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Determination of pressure in a flat channel of a solar water heater when water is freezing

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Abstract. The article considers the problem of modeling a flat channel of a rectangular section of a solar water heater filled with water when it freezes to estimate the pressure in the channel. It is assumed that when water freezes, due to the density difference between the solid and liquid phases, the volume it occupies in the channel changes, and under the influence of the pressure difference between the external environment and the internal region, the channel walls are deformed. The theory of elastic cylindrical bending of a two-layer plate is used for modeling. Calculation relations are obtained for determining the pressure in the channel depending on the thickness of the freezing ice. An example of calculating the pressure in a channel of rectangular cross section when water freezes is given.

1 Introduction

Closed liquid circulating systems are used to ensure the specified temperature conditions of the elements of machines and mechanisms, instruments and equipment. They are widely used in solar water heaters, as well as in the food and pharmaceutical industries, for industrial air conditioning of buildings and premises, to maintain the required temperature in the instrument compartments and life support systems of spacecraft and other technical industries.

At the same time, it is important to develop the designs of radiators and other elements of the liquid circulation system, which ensure the operability of the thermal control system in a wide range of temperature changes. Of great practical interest is the issue of maintaining the operability of the thermal control system during a temporary change in the state of aggregation of the coolant.

In published works concerning the design of various types of thermal control systems, thermal processes in the system are usually considered [1-4]. At the same time, issues of strength reliability of the system associated with the stress-strain state of thin-walled elements of the system are of great practical importance. The problem of creating a frost-resistant design of collectors of solar water heaters is discussed in [1,5]. Interest in this issue also arises among designers of spacecraft thermal control systems [2].

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The problem of the increase in pressure in flat channels when a liquid freezes is reflected in [3- 5]. In [3], the procedure for mathematical modeling of the stationary process of cooling and freezing of water in a pipeline covered with a layer of thermal insulation is considered. The maximum values of strains and stresses in the pipe wall are determined, which are useful in assessing the strength of the pipeline.

The purpose of this work is mathematical modeling of the deformation of the wall of a flat channel of a rectangular cross section of a solar water heater during partial freezing of water in the channel and obtaining analytical expressions for determining the pressure in the channel.

2 Materials and Methods

The design object is a separate flat channel of rectangular section, bounded by the upper and lower metal walls and side walls. Due to the design features, the bottom and side walls are considered thermally insulated, and the side walls are absolutely rigid. When the water temperature in the channel decreases, its state of aggregation changes and a solid phase appears in the form of ice. In this case, the condition is satisfied for water $\rho_L > \rho_T$, where ρ_L - density of water, ρ_T -ice density. Therefore, with the same mass, the volume of ice will be greater than the volume of water.

It is assumed that water freezes only on the inner surface of the upper channel plate, the water temperature is constant and equal to the crystallization temperature, and the interface between ice and water is flat and parallel to the metal wall. The assumption about the incompressibility of the solid and liquid phases is used.

With partial formation of ice, the volume that it occupies in the channel instead of water increases. This leads to an increase in pressure inside the channel and deformation of the lower and upper walls of the channel, which is considered in this paper in an elastic formulation.

3 Results

Since the increase in internal pressure by the value Δp occurs due to the formation of a solid phase, when modelling the upper wall of the channel, the model of a two-layer plate consisting of a metal wall and ice is used. The calculation scheme of a rectangular channel is shown in fig. 1, where the following designations are introduced: h1 – thickness of the upper metal wall; h2 – current ice thickness; h3 -thickness of the lower metal wall; b– channel width; w – normal displacement of the point of the coordinate surface of the reference plate; x, z – coordinates in the cross section of the channel. Ice does not form on the lower wall, but it is deformed by the same pressure difference as the upper one. Therefore, the deformation of the bottom plate is considered as the deformation of a single layer plate.

The calculation algorithm is reduced to the following. For a short period of time, the ice thickness at the upper plate increases by Δh_2 - which leads (assuming $\rho_{zh} > \rho_{\text{т}}$) to an increase in the volume of the inner region by ΔV and an increase in the pressure drop by Δp.

The change in displacements and stresses in the plates over the considered short period of time occurs only due to the additional load Δp. In this case, the bending stiffness of the two-layer upper plate is determined taking into account the entire thickness of the frozen layer h_2.

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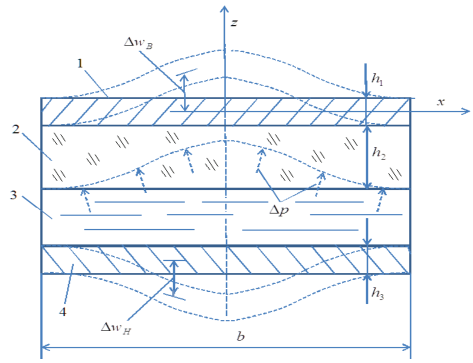

Fig. 1. Cross-section of a rectangular channel when the upper and lower plates are bent under the action of internal pressure: 1 - top plate; 2 - ice; 3 - water; 4 - bottom plate.

The change in the internal volume of the channel due to the ice formed during $\rho_{zh} > \rho_m$ can be written as [5].

$$\Delta V = Lb\Delta h_2 \left(\frac{\rho_{zh}-\rho_T}{\rho_{zh}}\right) \tag{1}$$

Since the channel length is much greater than its width, for the computational analysis of deformation, we assume that all values of the stress-strain state of the plate do not change along the channel length and consider the bending of plates of unit length ($L = 1$). The differential equation for cylindrical bending of each plate has the form [3]

$$\frac{d^4\Delta w}{dx^4} = \frac{\Delta p}{D_x} \tag{2}$$

Where D_x- bending stiffness of the plate.

In our case, the bending stiffness and pressure drop do Δpnot depend on the x coordinate.

The solution of equation (2) has the form

$$\Delta w(x) = \frac{\Delta p x^4}{24D} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \tag{3}$$

Where C_i $(i = \overline{1{,}4})$ - integration constants.

Using boundary conditions

$$\Delta w(0) = 0,\ \Delta w'(0) = 0,\ \Delta w(b) = 0,\ \Delta w'(b) = 0,\ (\cdot)' = \frac{d(\cdot)}{dx}, \tag{4}$$

we get the solution for the deflection

$$\Delta w(x) = \frac{\Delta p}{24D}(x^4 - 2bx^3 + b^2x^2) \tag{5}$$

The flexural rigidity of a two-layer upper plate is determined using the theory of layered plates [3]. To do this, in the plate section, along with the main coordinate system xyz associated with the reference plane xy, we introduce an auxiliary coordinate system ξη (Figure 2).

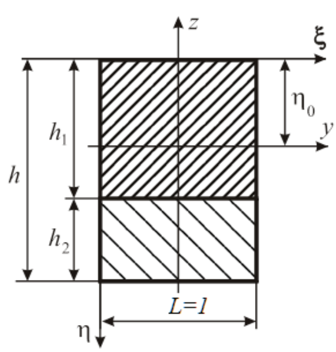

Fig. 2. Coordinate systems for the top plate.

As a result, expressions for the coordinates η0, determining the position of the reference plane, and the bending stiffness of the upper plate D_B:

$$\eta_0 = \frac{I_1}{I_0}, \quad D_B = I_2 - \eta_0 I_1, \tag{6}$$

$$I_0 = \overline{E}_1 h_1 + \overline{E}_2 h_2, \; I_1 = \frac{1}{2}\left[\overline{E}_1 h_1^2 + \overline{E}_2 (h^2 - h_1^2)\right], \tag{7}$$

$$I_2 = \frac{1}{3}\left[\overline{E}_1 h_1^3 + \overline{E}_2 (h^3 - h_1^3)\right], \tag{8}$$

$$\overline{E}_i = \frac{E_i}{1-\mu_i^2}, \; i = 1, 2,$$

Where E_1, μ_1 – modulus of elasticity and Poisson's ratio of the wall material;
E_2, μ_2 – modulus of elasticity and Poisson's ratio of ice.

Taking into account (8), we obtain an expression for the bending stiffness of the upper plate:

$$D_t = \frac{I_2 \cdot I_0 - I_1^2}{I_0} == \frac{\frac{1}{3}\left(\bar{E}_1 \cdot h_1^3 + \bar{E}_2 \cdot \left((h_1 + h_2)^3 - h_1^3\right)\right) \cdot (\bar{E}_1 \cdot h_1 + \bar{E}_2 \cdot h_2) - \frac{1}{4}\left(\bar{E}_1 \cdot h_1^2 + \bar{E}_2 \cdot \left((h_1 + h_2)^2 - h_1^2\right)\right)^2}{\overline{E}_1 h_1 + \overline{E}_2 h_2} =$$

$$= \frac{\bar{E}_2}{12}\left[\frac{h_2^4 + 4h_1 \cdot K_1 \cdot h_2^3 - 2h_1^2 \cdot K_1 \cdot h_2^2 + 4K_1 \cdot h_1^3 \cdot h_2 + K_1^2 \cdot h_1^4}{h_2 + K_1 \cdot h_1}\right] \tag{9}$$

Where $K_1 = \frac{\bar{E}_1}{\bar{E}_2}$

The bending stiffness of the lower single-layer plate has the form

$$D_b = \frac{\bar{E}_1 \cdot h_3^3}{12} \tag{10}$$

Based on the assumption that the liquid and solid phases are incompressib- le, it is assumed that the change in the internal volume of the channel due to the formation of ice is equal to the change in the internal volume due to the bending of the upper and lower walls of the channel. The change in linear volume of a channel of unit length during bending of the upper and lower plates can be determined by integrating expression (5) from the relation

$$\Delta V = \int_0^b \Delta w_B(x) dx + \int_0^b \Delta w_H(x) dx = \frac{\Delta p b^5}{720 D_B} + \frac{\Delta p b^5}{720 D_H} \tag{11}$$

Where Δw_B , Δw_Hdeflections of the upper and lower plates, respectively.

Equating relations (1) and (11), we obtain an expression for the change in pressure with an increase in ice thickness by the value Δh_2

$$\Delta p = \frac{720}{b^4}\left(1 - \frac{\rho_T}{\rho_{zh}}\right)\left(\frac{D_B D_H}{D_B + D_H}\right)\Delta h_2 \tag{12}$$

The total pressure drop between the inner and outer surfaces of the plates is determined by integrating expression (12).

$$\int_{p_0}^{p_k} dp = p_k - p_0 = \Delta p_k = \frac{720}{b^4}\left(1 - \frac{\rho_T}{\rho_{zh}}\right)\int_0^{h_{2k}}\left(\frac{D_B D_H}{D_B + D_H}\right) dh_2 \tag{13}$$

If we do not take into account the rigidity of ice and consider the upper plate as a single layer, then

$$D_B = \frac{\bar{E}_1 \cdot h_1^3}{12} \tag{14}$$

In this case, for the pressure drop, we obtain the expression

$$\int_{p_0}^{p_k} dp = p_k - p_0 = \Delta p_k = \frac{60}{b^4}\left(1 - \frac{\rho_T}{\rho_Ж}\right)\bar{E}_1\left(\frac{h_1^3 h_3^3}{h_1^3 + h_3^3}\right) h_{2k} \tag{15}$$

4 Discussion

As an example, consider the calculation of the steel collector channel of a solar water heater for the case $h_3 = h_1$.
Elastic characteristics of steel:

$$E_1 = 200\ GPa,\ \mu_1 = 0{,}29;$$

ice properties are taken according to the data of [10]:

$$E_2 = 5{,}5\ GPa,\ \ \mu_2 = 0{,}33.$$

The calculation used the value $\rho_T/\rho_{zh} = 0{,}917$

The results of numerical calculations using relations (5.6) and (9-11) are shown in fig. 3, which shows the dependence of the pressure drop acting on the channel plates on the dimensionless ice thickness $\bar{h}_2 = h_2/b$ for different values of the dimensionless wall thickness $\bar{h}_1 = h_1/b$. If the ice rigidity is not taken into account, the dependence of the pressure drop on the ice thickness in accordance with equation (11) is linear (straight lines $1^1, 2^1, 3^1$). Accounting for ice stiffness gives higher values of pressure drop (curves 1,2,3). This is explained by the fact that when taking into account the bending stiffness of the upper plate, a larger pressure drop is required.

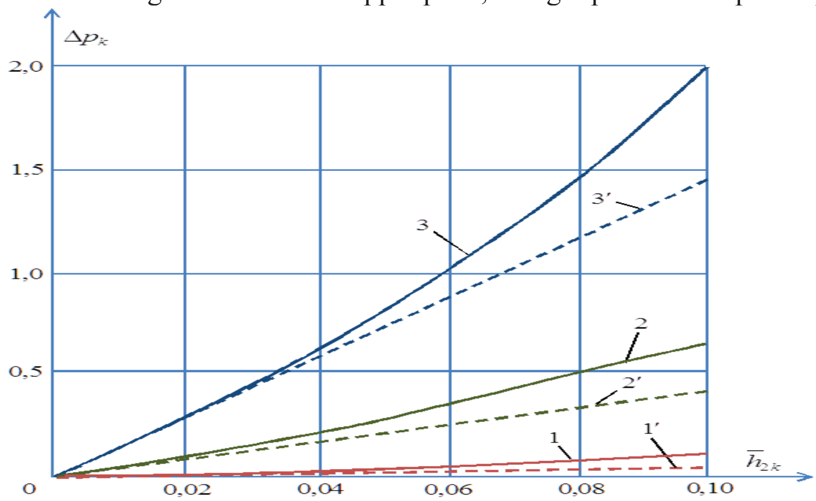

Fig. 3. Change in excess pressure inside the channel with increasing ice thickness.

for -1, $\bar{h}_1 = 0{,}01$, *for -2* $\bar{h}_1 = 0{,}02$; *for-3* $\bar{h}_1 = 0{,}03$;

1,2,3 - taking into account the rigidity of the ice on the upper plate;

$1^1, 2^1, 3^1$- without taking into account the stiffness of the ice on the top plate;

Thus, the presented calculated results allow us to estimate the pressure in the solar water heater channel as the water freezes. This is of practical importance for controlling the process of possible freezing of water in order to prevent the destruction of the water heater structure.

5 Conclusion

The problem of modeling the deformation of a flat channel of a solar water heater of rectangular cross section filled with water during its freezing is considered to estimate the pressure inside the channel. On the basis of the theory of elastic cylindrical bending of a two-layer plate, analytical calculation relations for determining the pressure in the channel are obtained. In accordance with the developed mathematical model, the pressure in the channel was calculated depending on the thickness of the formed ice for different thicknesses of the metal walls.

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The analytical dependencies obtained in the article can be used to assess the strength of the metal elements of a solar water heater and control the freezing process of water to avoid dangerous consequences.

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