively.

Taking into account (19), (20), (1) and some transformations [19-20], we obtain:

$$H_1 = B_1 \frac{H}{B} = \sum_{m=1,3,5...} a_m B_2 B^{m-1} = \sum_{m=1,3,5...} \sum_{k=m-1}^{0} a_m C_{\frac{m-1}{2}}^{1/2} B_2^k B_1^{m-k}$$

$$H_2 = B_2 \frac{H}{B} = \sum_{m=1,3,5...} a_m B_2 B^{m-1} = \sum_{m=1,3,5...} \sum_{k=m-1}^{0} a_m C_{\frac{m-1}{2}}^{1/2} B_1^k B_2^{m-k}$$

Where: K=m-1, m-3, … 2,0.

Or

$$i_1 = \frac{l_1}{W_1} H_1 = \sum_{m=1,3,5...} \sum_{k=m-1}^{0} \frac{a_m l_1 C_{\frac{m-1}{2}}^{k/2}}{W_1} B_2^k B_1^{m-k} \quad (21)$$

$$i_2 = \frac{l_2}{W_2} H_2 = \sum_{m=1,3,5...} \sum_{k=m-1}^{0} \frac{a_m l_2 C_{\frac{m-1}{2}}^{k/2}}{W_2} B_1^k B_2^{m-k} \quad (22)$$

If the characteristic of a nonlinear element is not symmetric with respect to the origin, then the approximating function has terms and about even powers [21-25]. And for even degrees, one can obtain the following identities:

$$x^m - y^m = \frac{1}{2^{m-1}} \sum_{k=m-1}^{1} C_m^k (x+y)^k (x-y)^{m-2}$$

Where: K=m-1; m-3; … 5; 3; 1;

$$x^m + y^m = \frac{1}{2^{m-1}} \sum_{k=m}^{0} C_m^k (x+y)^k (x-y)^{m-k}$$

Where: K=m; (m-2); (m-4); … 4;2;0.

3 Conclusion

In conclusion, the following conclusions can be drawn:

1. For all three magnetic circuits, dependences are obtained that convert the input and output currents i_1, i_2 into input and output inductions B_1, B_2 about an approximating function in the form of a polynomial of any degree;
2. Relations (13), (14) of a three-rod magnetic circuit, (17), (18) of a two-core one, (21), (22) - circuits with orthogonal fields differ only in the form of the coefficients (the term $\sum_{m=1,3,5} \frac{a_m l_2}{W_2} B_2^m$ in (14) is combined with the first term at K=0, which makes it possible to obtain generalized dependencies for similar magnetic circuits that are parts of certain electrical circuits

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