On Edge Proper Interval-Valued Complex Fuzzy Graphs

¹R. Venkateshwara¹, ²R.Sridevi

 ¹Ph.D. (Part Time)Research Scholar, PG and Research Department of Mathematics, Sri S.Ramasamy Naidu Memorial College(Autonomous),Sattur, Affiliated to Madurai Kamaraj University, Madurai
²Assistant Professor, PG and Research Department of Mathematics, Sri S.Ramasamy NaiduMemorial College(Autonomous), Sattur, Affiliated to Madurai KamarajUniversity, Madurai

ABSTRACT: This article summarizes about edge proper interval-valued complex fuzzy graph. Here, degree of an edge, total degree of an edge, edge proper and edge totally proper interval-valued complex fuzzy were introduced. A necessary and sufficient condition in which these two graphs are equivalent is provided. Many properties of edge proper interval-valued complex fuzzy graphs are examined and they were investigated for edge totally proper interval-valued complex fuzzy graphs,

Keywords:degree of a vertex in interval-valued complex fuzzy graph, total degree, proper interval-valued complex fuzzy graph, totally proper interval-valued complex fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph. A.M.S subject classification: 05C72

1. INTRODUCTION

global understanding Mathematics a powerful tool for provides effectivewayofbuildingnewthings.InMathematics,GraphTheoryisthestudyof graphs, which are mathematical structures used to model pairwise relationsbetween objects. Graphs are the basis for many new things, ideas, concepts, operations and process in everyday life. In many real applications, the membership degree in a fuzzy set cannot be lightly confirmed. It is more reasonable to give an interval-valued data to describe membership degree. In 2011, Akram and Dudek[1] defined the interval-valued fuzzy graph as an extension of fuzzy graph in which the values of the membership degrees are intervals of numbers instead of the number and described operations on it. R. Venkateshwara and R. Sridevi[10] defined the concept of interval-valued complex fuzzy graph. Throughout this paper ivcf graph denote interval-valued complex fuzzy graph.

2. PRELIMINARIES

Definition 2.1.A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions (σ,μ) , where $\sigma:V\to [0,1]$ is a fuzzy subset of a nonempty set V and $\mu:V\times V\to [0,1]$ is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\mu(u,v)\leq \sigma(u)\wedge \sigma(v)$ is satisfied. The underlying crisp graph of the fuzzy graph $G:(\sigma,\mu)$ is denoted as $G^*(V,E)$ where $E\subseteq V\times V$.

Definition 2.2.A interval-valued fuzzy graph with an underlying set V is defined to be the pair (A, B), where $A = (\mu_A^-, \mu_A^+)$ is an interval-valued fuzzy set on V and $B = (\mu_B^-, \mu_B^+)$ is an interval-valued fuzzy set on E such that $\mu_B^-(x,y) \le \min\{\mu_A^-(x), \mu_A^-(y)\}$ and $\mu_B^+(x,y) \le \min\{\mu_A^+(x), \mu_A^+(y)\}$ for all $(x,y \in E)$. Here, A is called interval-valued fuzzy vertex set on V and B is called interval-valued fuzzy edge set on E.

1Correspondingauthor: venkisippi@gmail.com

Definition 2.3. Let G: (A, B) be an interval-valued fuzzy graph, where $A = (\mu_A^-, \mu_A^+)$ and $B = (\mu_B^-, \mu_B^+)$. The positive degree of a vertex $u \in G$ is defined as $d^+(u) = \sum \mu_B^+(u, v)$, for $uv \in E$. The negative degree of a vertex $u \in G$ is defined as $d^-(u) = \sum \mu_B^-(u, v)$, for $uv \in E$ and $\mu_B^+(u, v) = \mu_B^-(u, v) = 0$ if uv not in E. The degree of a vertex u is defined as $d(u) = (d^-(u), d^+(u))$.

Definition 2.4. Let G:(A,B) be an interval-valued fuzzy graph on $G^*(V,E)$. If all the edges of G have the same degree, then G is said to be regular interval-valued fuzzy graph.

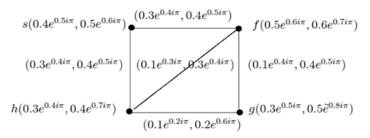
Definition 2.5. Let G:(A,B) be an interval-valued fuzzy graph on $G^*(V,E)$. The positive degree of an edge is defined as $d^+(uv) = d^+(u) + d^+(v) - 2\mu_B^+(uv)$. The negative degree of an edge is defined as $d^-(uv) = d^-(u) + d^-(v) - 2\mu_B^-(uv)$. The degree of an edge is defined as $d(uv) = (d^-(uv), d^+(uv))$. The minimum degree of an edge is $\delta_E(G) = \Lambda \{d(uv): uv \in E\}$. The maximum degree of an edge is $\Delta_E(G) = \Lambda \{d(uv): uv \in E\}$.

Definition 2.6. Let G:(A,B) be an interval-valued fuzzy graph on $G^*(V,E)$. The total positive degree of an edge is defined as $td^+(uv) = d^+(u) + d^+(v) - \mu_B^+(uv)$. The total negative degree of an edge is defined as $td^-(uv) = d^-(u) + d^-(v) - \mu_B^-(uv)$. The degree of an edge is defined as $td(uv) = (td^-(uv), td^+(uv))$. The minimum total degree of an edge is $t\delta_E(G) = \Lambda \{td(uv): uv \in E\}$. The maximum total degree of an edge is $t\Delta_E(G) = V\{td(uv): uv \in E\}$.

3. Edge Proper Complex Interval-Valued Fuzzy Graphs

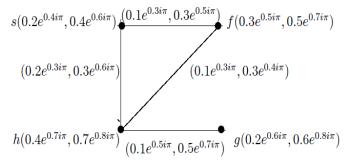
Definition 3.1.Let G:(S,T) be an interval-valued complex fuzzy graph on $G^*(F,I)$. The positive degree of an edge is defined as $d_I^+(uv) = d_I^+(u) + d_I^+(v) - 2\mu_T^+(uv)e^{i\sum \beta_T(uv)}$. The negative degree of an edge is defined as $d_I^-(uv) = d_I^-(u) + d_I^-(v) - 2\mu_T^-(uv)e^{i\sum \alpha_T(uv)}$. The degree of an edge is defined as $d_I^-(uv) = (d_I^-(uv), d_I^+(uv))$.

Example 3.2. Consider a ivef graph on $G^*(F, I)$.



Definition 3.3.Let G: (S,T) be an interval-valued complex fuzzy graph on $G^*(F,I)$. The positive total degree of an edge is defined as $td_I^+(uv) = d_I^+(u) + d_I^+(v) - \mu_T^+(uv)e^{i\sum\beta_T(uv)}$. The total negative degree of an edge is defined as $td_I^-(uv) = d_I^-(u) + d_I^-(v) - \mu_T^-(uv)e^{i\sum\alpha_T(uv)}$. The total degree of an edge is defined as $td_I^-(uv) + td_I^-(uv)$. It can also be defined as $td_I^-(uv) + T(uv)$.

Example 3.4. Consider a ivef graph on $G^*(F, I)$.



Definition 3.5.Let G:(S,T) be an ivcf graph on $G^*(F,I)$. If each edge in G has the same degree $(k_1e^{is_1\pi},k_2e^{is_2\pi})$, then G is said to be an edge proper ivcf-graph.

Definition 3.6.Let G:(S,T) be an ivcf graph on $G^*(F,I)$. If each edge in G has the same total degree $(c_1e^{it_1\pi}, c_2e^{it\pi})$, then G is said to be an edge totally proper ivcf-graph.

Theorem 3.7.Let G:(S,T) be an ivef graph on $G^*(F,I)$. Then T is constant function if and only if the following are equivalent

- (i) G is an $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ edge proper ivef graph.
- (ii) G is a totally $((r_1 + j_1)e^{i(s_1 + k_1)\pi}, (r_2 + j_2)e^{i(s_2 + k_2)\pi})$ edge proper ivef graph.

Proof.Assume T is constant function. Let $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ for all $gx \in I$. Suppose that G is $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ edge proper ivef graph. Then $d_l(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ for all $gx \in I$. Now $td_l(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ $d_{I}(gx) + T(gx)$ for all $gx \in I \Rightarrow td_{I}(gx) = (r_{1}e^{is_{1}\pi}, r_{2}e^{is_{2}\pi}) + (j_{1}e^{ik_{1}\pi}, j_{2}e^{ik_{2}\pi})$ for all $gx \in I \Rightarrow td_{I}(gx) = (r_{1}e^{is_{1}\pi}, r_{2}e^{is_{2}\pi})$ $((r_1+j_1)e^{i(s_1+k_1)\pi},(r_2+j_2)e^{i(s_2+k_2)\pi}) \text{ for all } gx \in I. \text{ Hence } G \text{ is a } ((r_1+j_1)e^{i(s_1+k_1)\pi},(r_2+j_2)e^{i(s_2+k_2)\pi})$ edge totally proper ivef graph. Thus (i) imply (ii) is proved. Now assume G is $((r_1 + j_1)e^{i(s_1 + k_1)\pi}, (r_2 + j_3)e^{i(s_1 + k_2)\pi})$ $(j_2)e^{i(s_2+k_2)\pi}$. Then $td_I(gx) = ((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi})$ for all $gx \in I \Rightarrow d_I(gx) + T(gx) = (r_1+j_1)e^{i(s_1+k_2)\pi}$ $(r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$ for all $gx \in I \Rightarrow d_I(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ for all $gx \in I$. So, G is $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper ivef graph. Thus (ii) imply (i) is proved. Hence (i) and (ii) are equivalent. Conversely, assume (i) and (ii) are equivalent. Let G be a $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper ivef graph and totally $((r_1+j_1)e^{i(s_1+k_1)\pi},(r_2+j_2)e^{i(s_2+k_2)\pi})$ - edge proper ivcf graph. Assume T is not a constant function, then $T(gx) \neq T(kl)$ for at least one pair of edges $gx, kl \in I$. Since G is $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper ivef graph. $d_I(gx) = d_I(kl) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) \Rightarrow td_I(gx) = d_I(gx) + T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + T(gx)$ and $td_I(kl) = d_I(kl) + T(kl) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + T(kl)$. Since $T(gx) \neq T(kl) \Rightarrow td_I(gx) \neq td_I(kl)$. Hence G is not totally $((r_1 + j_1)e^{i(s_1+k_1)\pi}, (r_2 + j_2)e^{i(s_2+k_2)\pi})$ - edge proper ivef graph. Which is a contradiction. Now, let G be a totally $((r_1 + j_1)e^{i(s_1 + k_1)\pi}, (r_2 + j_2)e^{i(s_2 + k_2)\pi})$ - edge proper ivef graph. Then $td_I(gx) = td_I(kl) \Rightarrow$ $d_I(gx) + T(gx) = d_I(kl) + T(kl) \Rightarrow d_I(gx) - d_I(kl) = T(kl) - T(gx) \neq 0 \Rightarrow d_I(gx) \neq d_I(kl)$. Thus G is not $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper ivef graph. Which is a contradiction. Hence T is constant function. Theorem 3.8.Let G:(S,T) be an ivef graph on $G^*(F,I)$. Then T is constant function if and only if the following are equivalent

- (i) G is a $(a_1e^{ib_1\pi}, a_2e^{ib_2\pi})$ proper ivef graph
- (ii) G is an $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ edge proper ivef graph.
- (iii) G is a totally $((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi})$ edge proper ivcf graph.

Proof: Proof of the theorem is similar to Theorem 3.7.

Theorem 3.9. If aivcf graph G is both edge proper and totally edge proper then T is constant function.

Proof: Let G be a $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper and totally $((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi})$ - edge proper ivef graph. Then $d_I(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ and $td_I(gx) = ((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi})$ for all $gx \in I$. Now $td_I(gx) = \left((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi}\right) \Rightarrow d_I(gx) + T(gx) = \left((r_1+j_1)e^{i(s_1+k_1)\pi}, (r_2+j_2)e^{i(s_2+k_2)\pi}\right)$ for all $gx \in I \Rightarrow (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + T(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$ for all $gx \in I$. Hence $T(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (j_1e^{ik_1\pi}, j_2e^{ik_2\pi}) - (r_1e^{is_1\pi}, r_2e^{is_2\pi}) = (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$. Thus T is constant function.

4. Properties of Edge Proper and Edge totally Proper Interval-Valued Complex Fuzzy Graph

Definition 4.1.Let G: (S, T) be an ivef graph on $G^*(F, I)$. If the underlying graph G^* is a proper graph, then G is said to be partially proper ivef graph.

Theorem 4.2.Let G: (S, T) be an ivef graph on $G^*(F, I)$ such that T is a constant function. Then G is a proper ivef graph if and only if G is a partially proper ivef graph.

Proof: Proof is Obvious.

Theorem 4.3.Let G:(S,T) be an ivef graph on $G^*(F,I)$. If G is both edge proper ivef graph and edge totally proper ivef graph, then G is proper ivef graph if and only if G^* is proper graph.

Proof: Assume that G is both edge proper and edge totally proper ivef graph then, T is a constant function. Hence by Theorem 4.2, the result follows.

Theorem 4.4.Let G: (S, T) be a proper ivef graph on $G^*(F, I)$. Then G is an edge proper ivef graph if and only if T is a constant function.

Proof.Let G: (S,T) be $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ — proper ivef graph. Then $d_I(j) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ for all $j \in F$. Assume that T is a constant function. Let $T(gx) = (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$ for all $gx \in I$. By definition, $d_I(gx) = d_I(g) + d_I(x) - 2T(gx) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (r_1e^{is_1\pi}, r_2e^{is_2\pi}) - 2(j_1e^{ik_1\pi}, j_2e^{ik_2\pi}) = 2((r_1 - j_1)e^{i(s_1-k_1)\pi}, (r_2 - j_2)e^{i(s_2-k_2)\pi})$ for all $gx \in I$. Hence G is an edge proper ivef graph. Conversely, assume that G is edge proper ivef graph. Let $d_I(gx) = (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$ for all $gx \in I$. By definition, $d_I(gx) = d_I(g) + d_I(x) - 2T(gx)$ $\Rightarrow (j_1e^{ik_1\pi}, j_2e^{ik_2\pi}) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (r_1e^{is_1\pi}, r_2e^{is_2\pi}) - 2T(gx)$ $\Rightarrow 2T(gx) = 2(r_1e^{is_1\pi}, r_2e^{is_2\pi}) - (j_1e^{ik_1\pi}, j_2e^{ik_2\pi})$ $\Rightarrow 2T(gx) = \left((2r_1 - j_1)e^{i(2s_1 - k_1)\pi}, (2r_2 - j_2)e^{i(2s_2 - k_2)\pi}\right)$. Hence T is a constant function.

Theorem 4.5.Let G:(S,T) be an ivef graph on $G^*(F,I)$ such that T is a constant function. If G is proper ivef graph, then G is edge totally proper ivef graph.

Proof. Given that T is a constant function. Take $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$, for all $gx \in I$. Assume that G is proper ivef graph. Then $d_I(j) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$, for all $j \in F$. Now, $td_I(gx) = d_I(g) + d_I(x) - T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = (2r_1 e^{i2s_1\pi}, 2r_2 e^{i2s_2\pi}) - (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = \left((2r_1 - j_1)e^{i(2s_1 - k_1)\pi}, (2r_2 - j_2)e^{i(2s_2 - k_2)\pi}\right)$. Hence, G is edge totally proper ivefgraph.

Theorem 4.6.Let G: (S, T) be an ivef graph on $G^*(F, I)$ such that T is a constant function. If G is proper ivef graph, then G is a $(r_1e^{is_1\pi}, r_2e^{is_2\pi})$ - edge proper ivef graph.

Proof.Proof is Obvious.

Remark 4.7. The converse of theorem 4.6 need not be true.

Theorem 4.8.Let G:(S,T) be an ivef graph on $G^*(F,I)$ with G^* is k-proper. Then T is a constant function if and only if G is both proper and edge totally proper ivef graph.

Proof.Let G:(S,T) be an ivef graph on $G^*(F,I)$. Let G^* be a k-proper. Assume that T is a constant function.Let $T(gx)=(j_1e^{ik_1\pi},j_2e^{ik_2\pi})$, for all $gx\in I$. Now, $d_I(j)=\sum T(gx)$, for all $j\in F\Rightarrow d_I(j)=(j_1e^{ik_1\pi},j_2e^{ik_2\pi})$, for all $j\in F\Rightarrow d(j)=(j_1e^{ik_1\pi},j_2e^{ik_2\pi})$. Hence G is edge totally proper ivef graph. Since G is edge proper, G is edge proper, G in G in G is edge proper, G in G in G in G in G in G is edge proper, G in G in

Conversely, assume G is both proper and edge totally proper ivef graph. Since G is edge proper, $d_I(f) = (r_1e^{is_1\pi}, r_2e^{is_2\pi})$ for all $f \in F$. Since G is edge totally proper, $td_I(gx) = (h_1e^{iw_1\pi}, h_2e^{iw_2\pi}) = (r_1e^{is_1\pi}, r_2e^{is_2\pi}) + (r_1e^{is_1\pi}, r_2e^{is_2\pi}) - T(gx)$, for all $gx \in I$. $T(gx) = 2(r_1e^{is_1\pi}, r_2e^{is_2\pi}) - (h_1e^{iw_1\pi}, h_2e^{iw_2\pi}) = \left((2r_1 - h_1)e^{i(2s_1 - w_1)\pi}, (2r_2 - h_2)e^{i(2s_2 - w_2)\pi}\right)$ for all $gx \in I$. Hence T is constant function.

Definition 4.9.Let G^* : (F, I) be a graph. Then G^* is said to be an edge proper graph if each edge in G^* has same degree.

Definition 4.10.If G is both an edge proper ivcf graph and partially edge proper ivcf graph, then G is said to be full edge proper ivcf graph.

Theorem 4.11.Let G: (S, T) be an ivef graph on $G^*(F, I)$ such that T is a constant function. If G is full proper ivef graph, then G is full edge proper ivef graph.

Proof. Since T is constant function, Let $T(gx) = (j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi})$ for all $gx \in I$. Assume that G is full proper ivef graph. Then $d_G(j) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$ and $d_{G^*}(j) = r$ for all $j \in F$. So, $d_{G^*}(gx) = d_{G^*}(g) + d_{G^*}(x) - 2 = 2r - 2$. Hence G^* is an edge proper ivef graph. Now, $d_I(gx) = d_I(g) + d_I(x) - 2T(gx) = (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) + (r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = 2(r_1 e^{is_1\pi}, r_2 e^{is_2\pi}) - 2(j_1 e^{ik_1\pi}, j_2 e^{ik_2\pi}) = 2(r_1 e^{is_1\pi}, r_2 e^{is_2\pi})$. Hence G is an edge proper ivef graph. Thus, G is full edge proper ivef graph.

Remark 4.12. The converse of the above theorem is need not be true.

5. Conclusion

In this paper, we have defined edge proper and edge totally proper ivcf graph and discussed some of its properties. In future we extend this property for improper ivcf graph and try to find real valued application.

References

- [1] AkramM.,and Dudek. W.A.,*Interval Valued Fuzzy Graph*,ComputersMathematicswithApplications,Volume61,Issue2,January2011,289-299.
- [2] HosseinRashmanlou., MostafaNouriJouybari., New Concepts of IntervalvaluedFuzzyGraphswithApplication,JournalofMathematicsandInformat-ics,Vol.8,2017,69-81.
- [3] Mohamed Ismayil and Mohamad Ali., A Note on Edge regular Interval Val-ued Fuzzy Graph, Jamal Academic Research Journal: An InterdisciplinarySpecialIssue(February2016),97-102.
- [4] NagoorGani.A., and Latha. S.R., On Irregular fuzzy graphs, Applied Mathe-matical sciences, 6(2012)(11), 517-523.
- [5] Nandhini. S.P and Nandhini. E, *Strongly OnIrregular Fuzzy Graphs*, International Journal of Mathematical Archieve, Vol. 5, No. 5 (2014), 110-114.
- [6] Radha. K., and Kumaravel. N., Some Properties of edge regular fuzzy graphs, Jamal Academic Research Journal, Specialissue, 2014, 121-127.
- [7] RaviNarayanan.S.,andSanthiMaheswari.N.R., *StronglyEdgeIrregularInterval valued Fuzzy Graphs*, International Journal of Mathematical ArchieveVol.7, No.1, (2016), 192-199.
- [8] Talebi A.A., Rashmanlou. H., and Bijan New Concepts of regular IntervalValued Fuzzy Graph, Journal of Applied Mathematics and Informatics Vol.35(2017), No.1-2,95-111.
- [9] Thirunavukarasu, P., Suresh, R.; Viswanathan, K.K. *Energy of a complexfuzzygraph*, Int. J. Math. Sci. Eng. Appl. 2016, 10, 243-248.
- [10] Venkateshwara, R., Sridevi, R., Operations on Interval-Valued Complex Fuzzy Graphs, South East Asian Journal of Mathematics and Mathematical Sciences (Accepted)
- [11]Zadeh.L.A., Fuzzysets, information and control, Vol. 8, No. 3, 338-353, 1965.