E3S Web of Conferences **387**, 04015 (2023) https://doi.org/10.1051/e3sconf/202338704015
ICSERET-2023

THEORYTICAL APPROACH ON APPLICATION OF GENERALIZED CLOSED SETS ENVIRONMENTAL LIFESTYLE USING r-NEIGHBOURHOOD IDEALSPACES

THARMAR S[1], *Veerasivaji* R[2] & *Senthil kumar* R[3]

[1,3]Departmet of Mathematics, Mohamed Sathak Engineering College, Kilakarai, Tamil Nadu, India.
[2]Assistant Professor ,Department of Mathematics,Sri Sankara Arts and Science College, Enathur Kanchipuram , India.

Abstract. We introduce in this paper, the new notion of R_I-g-closed sets and obtain some of its characterizations in ideal r-Neighbourhood space and give nice results on R_I-g-closed sets with examples. Finally, we discuss application of R_I-g-closed sets.

Keywords: Relation, I_r-closed set, R_I-g-closed set, R_I-g-open set.

1. Introduction

In 1970, the notion of generalized closed sets introduced by Levine [5] and various Generalized concepts in topology were introduced by Levine [5] and various authors were making modifications in generalized concepts. Lin[6] and Yao[9] introduced the concept of rough sets in neighbourhood system and Hosny[2] generated different topologies by the concept of idealization of j-approximation spaces.

In this paper, we introduce the new notion of R_g^I closed sets and obtain some of its characterizations in ideal r-Neighbourhood space and give nice results on R_g^I closed sets with example. Finally, we discuss application of R_g^I closed sets with an example.

1Correspondingauthor: tharmar11585@gmail.com * vracvag@gmail.com, srisenthil2011@gmail.com

2. Preliminaries

Definition 2.1. [3] A non-empty collection **I** of subsets of a set U is called an ideal on U, if it satisfies the following conditions.

(1) A∈I and B∈I ⇒ A∪B∈I.
(2) A∈I and B⊂A ⇒ B∈I.

Definition 2.2. [6] Let U be non-empty finite set and R be an arbitrary binary relation on U. The r-neighbourhood of x ∈ ∪ (N_r (x)) is defined as r-neighbourhood: $N_r(x) = \{y∈U: xRy\}$.

Definition 2.3. [6] Let U be a non-empty finite set and R be an arbitrary binary relation on U and $\sum_r$:U⟶P(U) be a mapping which assigns for each x_r in U its r-neighbourhood in P(U). The triple (U, R, $\sum_r$) is called a r-neighbourhood space (in briefly, r-NS).

Theorem 2.4. [6] Let U be a r-NS and A⊂U. Then, the collection $\tau_r = \{A⊂U : ∀p∈A, N_r(p)⊂A\}$ is a topology on U.

Definition 2.5. [2] Let U be a r-NS and a subset A⊂U is called r- open set if $A∈\tau_r$ and the complement of r-open set is called r-closed set.

Definition 2.6 [2] Let U be a r-NS and I be an ideal on U. Then, the collections $\tau_r^I = \{A⊂U : ∀p∈A, N_r(p)\cap A'∈I\}$ is a topology on U.

The r-Neighbourhood space with an ideal I is called ideal r-NS.

Definition 2.7 [2]

(1) Let U be an ideal r-NS and a subset A⊂U is called I_r-open set if $A∈\tau_r^I$ and the complement of I_r-open set is called I_r-closed set.
(2)$\underline{R_r^I}$(A)=∪$\{G ∈ \tau_r^I$:G⊂A}=int_r^I(A)(int_r^I is called I_r-lower approximations)
(3) $\overline{R_r^I}$(A) =∩$\{F ∈ (\tau_r^I)^c$: A⊂F}=$cl_r^I(A)$ (cl_r^I is called $\mathbf{I_r}$-upper approximations).

Lemma 2.8. [2] Let U be an ideal r-NS and A, B⊂U. Then

(1) $\underline{R_r^I}(A) ⊂ A ⊂ \overline{R_r^I}(A)$.
(2) $A ⊂ B \Longrightarrow \overline{R_r^I}(A) ⊂ \overline{R_r^I}(B)$
(3) $A ⊂ B \Longrightarrow \underline{R_r^I}(A) ⊂ \underline{R_r^I}(B)$
(4) $\overline{R_r^I}(\overline{R_r^I}(A)) = \overline{R_r^I}(A)$.
(5) $\underline{R_r^I}(\underline{R_r^I}(A)) = \underline{R_r^I}(A)$.

3. RI- g-CLOSED SETS

Definition 3.1 Let U be an ideal r-NS and a subset A_r is said to be R_I-g-closed set if $\overline{R_r^I}(A_r)$ ⊂O_r whenever A_r⊂O_r and O_r is I_r-open.
The complement of R_I-g-closed set is called R_I-g-open set.

Example 3.2 Let U={Food, Health, Exercise, Sleep} and we get the relation R={(Food, Food), (Food, Health), (Health, Health), (Health, Exercise), (Exercise, Exercise), (Exercise, Sleep), (Exercise, Health), (Sleep, Sleep), (Sleep, Health)} and the ideal I={∅, Health}.

The neighbourhood of Food is {Food, Health}, neighbourhood of Health is {Health, Exercise}, neighbourhood of Exercise is {Health, Exercise, Sleep} and neighbourhood of Sleep is {Health, Sleep}.

The members of I_r-open sets are {Food}, {Sleep}, {Food, Sleep}, {Exercise, Sleep}, {Food, Exercise, Sleep}, {Health, Exercise, Sleep}, ∅, U and the members of I_r-closed sets are {Food}, {Health}, {Food, Health}, {Health, Exercise}, {Food, Health, Exercise}, {Health, Exercise, Sleep}, ∅, U.

Subset of U (A_r)	**$N_r(P)$, p∈ A_r**	**$\overline{R_r^I}(A_r)$**	**$A_r \subset O_r$ and O_r is I_r-open**	**Member of R_g^I closed**
{Food}	{ Food, Health }	Singleton member {Food}	{Food}, {Food, Sleep}, {Food, Exercise, Sleep}	Yes
{Health}	Two members { Exercise, Health}	Singleton member {Health}	{ Exercise, Health, Sleep}	Yes
{Exercise}	{Exercise, Health, Sleep}	{ Health, Exercise}	{Exercise, Sleep} and {Food, Exercise, Sleep} and {Health, Exercise, Sleep}	No
Subset of U (A_r)	**$N_r(P)$, p∈ A_r**	**$\overline{R_r^I}(A_r)$**	**$A_r \subset O_r$ and O_r is I_r-open**	**Member of R_g^I closed**
{Sleep}	{Health, Sleep}	{ Exercise, Sleep, Health,}	{Sleep}	No
{ Food, Health}	{ Food, Health } and { Health, Exercise}	{ Food, Health }	U	Yes
{Food, Exercise}	{ Food, Health } and {Health, Exercise, Sleep}	{Food, Health, Exercise}	{Food, Exercise, Sleep} and U	No
{Food, Sleep}	{ Food, Health } and {Health, Sleep}	U	{Food, Sleep} and {Food, Exercise, Sleep}, U	No
{ Exercise	{ Exercise,	{ Exercise,	{Exercise,	Yes

Health }	Health}, {Exercise, Health, Sleep}	Health}	Health, Sleep}, U	
{Health, Sleep}	{ Exercise, Health}, {Health, Sleep}	{ Exercise, Health, Sleep}	{Exercise, Health, Sleep}, U	Yes
{Exercise, Sleep}	{Health, Sleep}, {Health, Exercise, Sleep}	Health, Exercise, Sleep}	{Exercise, Sleep}, {Food, Exercise, Sleep},{Health, Exercise, Sleep, U	No
{Food, Exercise, Health}	{ Exercise, Food, Health}, { Health},{Health, Sleep, Exercise}	{Food, Exercise, Health}	U	Yes
{Food, Exercise, Sleep}	{ Food, Health }, {Health, Sleep} {Sleep, Exercise, Health}	U	{Food, Sleep, Exercise}, U	No
{ Health, Sleep, Exercise}	{Exercise, Health, Sleep}, { Health, Exercise},{Health, Sleep}	{Exercise, Health, Sleep}	{ Sleep, Exercise, Health}, U	Yes
{ Health, Food, Sleep}	{ Exercise, Health},{Health, Sleep}, {Food, Health}	U	U	Yes
Subset of U (A_r)	**N_r(P), p∈ A_r**	**$\overline{R_r^I}(A_r)$**	**A_r⊂O_r and O_r is I_r-open**	**Member of R_g^I closed**
∅	∅	∅	All I_r-open sets	Yes
U	U	U	U	Yes

Theorem 3.3 Every I_r-closed set is R_I-g-closed.
Proof. The indication comes from the reality that $\overline{R_r^I}(A_r)=A_r$.

Remark 3.4 In the Example 3.2, the member {Exercise} in R_I-g-closed set but not in I_r-closed.

Theorem 3.5 Let U be an ideal r-NS and $A_r \subset U$. Then $x_r \in \overline{R_r^I}(A_r)$ if and only if $O_r \cap A_r \neq \emptyset$ for every I_r-open set O_r containing x_r.

Proof. Let O_r be an I_r-open set such that $x_r \in O_r$ and $O_r \cap A_r = \emptyset$. Then $A_r \subset U - O_r$ and $U - O_r$ is I_r-closed set and hence $\overline{R_r^I}(A_r) \subset U - O_r$. Since $x_r \notin U - O_r$ which implies $x_r \notin \overline{R_r^I}(A_r)$. Conversely, let $x_r \notin \overline{R_r^I}(A_r)$. Then there exists an I_r-closed set F_r such that $A_r \subset F_r$ and $x_r \notin F_r$. Hence $(U - F_r) \cap A_r = \emptyset$.

Theorem 3.6 If A_r and B_r are R_I-g-closed then

(1) $A_r \cup B_r$ is R_I-g-closed.

(2) $A_r \cap B_r$ is R_I-g-closed.

Proof. The reality, $\overline{R_r^I}(A_r \cup B_r) = \overline{R_r^I}(A_r) \cup \overline{R_r^I}(B_r)$ and $\overline{R_r^I}(A_r \cap B_r) \subset \overline{R_r^I}(A_r) \cap \overline{R_r^I}(B_r)$ gives the proof.

Remark 3.7 The collection of R_I-g-closed sets form a topology.

Theorem 3.8 If A_r is R_I-g-closed and $A_r \subset B_r \subset \overline{R_r^I}(A_r)$ then B_r is R_I-g-closed.

Proof. Let $B_r \subset O_r$ and O_r be I_r-open. Then $A_r \subset U$ and A_r is R_I-g-closed. Therefore $\overline{R_r^I}(A_r) \subset U$ which implies $\overline{R_r^I}(B_r) \subset U$. Hence B_r is R_I-g-closed.

Theorem 3.9 Let U be an ideal r-NS and $A_r \subset U$. Then A_r is R_I-g-open if and only if $F_r \subset \underline{R_r^I}(A_r)$ whenever $F_r \subset A_r$ and F_r is I_r-closed.

Proof. Let A_r be R_I-g-open and $F_r \subset A_r$ and F_r be I_r-closed. Then $U - A_r \subset U - F_r$ and $U - F_r$ is I_r-open. Since $U - A_r$ is R_I-g-closed, $\overline{R_r^I}(U - A_r) \subset U - F_r$ and $U - \underline{R_r^I}(A_r) = \overline{R_r^I}(U - A_r) \subset U - F_r$. Hence $F_r \subset \underline{R_r^I}(A_r)$.

Conversely, let $U - A_r \subset O_r$ where O_r is I_r-open. Then $U - O_r \subset A_r$ and $U - O_r$ is I_r-closed. By hypothesis, we have $U - O_r \subset \underline{R_r^I}(A_r)$ and hence $\overline{R_r^I}(U - A_r) = U - \underline{R_r^I}(Ar) \subset O_r$. Hence A_r is R_I-g-open.

Theorem 3.10 Let U be an ideal r-NS and $A_r \subset U$.

(1) A_r is R_I-g-closed,

(2) $\overline{R_r^I}(A_r) \subset O_r$ such that $A_r \subset O_r$ and O_r is I_r-open,

(3) $\overline{R_r^I}(A_r) \cap F_r = \emptyset$ whenever $A_r \cap F_r = \emptyset$ and F_r is I_r-closed.

The statements (1), (2) and (3) are equivalent.

Proof. (1) $\Leftrightarrow$ (2) Directly we get by the definition3.1.

(2) $\Longrightarrow$ (3) Let $A_r \cap F_r = \emptyset$ and F_r be I_r-closed. Then $A_r \subset U - F_r$ and $U - F_r$ is I_r-open. By (2), $\overline{R_r^I}(A_r) \subset U - F_r$. Hence $\overline{R_r^I}(A_r) \cap F_r = \emptyset$.

(3)$\Longrightarrow$(1) Let $A_r \subset O_r$ where O_r is I_r-open. Then $A_r \cap (U - O_r) = \emptyset$ and $U - O_r$ is I_r-closed. By (3), $\overline{R_r^I}(A_r) \cap (U - O_r) = \emptyset$ which implies that $\overline{R_r^I}(A_r) \subset O_r$. Hence A_r is R_I-g-closed.

4. Application

In the example 3.2, the members of I_r-open sets are {Food}, {Sleep}, {Food, Sleep}, {Exercise, Sleep}, {Food, Exercise, Sleep}, {Health, Exercise, Sleep}, ∅, U and the members of I_r-closed sets are {Food}, {Health}, {Food, Health}, {Health, Exercise}, {Food, Health, Exercise}, {Health, Exercise, Sleep}, ∅, U.

https://doi.org/10.1051/e3sconf/202338704015

The members of R_I-g-open sets are {Food}, {Exercise}, {Sleep}, {Food, Exercise}, {Food, Sleep}, {Exercise, Sleep}, {Food, Exercise, Sleep}, {Health, Exercise, Sleep}, ∅, U and the members of I_r-closed sets are {Food}, {Health}, {Food, Health}, {Health, Exercise}, {Food, Health, Exercise}, {Health, Exercise, Sleep}, ∅, U.

Therefore the members in difference of I_r-open sets and R_I-g-open sets are {Exercise} and {Food, Exercise} and hence the common activity is Exercise and hence Exercise is base for all human activities.

5. Conclusion

Further study about r-neighbourhood space, may give many solutions for the real life problems.

References

[1] M. E. Abd El-Monsef, A. S. Salama and O.A. Embaby, Granular computing using mixed neighborhood systems, Jour. of Inst. of Mathematics & Computer Science (Computer Science Ser.), 20(2009), 233–243.
[2] M. Hosny, Idealization of j-Approximation spaces, Faculty of sciences and mathematics, 34(2)(2020), 287-301.
[3] D. Jankovic, T. R. Hamlet, New topologies from old via ideals, Amer.Math.Monthly 97 (1990) 295 310.
[4] A. M. Kozae, On topology expansions by ideals and applications, Chaos, Soli- tons and Fractals, 13(2002), 55–60.
[5] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2)(19)(1970), 89-96.
[6] T. Y. Lin, Granular computing on binary relations II: Rough set representations and belief functions, in: A. Skowron, L. Polkowski (Eds.), Rough Sets In Knowledge Discovery, Physica-Verlag, (1998), 121–140.
[7] A. S. Salama, Some topological properties of rough sets with tools for data mining, Int. J. Compu. Sci., 8(2011), 588–595.
[8] O. A. E. Tantawy, Heba. I. Mustafa, On rough approximations via ideal, Inform. Sci., 251(2013), 114–125.
[9] Y. Y. Yao, Rough sets, neighborhood systems and granular computing, Proceedings of the IEEE Canadian Conference on Electrical and Computer Engineering, (1999), 1553–1559.
[10] W. Zhu, Topological approaches to covering rough sets, Inform. Sci., 177(2007), 1499–15.