Mathematical modelling and numerical methods for studying the dynamic stability of the movement of cotton harvesters on slopes

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Abstract. The article deals with the issues of modelling and stability of the movement of semi-mounted cotton harvesters on slopes. To solve this problem, the work has compiled the equations of motion of a cotton harvester (CH), using the example of the MX-2.4 brand, using the Lagrange equations of the second kind. The numerical method Runge-Kutta solved the equations of vertical oscillations of the machine when driving on slopes. Taking into account the uneven distribution of mass between the front driving and back steered wheels, an equation was drawn up and the reaction of the sides, the distribution of mass on the wheels, the coefficients of stiffness and viscous resistance of the tires of the wheel on the slope, the dynamic radii of the wheel, the pressure on the tire tires, the power of the vibration of the machine and the wheels, the resistance rolling of the left back steered wheel, maximum angles of inclination in the front and back parts of the wheels of the machine, tipping moments of the front and back axles, stabilizing moments of the machine, moments of overturning and withdrawal of the machine on slopes.

1 Introduction

One of the important performance indicators of the machine's patency is stability, which characterizes its ability to work on longitudinal and transverse slopes without tipping over. The assessment of the stability of machine-tractor units in dynamics presents great difficulties due to the large number of interacting factors affecting the stability of the system movement [1-9,10,12,13,14,17].

Determination of equilibrium under various conditions is one of the most important factors in the testing processes of cotton harvesters. In this case, the equilibrium depends on the position of the machine and the acting forces. Knowing the position and forces can determine how the cotton harvester will move on slopes.

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2 Materials and methods

In [1,4-9], mathematical models and an algorithm for optimal control of the movement of cotton harvesters under various conditions of the movement were developed. Based on these models and algorithms, we consider the limiting cases of lateral stability loss of cotton harvesters on a slope. The beginning of the overturning is preceded by the redistribution of the support reactions of the machine along the sides.

The limit position occurs at the moment when the vertical plane passing through the centre of mass passes through the tipping axis. On figure 2 a dynamic model of testing a cotton harvester on a cross slope is presented.

For the transverse tipping of cotton harvesters, a smaller value is taken, connecting the centres of the contact areas of the back wheels of one side (Figure 1). The reaction of the sides located down and up the slope with the static position of the machine on the transverse slope with an angle β, is determined from the condition of equilibrium of the machine relative to the axis, passing through the centres of contacts of the wheels of one side [11-16]

$$R_A = \frac{1}{2}G\cos\beta + \frac{h_c G}{B}\sin\beta \quad R_B = \frac{1}{2}G\cos\beta - \frac{h_c G}{B}\sin\beta \tag{1}$$

where: B is the gauge of the cotton harvester varies within 1.2-2.4 m; h_c - the height of the centre of mass of the cotton harvester, 2 м; G= mm g - gravity machine, mm - machine weight, 7745 kg; g is the free fall acceleration, 9.81 m/s 2.

It can be seen from (1) that the load on the board located down the slope, i.e., on the left wheel increases, and the load on the right wheel decreases. The redistribution of normal reactions along the sides depends not only on the track of the machine but also on the vertical coordinate of the centre of mass [11-16].
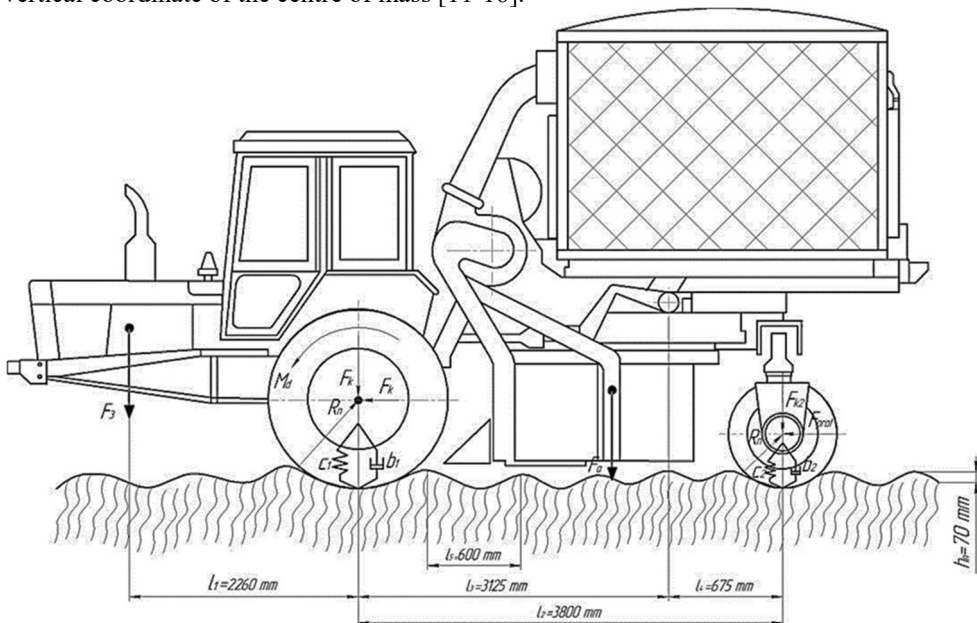

Fig. 1. Calculation scheme CH MX-2.4.

A cotton harvester on a slope is in a statically critical state when the unloaded side response becomes zero, i.e., R B =0.

Based on (1), we determine the reactions of the sides on each wheel of the cotton harvester

$$R_{sfl} = \frac{1}{2} m_2 g \cos \beta + \frac{h_c m_2 g}{B_f} \sin \beta \quad R_{sfr} = \frac{1}{2} m_3 g \cos \beta - \frac{h_c m_3 g}{B_f} \sin \beta \quad , \quad (2)$$

$$R_{sbl} = \frac{1}{2} m_4 g \cos \beta + \frac{h_c m_4 g}{B_b} \sin \beta \quad R_{sbr} = \frac{1}{2} m_5 g \cos \beta - \frac{h_c m_5 g}{B_b} \sin \beta \quad , \quad (3)$$

where: m2, m3, m4, m5 - initial mass distribution on the front left and right, back left and right wheels.

Based on the reactions of the sides, we determine the mass distribution over the wheels on the slope.

$$m_{wfl} = \frac{4R_{sfl}}{g}, \; m_{wfr} = \frac{4R_{sfr}}{g}, \; m_{wbl} = \frac{4R_{sbl}}{g}, \; m_{wbr} = \frac{4R_{sbr}}{g}.$$

We determine the stiffness and viscous resistance coefficients of the wheel tires on the slope

$$c_{fl} = \frac{m_2 \cdot g}{h_w \cdot \dfrac{m_{wfl}}{m_2}}, \; c_{fr} = \frac{m_3 \cdot g}{h_w \cdot \dfrac{m_3}{m_{wfr}}}, \; c_{bl} = \frac{m_4 \cdot g}{h_w \cdot \dfrac{m_{wbl}}{m_4}}, \; c_{br} = \frac{m_5 \cdot g}{h_w \cdot \dfrac{m_5}{m_{wbr}}}$$

$$b_{fl} = \frac{0.64 \cdot c_{fl}}{2\pi V_m}, \; b_{fr} = \frac{0.64 \cdot c_{fr}}{2\pi V_m}, \; b_{bl} = \frac{0.64 \cdot c_{bl}}{2\pi V_m}, \; b_{br} = \frac{0.64 \cdot c_{br}}{2\pi V_m},$$

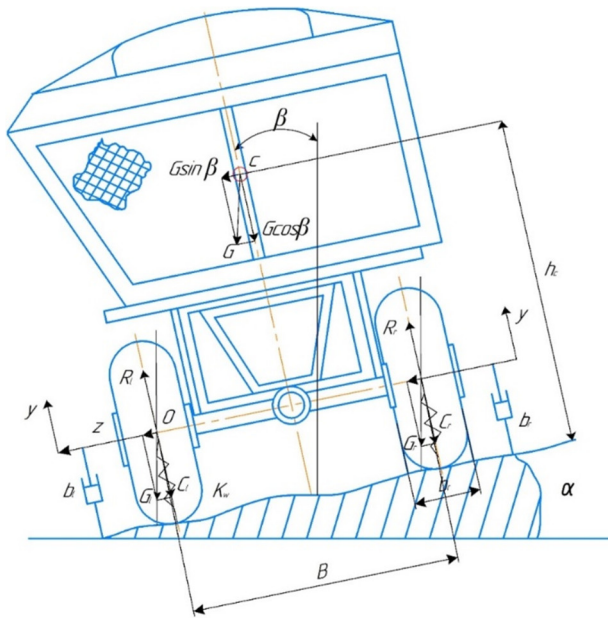where: h w - deflection of the tire of the wheel of the machine.

Fig. 2. Dynamic model of testing a cotton harvesting machine on a transverse slope.

Solution. Based on the Lagrange equations of the second kind, we compose a mathematical model for the vertical oscillation of a cotton harvester in the form [1-9]

$$\left.\begin{aligned}
&m_m \ddot{y}_m = F_m \cdot \sin\alpha - b_{fl}(\dot{y}_m - \dot{y}_{wfl}) - c_{fl}(y_m - y_{wfl}) - b_{fr}(\dot{y}_m - \dot{y}_{wfr}) - c_{fr}(y_m - y_{wfr}) \\
&m_{wfl}\ddot{y}_{wfl} = b_{fl}(\dot{y}_m - \dot{y}_{wfl}) + c_{fl}(y_m - y_{wfl}) - m_{wfl}\frac{2\pi^2 V_{wfl}^2}{l_5^2}h_f\left(1 - \cos\frac{2\pi V_{wfl}}{l_5}t\right) \\
&m_{wfr}\ddot{y}_{wfr} = b_{fr}(\dot{y}_m - \dot{y}_{wfr}) + c_{fr}(y_m - y_{wfr}) - m_{wfr}\frac{2\pi^2 V_{wfr}^2}{l_5^2}h_f\left(1 - \cos\frac{2\pi V_{wfr}}{l_5}t\right) \\
&(m_{wbl} - 0.5 \cdot m_r)\ddot{y}_{wbl} = b_{bl}(\dot{y}_m - \dot{y}_{wbl}) + c_{bl}(y_m - y_{wbl}) - \\
&\qquad\qquad - (m_{wbl} - 0.5 \cdot m_r)\frac{2\pi^2 V_{wbl}^2}{l_5^2}h_f\left(1 - \cos\frac{2\pi V_{wbl}}{l_5}t\right) \\
&(m_{wbr} - 0.5 \cdot m_r)\ddot{y}_{wbr} = b_{br}(\dot{y}_m - \dot{y}_{wbr}) + c_{br}(y_m - y_{wbr}) - \\
&\qquad\qquad - (m_{wbr} - 0.5 \cdot m_r)\frac{2\pi^2 V_{wbr}^2}{l_5^2}h_f\left(1 - \cos\frac{2\pi V_{wbr}}{l_5}t\right)
\end{aligned}\right\}$$
(4)

where: $\dot{y}_i$ and $\ddot{y}_i$ are the linear speeds and acceleration of the machine, front and back wheels; α is the angle of inclination of the tangent to the unevenness at the point of contact of the wheel with the road; bfl, bfr, bbl, bbr, cfl, cfr, cbl, cbr, - coefficients of viscous resistance and tire stiffness of the front left, right, back left, right wheels of the machine; mwfl, mwfr, mwbl, mwbr, mr, is the distribution of mass on the front left, right, back left, right wheels and the engine of the machine on a slope; hf - the height of the roughness of the cotton field road; h c - height of the center of mass of the machine; Vwfl, Vwfr, Vwbl, Vwbr - the speed of movement of the front left, right, back left, right wheels; l 5 - distances between supports and irregularities.

When a wheel (rigid or elastic) rolls on a slope (on a deformable or non-deformable surface), unstabilized and stabilized wheels roll. An example of the rolling of an unstabilized wheel can be the rolling of wheels of a conventional flat type when the longitudinal plane of the wheel is located normally to the surface and at an angle $90^0 - \beta$ to the horizontal plane [11,16].

When an unstabilized elastic wheel rolls on a slope, under the action of a lateral load component, the contact patch of the tire with the slope surface is distorted and takes the form of a curved ellipse. The point of application of the resultant reactions of the soil is shifted by a value e from the longitudinal plane of symmetry of the wheel. In this case, tire side slip occurs, which is estimated by the coefficient k_{sw}. It can be assumed with a sufficient degree of accuracy that the displacement e is directly proportional to the side slip of the tire k_{sw} [11,16].

$$M_{wf}^{frame} = m_{wfl} \cdot \ddot{y}_{wfl}(e_f \cdot \cos\beta + r_{rwfl} \cdot \sin\beta), \quad e_f = \frac{b_f}{6 \cdot h_{max}}(h_{max} - h_0),$$

$$M_{wb}^{frame} = m_{wbl} \cdot \ddot{y}_{wbl}(e_b \cdot \cos\beta + r_{rwbl} \cdot \sin\beta), \quad e_b = \frac{b_b}{6 \cdot h_{max}}(h_{max} - h_0)$$

where: bf , bb - the width of the front and back wheels.

Determine the maximum height of the road roughness [11-16]

$$h_{max} = h_0 + tg\beta, \quad h_0 = \frac{h_f}{2}$$

Under the stabilization of the wheel on the slope is understood as moving it in the transverse plane until the frame of the machine is installed in a vertical position relative to the horizontal plane.

The stabilizing moment of the machine on slopes is

$$M_{\text{wfl}}^{\text{st}} = h_{\text{c}} \cdot m_{\text{wfl}} \cdot \ddot{y}_{\text{wfl}}, \; M_{\text{wfr}}^{\text{st}} = h_{\text{c}} \cdot m_{\text{wfr}} \cdot \ddot{y}_{\text{wfr}},$$

$$M_{\text{wbl}}^{\text{st}} = h_{\text{c}} \cdot m_{\text{wbl}} \cdot \ddot{y}_{\text{wbl}}, \; M_{\text{wbr}}^{\text{st}} = h_{\text{c}} \cdot m_{\text{wbr}} \cdot \ddot{y}_{\text{wbr}}.$$

In real conditions, when a moving car passes over bumps in the road, an additional inertia force arises, which, with its appropriate direction, can increase the overturning moment.

Then the overturning moment and the withdrawal of the machine on the slopes is equal to

$$M_{\text{wf}}^{\text{overt}} = h_{\text{c}}(m_{\text{wfl}} \cdot \ddot{y}_{\text{wfl}} - m_{\text{wfr}} \cdot \ddot{y}_{\text{wfr}}), \; M_{\text{wb}}^{\text{overt}} = h_{\text{c}}(m_{\text{wbl}} \cdot \ddot{y}_{\text{wbl}} - m_{\text{wbr}} \cdot \ddot{y}_{\text{wbr}}),$$

$$M_{\text{wd}} = \frac{m_1 \ddot{y}_{\text{m}} \cdot B}{2} \cos \beta.$$

Wheel slip resistance coefficients are equal to [15].

$$K_{\text{wsf}} = 780(d_{\text{f}} + 2b_{\text{f}})b_{\text{f}}(98 + \frac{s_i}{1000}), \; K_{\text{wsb}} = 780(d_{\text{b}} + 2b_{\text{b}})b_{\text{b}}(98 + \frac{s_i}{1000}).$$

where: df, db - landing diameters of the tires of the front and back wheels; si - tire pressure.

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where d_{f}, d_{b} - landing diameters of the tires of the front and back wheels; s_i - tire pressure.

3 Results and discussion

System (4) and calculation formulas are solved using Runge–Kutta numerical methods under initial conditions t =0. The computational experiment was carried out using the software package [9] and the following parameter values were obtained:

- with tire deflection ht = 30 mm=0.03 m and slope angle α=6 0 35':

cfl = 726466.227 N/m;

r1 = 0.785 m;

bfl =105710.346 Ns/m;

r2 = 0.43 m;

cfr = 95410.672 N/m;

si =103100 atm ;

bfr = 13883.502 Ns/m;

Bf = 2.4 m;

cbl =1497439.277 N/m;

Vf = 1.2 m ;

bbl =217897.018 Ns/m;

bf = 0.394 m;

cbr =196666.661 N/m;

bb = 0.254 m;

bbr =28617.574 Ns/m;

hf =0.07 m;

m1 = m m = 7745 kg;

ht =0.03 m;

m 2 =m 3=2557 kg;

Vm =1.2 m/s;

mwfl = 2221.61 kg;

β = 6 0 35 '

mwfl = 291.77 kg;

$F_y = F_m \sin 45^0 = 11433.9 N;$

m4 =m5=1300 kg;

F m =16170 N.

mwbl = 4579.325 kg;

mwbr = 601.427 kg;

mr = 1262 kg;

Table 1. Font styles for a reference to a journal article.

| T, s	$\dot{y}_m$, m/s	$\ddot{y}_m$, m/s 2	$\dot{y}_{wfl}$, m/s	$\ddot{y}_{wfl}$, m/s 2	$\dot{y}_{wfr}$, m/s	$\ddot{y}_{wfr}$, m/s 2	$\dot{y}_{wbl}$, m/s	$\ddot{y}_{wbl}$, m/s 2	$\dot{y}_{wbr}$, m/s	$\ddot{y}_{wbr}$, m/s 2
1	2	3	4	5	6	7	8	9	10	11
0	0	1.486	0	-0.369	0	-0.369	0	-0.368	0	-0.37
0.1	0.084	0.776	0.0748	0.847	0.0748	0.847	0.076	0.837	0.084	0.776
0.2	0.163	0.790	0.158	0.825	0.158	0.825	0.159	0.819	0.163	0.791
0.3	0.242	0.790	0.24	0.8	0.24	0.8	0.24	0.8	0.242	0.79
0.4	0.32	0.757	0.318	0.755	0.318	0.755	0.318	0.755	0.32	0.757
0.5	0.39 2	0.670	0.389	0.66	0.389	0.66	0.389	0.66	0.39 2	0.67
0.6	0.45	0.556	0.4 5	0.554	0.4 5	0.554	0.45	0.554	0.453	0.556
0.7	0.5	0.501	0.5	0.5 19	0.5	0.5 19	0.5	0.517	0.5	0.5
0.8	0.558	0.585	0.558	0.624	0.558	0.6247	0.558	0.619	0.558	0.585
0.9	0.626	0.786	0.63	0.799	0.63	0.799	0.63	0.796	0.626	0.786
1	0.7	0.675	0.7	0.584	0.7	0.584	0.7	0.595	0.7	0.675
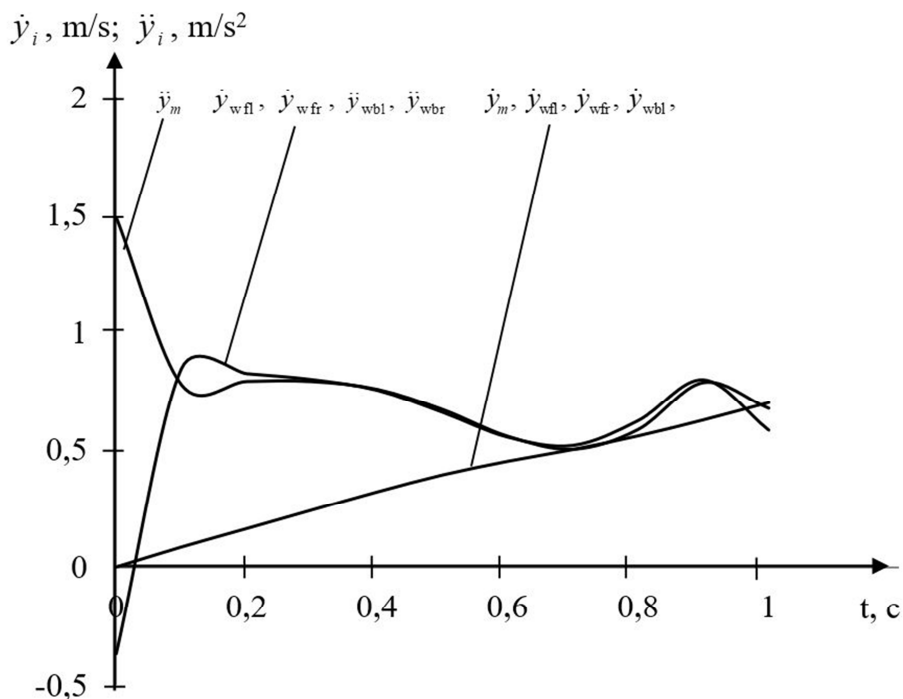
Fig. 3. The nature of the change in the movement parameters CH MX-2.4 for vertical vibrations at ht =30 mm=0.03 m on slopes.

Table 2. The values of the driving forces CH MX-2.4 and on wheels for vertical vibrations at ht =30 mm=0.03 m on slopes α=6 0 35'

| T, s	F_m, N	F_{wfl}, N	F_{wfr}, N	F_{wbl}, N	F_{wbr}, N
1	2	3	4	5	6
0	11433.9	-642.754	-286.14	-1467.694	-0.533
0.1	5971.5	1473.966	656.179	3330.99	1.107
0.2	6085.84	1435.379	639.001	3260.139	1.128
0.3	6082.67	1397.814	622.278	3189.535	1.128
0.4	5829.72	1314.258	585.057	3007.458	1.081
0.5	5161.018	1148.45 7	511.269	2633.464	0.957
0.6	4283.279	963.56	428.958	2207.828	0.794
0.7	3856.957	903.499	402.219	2060.04	0.715
0.8	4506.566	1086.51	483.69	2465.19	0.83 6
0.9	6049.444	1389.97	618.786	3167.76	1.122
1	5200.7	1016.03 6	452.318	2367.354	0.964
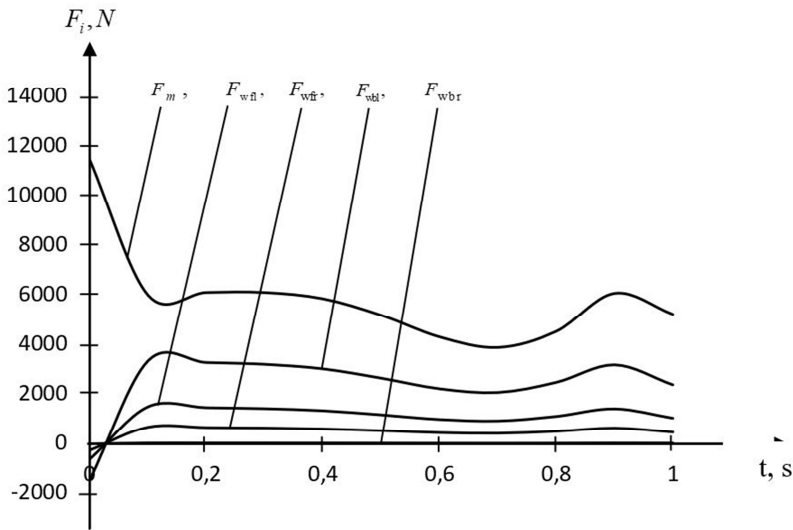
Figure 4. The nature of the change in the driving forces of CH MX-2.4 and on wheels for vertical vibrations at ht =30 mm=0.03 m on slopes α=6 0 35'.

Table 3. Values with stabilizing overturning moments and withdrawal CH MX-2.4 on slopes for vertical oscillations at ht =30 mm=0.03 m on slopes α=6 0 35'

T, s	M_{wf}^{frame}, N	M_{wb}^{frame}, N	M_{wf}^{overt}, N	M_{wb}^{overt}, N	M_{wd}, N
1	2	3	4	5	6
0	-89.888	-137.57	-713.226	-2928. 6	13720.7
0.1	206.132	312.229	1635.574	6732.796	7165.8
0.2	200.73 6	305.587	1592.755	6551.906	7303.01

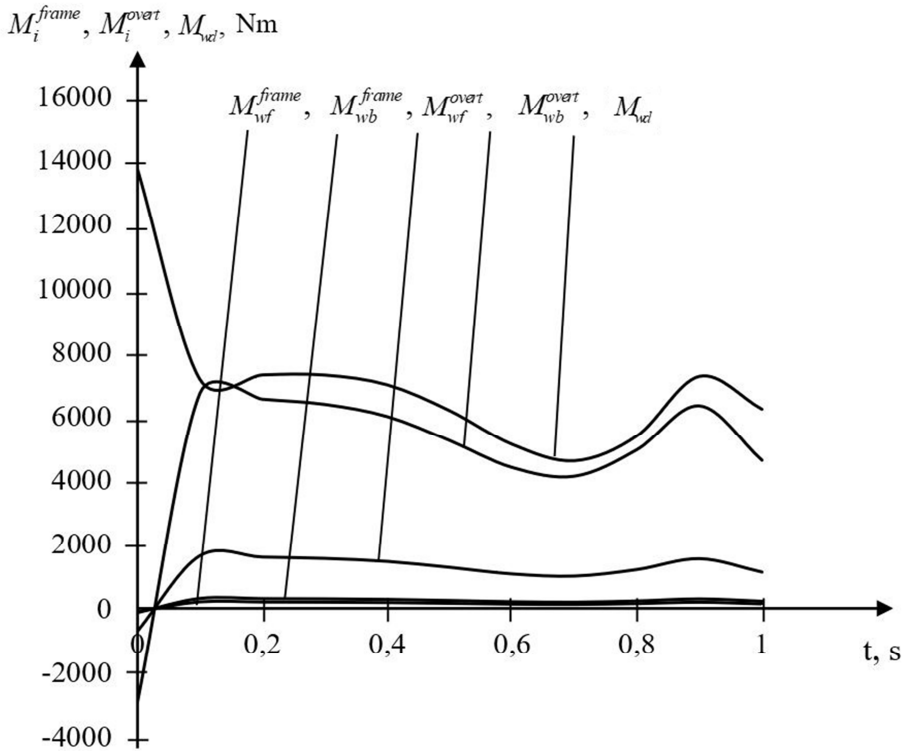| 0.3	195.48	298.969	1551.07	6389.945	7299.2
0.4	183.79	281.902	1458.29 7	6010.46	6995.6 7
0.5	160.61	246.846	1274.375	5254.248	6193.222
0.6	134.753	206.9 5	1069.208	4411.813	5139.935
0.7	126.353	193.09 7	1002.56	4138.272	4628.348
0.8	151.947	231.073	1205.637	4969.1 7	5407.8 8
0.9	194.385	296.928	1542.368	6345.064	7259.33
1	142.09	221.9	1127.435	4635.813	6240.847

Fig. 5. The nature of the change in the stabilizing, tipping moments and withdrawal of CH MX-2.4 on the slopes for vertical oscillations at ht =30 mm=0.03 m on slopes α=6 0 35'.

4 Conclusion

Mathematical models of the dynamic stability of the movement of the cotton harvester on the slopes have been developed. The motion parameters are determined, with stabilizing, overturning moments and withdrawal of CH MX-2.4 on slopes for vertical oscillations at ht =30 mm=0.03 m on slopes α=6 0 35'. In this case, the reactions of the sides on each wheel of the cotton harvester are:

Rsfl = 4265.267 N, Rsfr = 1898.81 N, Rsbl = 11230.79458 N, Rsbr = 1474.99 N.

Slip resistance coefficients:

Kwsf = 108351.357 N/m, Kwsb = 36431.46 N/m.

The forces of vertical vibrations are:

$$F_m = 5200.7...11433.9 \text{ N},$$

$$F_{wfl} = -642.754...1473.966 \, \text{N}, \qquad F_{wbl} = -0.533...1.128 \, \text{N},$$

$$F_{wfr} = -286.14...656.179 \, \text{N}, \qquad M_{wfl}^{st} = 628.478 \, \text{Nm}$$

$$F_{wbl} = 1467.694...3330.99 \, \text{N},$$

$$M_{wfr}^{st} = 136.229 \, \text{Nm}$$ mean that, CH MX-2.4 are at the maximum slope angle.

Thus, the developed mathematical model, computational algorithm and software package for optimal control of vertical oscillations during movements on a slope make it possible to optimize the design parameters of the cotton harvester and significantly reduce the time for experimental finishing.

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