Divided Square Difference Cordial Labeling Of Join Some Spider graphs

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AbstractLet G be a graph with itsvertices and edges. On defining bijective functionρ:V(G) →{0,1,...,p}. For each edge assign the label with 1 if ρ*(ab)=$\left|\frac{\rho(a)^2-\rho(b)^2}{\rho(a)-\rho(b)}\right|$ is odd or 0 otherwise such that $|e_\rho(1) -$ *eρ0≤1* then the labeling is called as divided square difference cordial labeling graph. We prove in this paper for relatively possible set of spider graphs with atmost one legs greaterthan one namely J(SP(1^m,2^n)) ,J(SP(1^m,2^n,3^1)), (SP(1^m,2^n,3^2)),J(SP(1^m,2^n,4^1)),J(SP(1^m,2^n,5^1).
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1. Introduction

Graph theory plays a significant role in computers and research area in graph labeling finds a suitable study on constructing algorithm . We consider in our discussion finite graph which constitutes the vertices and edges . Different labeling models have been given in Gallian JA[1] 'A dynamic Survey of graph labeling'. Most of graph labeling techniques resultsfrom the paper by A.Rosa[2].

A graph with difference labeling is summarised by assigning integer values being difference among its vertices where edges are associated with its absolute difference which are associated with each pair of distinct vertices.

The square difference labeling was introduced by Shiama[3] .Alfred Leo[4] identified divided square difference labeling and labeled the graphs and its classes with the divided square difference labeling. Further study on the labeling and its behaviour is extensively found in many papers by various authors [5] and [6].

2. Preliminaries

Definition 2.1

A treeis spider with center vertex C having degree and other vertices are either degree 2 or a leaf.

Definition 2.2

A bijective function ρ:V(G) →{0,1,...,p}. For each edge assign the label with 1 if ρ*(ab)=$\left|\frac{\rho(a)^2-\rho(b)^2}{\rho(a)-\rho(b)}\right|$ is odd or 0 otherwise such that $\left|e_\rho(1) - e_\rho(0)\right| \leq 1$ then the labeling is called as divided square difference cordial labeling graph.

Definition 2.3: Construction of Join of SP(1^m,2^n)

Spider graph SP(1^m,2^n) is defined as a tree with m legs of length n. Now Let us construct Join of Spider graph SP(1^m,2^n) by attaching another graph SP(1^m,2^n) by attaching centre vertex.

We call the graph so obtained as Join of Spider graph SP(1^m,2^n) and denoteit as J(SP(1^m,2^n))

Definition 2.4: Construction of Join of SP(1^m,2^n,3^1)

Spider graph SP(1^m,2^n, 3^1) is defined as a tree with m legs of length n and 3^1. Now Let us constructJoin of Spider graph SP(1^m,2^n,3^1) by attaching another graph SP(1^m,2^n,3^1) by attaching centre vertex.

We call the graph soobtained as Join of Spider graph SP(1^m,2^n,3^1) and denoteit as J(SP(1^m,2^n,3^1)).

Definition 2.5: Construction of Join of SP(1^m,2^n,3^2)

Spider graph SP(1^m,2^n,3^2) is defined as a tree with m legs of length n and 3^2 .Now Let us construct Join of Spider graph SP(1^m,2^n,3^2) by attaching another graph SP(1^m,2^n,3^2) by attaching centre vertex.

We call the graph so obtained as Join of Spider graph SP(1^m,2^n,3^2) and denoteit as J(SP(1^m,2^n,3^2)).

Definition 2.6: Construction of Join of SP(1^m,2^n,4^1)

Spider graph SP(1^m,2^n,4^1) is defined as a tree with m legs of length n and 4^1. Now Let us construct Join of Spider graph SP(1^m,2^n,4^1) by attaching another graph SP(1^m,2^n,4^1) by attaching centre vertex.

We call the graph so obtained as Join of Spider graph SP(1^m,2^n,4^1) and denote it as J(SP(1^m,2^n,4^1)).

Definition 2.7: Construction of Join of SP(1^m,2^n,5^1)

Spider graph SP(1^m,2^n,5^1)is defined as a tree with m legs of length n and 5^1. Now Let us construct Join of Spider graph SP(1^m,2^n,5^1) by attaching another graph SP(1^m,2^n,5^1) by attaching centre vertex.;

We call the graph so obtained as Join of Spider graph SP(1^m,2^n,5^1) and denoteit as J(SP(1^m,2^n,5^1)).

Theorem 2.1

Join of Spider graph J(SP(1^m,2^n)) is divided square cordial labeling graph ∀ m and n being integers.

Proof :

Let G= J(SP(1^m,2^n)) with vertices and edges where

V[J(SP(1^m,2^n)) = $\{a, a', a_i, a_i', b_j, b_j', c_j, c_j'; \quad 1\leq i, j \leq n\}$

E[J(SP(1^m,2^n)) = $\{e_i = aa_i, 1 \le i \le n\} \cup \{e_i' = ab_j; 1 \le j \le n\} \cup \{e_i''' = b_jc_j; 1 \le j \le n\} \cup \{e_i'''' = a'a_i'; 1 \le i \le n\} \cup \{e_i''''' = a'b_j'; 1 \le j \le n\} \cup \{e_i'''''' = b_j'c_j'; 1 \le j \le n\} \cup \{e_i''''''' = aa'\}$

Now let us prove that the graph G is divided square difference cordial labeling graph for all values of m and n.

Weconsider the vertices to take the label as below.

$\rho(a) = 1$ $\rho(a_1) = 2$ $\rho(a_{i+1}) = 6i+4$; $1 \le i \le n$

$\rho(b_1) = 3$ $\rho(b_{j+1}) = 6j+3$; $1 \le j \le n$

$\rho(c_1)=4$ $\rho(c_{j+1}) = 6j+5$; $1 \le j \le n$

$$\rho(a_i') = 6i \;\; 1 \le i \le n \qquad \rho(a') = 5$$

$$\rho(b_j') = 6j + 1 \qquad \rho(c_j') = 6j + 21 \le j \le n$$

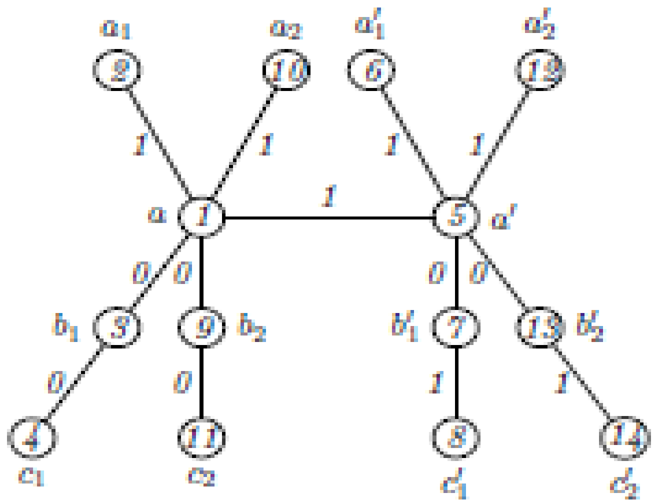
The edges are computed as below

$$\rho^*(aa') = 1 \quad \rho^*(aa_i) = 1 \;\; \rho^*(a'a_i') = 1 \qquad 1 \le i \le n$$
$$\rho^*(ab_j) = 0 \qquad \rho^*(b_jc_j) = 0 \qquad 1 \le j \le n$$
$$\rho^*(ab_j') = 0 \qquad \rho^*(b_j'c_j') = 11 \le j \le n$$

We find that induced edge labeling satisfies the condition

$$|e_\rho(1) - e_\rho(0)| \le 1$$

Example 2.1:Join of Spider graph J(SP(1^2,2^2))



Theorem 2.2

Join of Spider graph J(SP(1^m,2^n,3^1)) isdivided square cordial labeling graph ∀ m and n being integers.

Proof :

Let G= J(SP(1^m,2^n,3^1)) with vertices and edges where
V[J(SP(1^m,2^n,3^1)) = $\{a, a', a_i, a_i', b_j, b_j', c_j, c_j', l_k', l_k; \quad 1 \le i, j \le n; 1 \le k \le 3\}$

E[J(SP(1^m,2^n,3^1)) $= \{e_i = aa_i, 1 \leq i \leq n\} \cup \{e_i' = ab_j; 1 \leq j \leq n\} \cup \{e_i'' = b_j c_j; 1 \leq j \leq n\} \cup \{e_i''' = a'a_i'; 1 \leq i \leq n\} \cup \{e_i'''' = a'b_j'; 1 \leq j \leq n\} \cup \{e_i''''' = b_j' c_j'; 1 \leq j \leq n\} \cup \{e_i'''''' = (aa')(al_1)(l_1 l_2)(l_2 l_3)(a'l_1')(l_1' l_2')(l_2' l_3')\}$

Now let us provethat the graph G isdivided square difference cordial labeling graph for all values of m and n.

Weconsider the vertices to take the label as below.

$\rho(a) = 1$ $\rho(a_1) = 2$ $\rho(a_{i+1}) = 6i+10$; $1 \leq i \leq n$

$\rho(a') = 8$ $\rho(a_1') = 9$ $\rho(a_{i+1}') = 6i+12$; $1 \leq i \leq n$

$\rho(b_1) = 3$ $\rho(b_{j+1}) = 6j+9$; $1 \leq j \leq n$

$\rho(b_1') = 10$ $\rho(b_{j+1}') = 6j+13$; $1 \leq j \leq n$

$\rho(c_1) = 4$ $\rho(c_{j+1}) = 6j+11$; $1 \leq j \leq n$

$\rho(c_1') = 11$ $\rho(c_{j+1}') = 6j+14$; $1 \leq j \leq n$

$$\rho(\mathrm{l}_1) = 5 \quad \rho(\mathrm{l}_2) = 7 \quad \rho(\mathrm{l}_3) = 6$$

$$\rho(\mathrm{l}_1') = 12 \qquad \rho(\mathrm{l}_2') = 14 \quad \rho(\mathrm{l}_3') = 13$$

The edges are computed as below

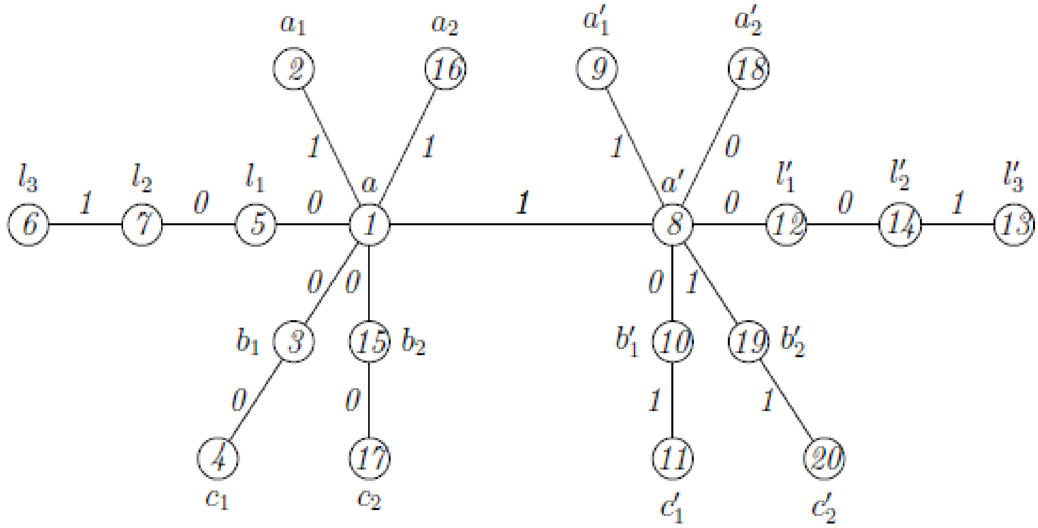
$$\rho^*(aa') = 1 \quad \rho^*(aa_i) = 1 \quad \rho^*(a'a_1') = 1 \quad \rho^*(a'a_{i+1}') = 0 \qquad 1 \leq i \leq n$$
$$\rho^*(ab_j) = 0 \qquad \rho^*(b_1 c_1) = 1 \qquad \rho^*(b_{j+1} c_{j+1}) = 0 \qquad 1 \leq j \leq n$$
$$\rho^*(a'b_1') = 0 \qquad \rho^*(a'b_{j+1}') = 1 \qquad \rho^*(b_j' c_j') = 11 \leq j \leq n$$
$$\rho^*(a'l_1') = 0 \quad \rho^*(l_1' l_2') = 0 \quad \rho^*(l_2' l_3') = 1$$
$$\rho^*(al_1) = 0 \;\; \rho^*(l_2 l_3) = 1 \; \rho^*(l_1 l_2) = 0$$

We find that induced edge labeling satisfies the condition

$$|e_\rho(1) - e_\rho(0)| \leq 1$$

Example 2.2:Join of Spider graph J(SP(1^2,2^2,3^1))

 https://doi.org/10.1051/e3sconf/202338705006



Theorem 2.3

Join of Spider graph J(SP(1^m,2^n,3^2)) is divided square cordial labeling graph ∀ m and n being integers.

Proof :

Let G= J(SP(1^m,2^n,3^2)) withvertices and edgeswhere

V[J(SP(1^m,2^n,3^2)) = $\{a, a', a_i, a'_i, b_j, b'_j, c_j, c'_j, l'_k, l_k; \quad 1 \le i, j \le n; 1 \le k \le 6\}$

E[J(SP(1^m,2^n,3^2)) = $\{e_i = aa_i, 1 \le i \le n\} \cup \{e'_i = ab_j; 1 \le j \le n\} \cup \{e''_i = b_jc_j; 1 \le j \le n\} \cup \{e'''_i = a'a'_i; 1 \le i \le n\} \cup \{e''''_i = a'b'_j; 1 \le j \le n\} \cup \{e'''''_i = b'_jc'_j; 1 \le j \le n\} \cup \{e''''''_i = (aa')(al_1)(al_4)(l_1l_2)(l_2l_3)(l_4l_5)(l_5l_6)\} \cup$
$\{e'''''''_i = (a'l'_1)(l'_1l'_2)(l'_2l'_3)(a'l'_4)(l'_4l'_5)(l'_5l'_6)\}$

Now let us prove that the graph G is divided square difference cordial labeling graph for all values of m and n.

Weconsider the vertices to take the label as below.

$\rho(a)$= 1 $\quad \rho(a_1)$ = 2 $\quad \rho(a_{i+1})$ = 6i+15; $\quad 1 \le i \le n$

$\rho(a')$= 11 $\quad \rho(a'_1)$ = 12 $\quad \rho(a'_{i+1})$ = 6i+18; $\quad 1 \le i \le n$

$\rho(b_1)$ = 3 $\quad \rho(b_{j+1})$ = 6j+17; $\quad 1 \le j \le n$

$\rho(b'_1)$ = 13 $\quad \rho(b'_{j+1})$ = 6j+19; $\quad 1 \le j \le n$

$\rho(c_1)$ = 4 $\quad \rho(c_{j+1})$ = 6j+16; $\quad 1 \le j \le n$

$\rho(c'_1)$ = 14 $\quad \rho(c'_{j+1})$ = 6j+20; $\quad 1 \le j \le n$

$\rho(\mathrm{l}_k) = 3k + 2 \qquad \rho(\mathrm{l}_{k+2}) = 3k + 4 \qquad \rho(\mathrm{l}_{k+4}) = 3k + 3 \qquad 1 \le k \le 2$
$\rho(\mathrm{l}'_k) = 3k + 12 \qquad \rho(\mathrm{l}'_{k+2}) = 3k + 14 \quad \rho(\mathrm{l}'_{k+4}) = 3k + 131 \le k \le 2$

The edges are computed as below

ρ^*(aa') = 1 $\rho^*(aa_1) = 1$ $\rho^*(aa_{i+1}) = 0$ $\rho^*(a'a_i') = 1$ $1 \le i \le n$

$\rho^*(ab_j) = 0$ $\rho^*(b_j c_j) = 1$ $1 \le j \le n$

$\rho^*(a'b_j') = 0$ $\rho^*(b_j'c_j') = 11 \le j \le n$

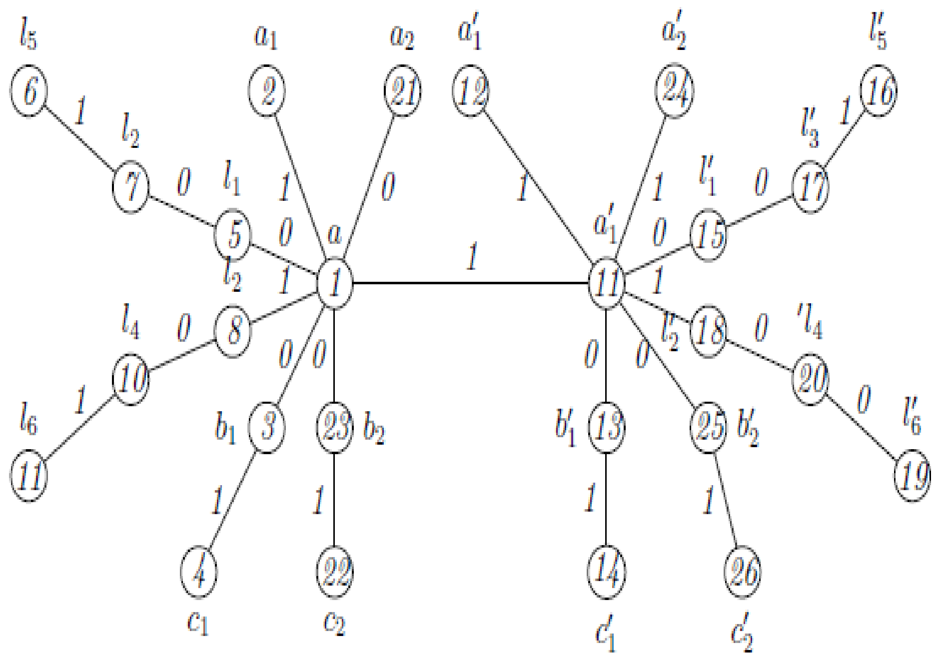
$\rho^*(al_1) = 0$ $\rho^*(al_2) = 1$ $\rho^*(l_1l_3) = 0$ $\rho^*(l_2l_4) = 0$ $\rho^*(l_3l_5) = 1$ $\rho^*(l_4l_6) = 1$

$\rho^*(a'l_1') = 0$ $\rho^*(a'l_2') = 1$ $\rho^*(l_1'l_3') = 0$ $\rho^*(l_3'l_5') = 1$ $\rho^*(l_2'l_4') = 0$ $\rho^*(l_4'l_6') = 0$

We find that induced edge labeling satisfies the condition

$$\left|e_\rho(1) - e_\rho(0)\right| \le 1$$

Example 2.3:Join of Spider graph J(SP(1^2,2^2,3^2))



Theorem 2.4

Join of Spider graph J(SP(1^m,2^n,4^1)) is divided square cordial labeling graph ∀ m and n being integers.

Proof :

Let G= J(SP(1^m,2^n,4^1)) withvertices and edges where

V[J(SP(1^m,2^n,4^1)) = $\{a, a', a_i, a_i', b_j, b_j', c_j, c_j', l_k', l_k;\quad 1 \le i, j \le n; 1 \le k \le 4\}$

E[J(SP(1^m,2^n,4^1)) = $\{e_i = aa_i, 1 \le i \le n\} \cup \{e_i' = ab_j; 1 \le j \le n\} \cup \{e_i'' = b_j c_j; 1 \le j \le n\} \cup \{e_i''' = a'a_i'; 1 \le i \le n\} \cup \{e_i'''' = a'b_j'; 1 \le j \le n\} \cup \{e_i''''' = b_j'c_j'; 1 \le j \le n\} \cup \{e_i'''''' = (aa')(al_1)(l_1l_2)(l_2l_3)(l_3l_4)\} \cup \{e_i''''''' = (a'l_1')(l_1'l_2')(l_2'l_3')(l_3'l_4')\}$

Now let us prove that the graph G is divided square difference cordial labeling graph for all values of m and n.

We consider the vertices to take the label as below.

$\rho(a)$= 1 $\rho(a_1)$ = 2 $\rho(a_{i+1})$ = 6i+12; $1 \le i \le n$

$\rho(a')$= 9 $\rho(a'_1)$ = 10 $\rho(a'_{i+1})$ = 6i+14; $1 \le i \le n$

$\rho(b_1)$ = 3 $\rho(b_{j+1})$ = 6j+11; $1 \le j \le n$

$\rho(b'_1)$ = 11 $\rho(b'_{j+1})$ = 6j+15; $1 \le j \le n$

$\rho(c_1)$ = 4 $\rho(c_{j+1})$ = 6j+13; $1 \le j \le n$

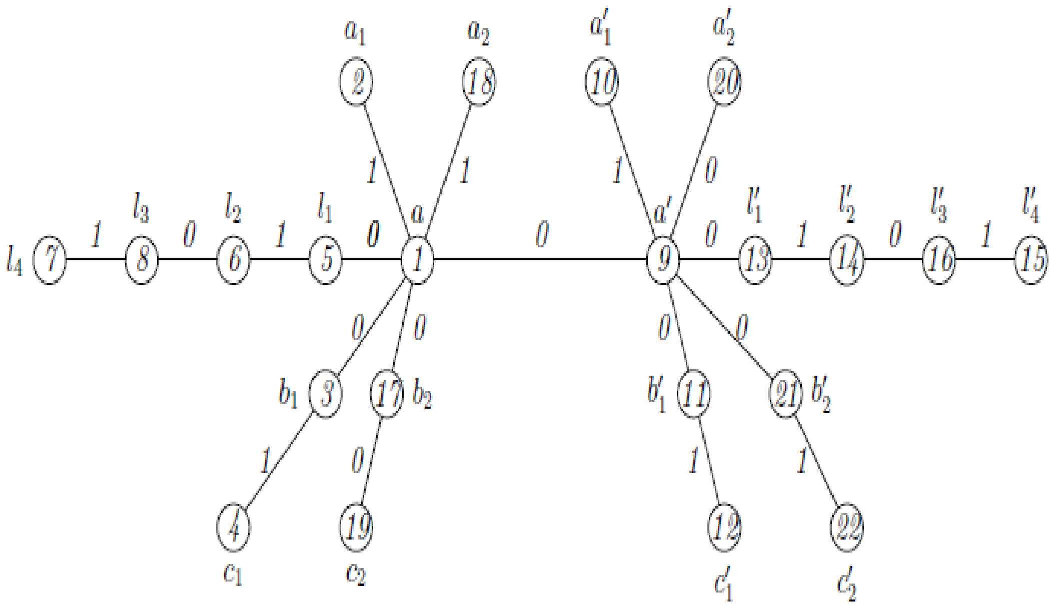
$\rho(c'_1)$ = 12 $\rho(c'_{j+1})$ = 6j+16; $1 \le j \le n$

$\rho(l_1) = 5 \quad \rho(l_2) = 6 \quad \rho(l_3) = 8 \quad \rho(l_4) = 7$
$\rho(l'_1) = 13 \quad \rho(l'_2) = 14 \quad \rho(l'_3) = 16 \; \rho(l'_4) = 15$

The edges are computed as below
ρ^*(aa') = 0 $\rho^*(aa_i) = 1$ $\rho^*(a'a'_i) = 1 \quad 1 \le i \le n$
$\rho^*(ab_j) = 0 \quad \rho^*(b_1c_1) = 1 \quad \rho^*(b_{j+1}c_{j+1}) = 0 \quad 1 \le j \le n$
$\rho^*(a'b'_j) = 0 \quad \rho^*(b'_jc'_j) = 1 1 \le j \le n$
$\rho^*(al_1) = 0 \quad \rho^*(l_1l_2) = 1 \quad \rho^*(l_2l_3) = 0 \quad \rho^*(l_3l_4) = 1$
$\rho^*(a'l'_1) = 0 \quad \rho^*(l'_1l'_2) = 1 \quad \rho^*(l'_2l'_3) = 0 \quad \rho^*(l'_3l'_4) = 1$
We find that induced edge labeling satisfies the condition

$$|e_\rho(1) - e_\rho(0)| \le 1$$

Example 2.4:Join of Spider graph J(SP(1^2,2^2,4^1))



Theorem 2.5

Join of Spider graph J(SP(1^m,2^n,5^1)) isdivided square cordial labeling graph ∀ m and n being integers.

Proof :

Let G= J(SP(1^m,2^n,5^1)) withvertices and edgeswhere

V[J(SP(1^m,2^n,5^1)) = $\{a, a', a_i, a_i', b_j, b_j', c_j, c_j', l_k', l_k; \quad 1\le i, j \le n; 1 \le k \le 5\}$

E[J(SP(1^m,2^n,5^1)) = $\{e_i = aa_i, 1 \le i \le n\} \cup \{e_i' = ab_j; 1 \le j \le n\} \cup \{e_i'' = b_jc_j; 1 \le j \le n\} \cup \{e_i''' = a'a_i'; 1 \le i \le n\} \cup \{e_i'''' = a'b_j'; 1 \le j \le n\} \cup \{e_i''''' = b_j'c_j'; 1 \le j \le n\} \cup \{e_i'''''' = (aa')(al_1)(l_1l_2)(l_2l_3)(l_3l_4)(l_4l_5)\} \cup \{e_i''''''' = (a'l_1')(l_1'l_2')(l_2'l_3')(l_3'l_4')(l_4'l_5')\}$

Now let us prove that the graph G is divided square difference cordial labeling graph for all values of m and n.

Weconsider the vertices to take the label as below.

$\rho(a)$= 1 $\rho(a_1)$ = 2 $\rho(a_{i+1})$ = 6i+13; $1 \le i \le n$

$\rho(a')$= 10 $\rho(a_1')$ = 11 $\rho(a_{i+1}')$ = 6i+16; $1 \le i \le n$

$\rho(b_1)$ = 3 $\rho(b_{j+1})$ = 6j+14; $1 \le j \le n$

$\rho(b_1')$ = 12 $\rho(b_{j+1}')$ = 6j+18; $1 \le j \le n$

$\rho(c_1)$ = 4 $\rho(c_{j+1})$ = 6j+15; $1 \le j \le n$

$\rho(c_1')$ = 13 $\rho(c_{j+1}')$ = 6j+17; $1 \le j \le n$

$$\rho(\mathrm{l}_1) = 5 \quad \rho(\mathrm{l}_2) = 7 \quad \rho(\mathrm{l}_3) = 6 \quad \rho(\mathrm{l}_4) = 8 \quad \rho(\mathrm{l}_5) = 9$$
$$\rho(\mathrm{l}_1') = 14 \quad \rho(\mathrm{l}_2') = 16 \quad \rho(\mathrm{l}_3') = 15 \quad \rho(\mathrm{l}_4') = 17 \quad \rho(\mathrm{l}_5') = 18$$

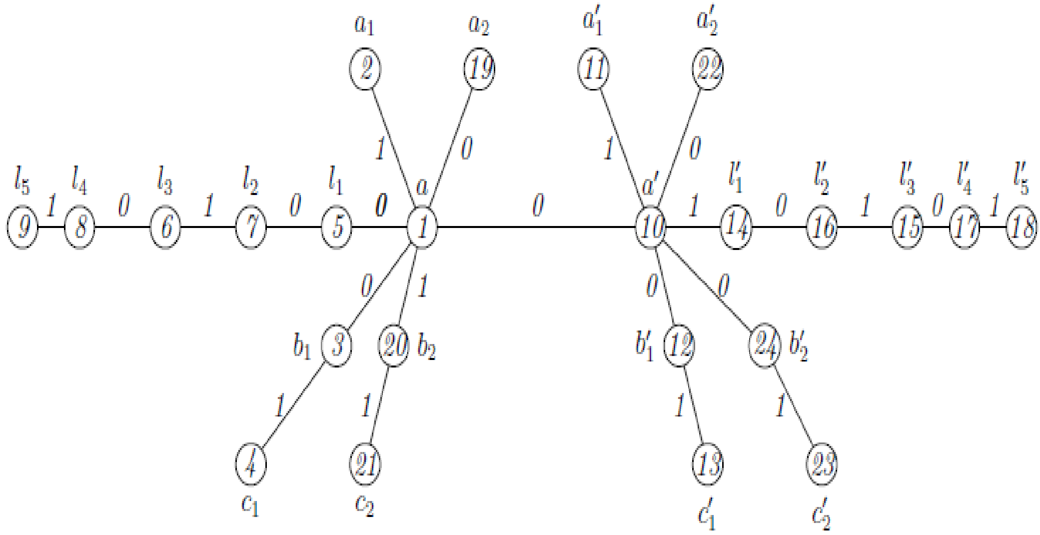
The edges are computed as below

$$\rho^*(\mathrm{aa'}) = 0 \rho^*(aa_1) = 1 \quad \rho^*(aa_{i+1}) = 0 \quad 1 \le i \le n$$
$$\rho^*(a'a_1') = 1 \quad \rho^*(a'a_{i+1}') = 0 \quad 1 \le i \le n$$
$$\rho^*(ab_1) = 0 \quad \rho^*\left(ab_{j+1}\right) = 1 \quad \rho^*\left(b_jc_j\right) = 1 \quad 1 \le j \le n$$
$$\rho^*\left(a'b_j'\right) = 0 \quad \rho^*\left(b_j'c_j'\right) = 11 \le j \le n$$
$$\rho^*(al_1) = 0 \quad \rho^*(l_1l_2) = 0 \quad \rho^*(l_2l_3) = 1 \quad \rho^*(l_3l_4) = 0 \quad \rho^*(l_4l_5) = 1$$
$$\rho^*(a'l_1') = 0 \quad \rho^*(l_1'l_2') = 0 \quad \rho^*(l_2'l_3') = 1 \quad \rho^*(l_3'l_4') = 0 \quad \rho^*(l_4'l_5') = 1$$

We find that induced edge labeling satisfies the condition

$$\left|e_\rho(1) - e_\rho(0)\right| \le 1$$

Example 2.5: Join of Spider graph J(SP(1^2,2^2,5^1))



3.Conclusion

In understanding the different labeling techniques and its applications we in this paper have found some classes of spider graph satisfying the divided square difference cordial labeling conditions. We in our future discussion intend to find applications to the labeling techniques employed through Join spider graphs in this paper. We also are in the process of identifying some special classes of graphs which can be proved to be divided square difference cordial labeling. Divided square difference cordial labeling provides a base among the labelingtechnqiues and in the process of identifying graphs which are suitable for this labeling we found that there are some more special graphs which are likely to satisfy this labeling. We also further look for the applications through this labeling in computers.

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