Mathematical model of the movement of dust-contained air flows in the air filter of hydraulic systems

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Abstract. This article discusses the development of a mathematical model that allows you to set the appropriate parameters of the air filter to ensure efficient air filtration due to the ability to retain fine particles. Based on the developed mathematical model, it is possible to improve the air filtration systems supplied to the hydraulic tank of the hydraulic system of mining hydraulic excavators.

1 Introduction

It is known that the behavior of working air flows depends on the physical and mechanical properties of the dispersed phase. Industrial dusts are polydisperse, therefore, analytical expressions for particle size distribution functions are an important element of the mathematical description of the physical and mechanical properties of industrial dust.

1.1 Related works

In [1-3] discusses the theory of particle deposition to evaluate the efficiency of filters. However, mathematical models in them are recommended for special cases, and most often, filter characteristics that are used in practice were used to build analytical models. Also, prototypes of flows are used for the analytical model of air movement (air is passed through specially prepared sorbents). This circumstance predetermined the limitations of their practical application, which does not allow one to take into account the action of various influences during the operation of the filter at the same time. In addition, they do not take into account the effect of particles that have already settled on trapping subsequent ones, so the filter life is definitely exaggerated from this point of view. You can specify one more direction, providing a reduction in energy consumption when air passes through the filter. This direction is connected with the development of the geometry of filtering objects. In general, structures are proposed in which the filter material is deployed, for example, based on the pleating of a thin-layered air filter or by receiving electrostatic actions that are initially carried out in air filters.

The above analytical descriptions characterize only the periods of movement of dusty air flows through the filter, and there is no complete mathematical description that takes into

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account the physical processes inside the filter, changes in the speed and direction of flows. The above problems require a detailed consideration of descriptions and the development of a new mathematical model.

2 Problem statement

It has been established that the dispersed structures are not disturbed; do not encounter passive or aerodynamic dust topology and materials can be described by a single four-parameter gas distribution function that generalizes the log-normal distribution, including the variable dispersion distribution and the Swenson-Avdeev distribution [4-6]:

$$D(\delta) = A + (1 - A)\frac{\gamma(C,t)}{\Gamma(C)} \quad (1)$$

Where $\gamma(C,t), \Gamma(C)$– are the partial and absolute gamma functions;

$t = b\delta, \delta$ – is the particle scale;

$c = \frac{a}{p}$ and the parameters A, a, b and p – are determined from experimental data using a special program.

The modeling process is the basis of a scientific experiment, the purpose of which is to obtain a picture of the flow of dusty air flows in the filter elements of the hydraulic system of a mining excavator, to express them mathematically in the form of differential equations, to obtain kinematic and dynamic characteristics as continuous functions of coordinates and time.

For the mathematical apparatus of the dispersed composition of dust that has undergone mechanical or aerodynamic influences, local approximations of the distribution function in separate sections of the segment of the second and third degrees were used. Stable particles of industrial spray aerosols have a non-spherical appearance. The power of the aerodynamic relationship of irregular-type particles with air movements depends on their size, type, flow regime and is characterized by the shape index:

$$\Phi = K_M K_D, mm \quad (2)$$

where $K_M = 4S_{mp} / \pi\delta_M^2, K_D = C / C_D, S_{mp}$ – is the area of the largest cross section of particles, mm^2; δ_e – is the equivalent particle size, mm;

C, C_D– are the coefficients of aerodynamic drag of particles and spheres of similar volume.

For the intermediate section $(0.005 < \mathrm{Re}_p < 2000)$, based on the probable processing of experimental data, the dependence of the dynamic shape factor K_D on the Reynolds number and the geometric shape factor was found $K_S = \frac{S_p}{\pi\delta_M^2}$:

$$K_D = \frac{1}{K_S} + \left(1 - \frac{1}{K_S}\right)\left(1.4 + 0.304\mathrm{Re}^{0.466}\right), \; Pa \cdot s \quad (3)$$

Characteristics of non-circular particles are approximated by approximating their shape using geometric bodies: no-hedron (for spherical particles) and polyhedron (for angular particles).

The high efficiency of dust-collecting filter systems is significantly affected by particle coagulation processes, which lead to their expansion. The system of equations describing the increase in the average particle size as a result of a decrease in their calculated concentration n is derived from the situation of conservation of the weight of the dispersed phase:

$$\frac{d\delta}{dt} = \frac{k\delta(t)}{3}\left(1 + 2\delta(t)\cdot\sqrt{\frac{\delta(t)}{kt}}\right)$$

$$\frac{dn}{dt} = -k\left(1 + 2\delta(t)\cdot\sqrt{\frac{\delta(t)}{kt}}\right)n^2 \quad (4)$$

where k – is the constancy of coagulation, equal to the set of constants associated with its various special mechanisms.

Consequently, refined designations for the gradient and turbulent constants have been produced, taking into account the peculiarities of the separation procedure in heavy gas and dust flows. The action of a coarsely dispersed aerosol in a primary form is the dynamics of the movement of single particles under the action of aerodynamic forces. Taking into account the separation procedures, due to which the weight of the particle is constantly increasing, the equation of their movement will take the form:

$$\frac{dV}{dt} = -\frac{3\Phi C_D p|V-U|(V-U)}{4p_p\delta_e} - knv\left(1 + 2\delta_e\cdot\sqrt{\frac{\delta_e}{kt}}\right) + g + \frac{6F_e}{\pi\delta_e^3 p_p} \quad (5)$$

Where F_e – is the consequence of external forces acting on elementary particles, N.

These equations must be solved together with equations (4). Thus, the processes of organized movement and coagulation of particles are connected. A change in the size and mass of particles actually affects their aerodynamic qualities and dynamics, on the other hand, the movement of particles determines the duration of their stay in the coagulation zone, and hence the degree of their enlargement.

3 Results and discussion

Such a task can be performed by two analytical research methods - Lagrange or Euler. According to the Lagrange method, the movement of air flows is analyzed by the movement of specific particles through the flows of space and by tracking in time the change in their kinematic characteristics.

A certain approach to the analytical modeling of the motion of the dispersed phase of industrial aerosols can only be used for much larger particles. The method of trajectories does not consider the unpredictable effects of the movement of fine particles from the turbulent gas-dust environment, leading to the blurring of their trajectories. Mathematical modeling of the motion of solid particles in turbulent dusty air flows is done in accordance with the Monte Carlo method and scattering on turbulent fluctuations is reproduced using the dependencies:

$$x' = x + l\cos\theta,\ V' = pV_x\cos\theta\,\theta = 2\pi\gamma,$$
$$y' = y + l\sin\theta,\ V_y' = pV_y\ ,\ l = -\Delta l\ln\gamma, \quad (6)$$

where θ, l – are arbitrary values of the angle of dispersion and movement of particles;

Δl – is the root-mean-square movement of particles under the action of turbulent pulsation;

$p \le 1$ – relative indicator of dispersion $\gamma \in (0,1)$ of pseudo-random numbers.

The effect of turbulent air pulsations is shown in (Figure 1)

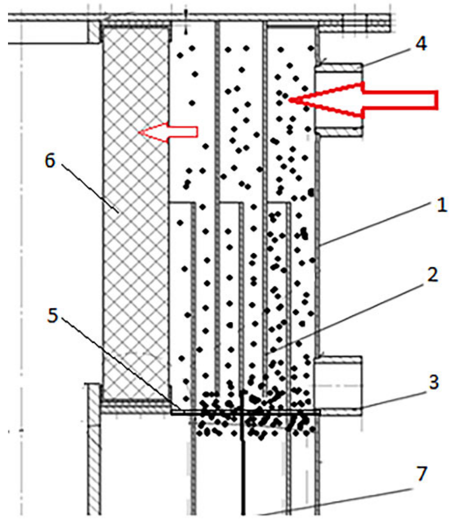

Fig. 1. Movement of fine dust in the hydraulic tank filter. 1 - filter housing; 2 - lamellar partitions; 3 - oil drain hole; 4 - air inlet holes; 5 - grid; 6 - filter with a cotton layer; 7 - oil bath.

An important characteristic of gas-dispersed flows is the distribution of the concentrations of the dispersed phase. Particle concentration fields were studied within the framework of the model of a quasi-continuous medium of moving particles [7-10]. The special saturation distribution of the coarsely dispersed aero dust was calculated by integrating the equality of the continuity of the "aerosol" medium along the particle trajectories.

Because of this, the Euler method was taken to analyze the movement of the dust and gas flow in the air filter, on the basis of this, the study of the kinematics of the filter of the hydraulic tank of the hydraulic system of a mining excavator shows that a one-time account of all the influences that affect the flow of the gas and dust mixture in the above mathematical forms greatly complicates their design.

To create a mathematical model for the movement of the dust-gas mixture flow, differential equations were used with some assumptions. The combination “gas - solid particles” is perceived as an integral medium with a continuous distribution of concentration.

Based on the Navier-Stokes equation, we define a mathematical model for the motion of a dusty air flow [3,4]:

$$\frac{\partial \vec{V}}{\partial t} + (V\Delta)\vec{V} = -\frac{1}{p}\nabla P(v_m + v_t \cdot)\Delta\vec{V} + \vec{g}\beta T, \qquad (7)$$

where $\vec{V}$ – is the three-dimensional velocity gradient; ∇ – is the Hamilton operator; Δ – is the Laplace operator; t – time, s; P – pressure, Pa; v_t – is the turbulent viscosity of the incompressible medium, mm^2/c; v_m – is the molecular viscosity of the incompressible medium, mm^2/c; p – density, $г/mm^3$; β – is the degree of spatial increase in aerosol; $\vec{g}$ – is the acceleration due to gravity ($\vec{g} = 9{,}81 m/c^2$).

Since the speed of movement of the air flow in the filter is small, it can be assumed that the dust-gas mixture is not compressed. And also, let's add the equality of continuity (continuity) - with this or that movement of air, the volume of gas remains unchanged:

$$div\vec{V} = 0 \qquad (8)$$

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The calculations take into account the condition of temperature increase due to the influence of air heating. We derive an auxiliary equation that displays the heat transfer:

$$\frac{\partial T}{\partial t}+(V\nabla)T=c\cdot(v_m-v_t)\Delta T, \tag{9}$$

where T – is temperature, ^oC degrees;

λ – thermal conductivity, $B\text{м} / m$;

c– is the coefficient of thermal diffusivity.

Acceptability is based on the calculated small particle size and the applied averaging technique for the density of a two-component mixture.

In order to more accurately simulate the movement of a dusty air flow, it is necessary to take into account that the plate-like partitions of the filter conduct thermal energy. To do this, we include a formula that describes the transfer of thermal energy in plate partitions:

$$\frac{\partial T}{\partial t}=a^2\Delta T, \tag{10}$$

where T – is temperature, degrees;

a – an indicator of the degree of heating.

1. The Navier-Stokes formula in vector form for an incompressible medium is written as follows:

$$\begin{gathered}
\frac{\partial \vec{V}_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}=-\frac{1}{p}\frac{\partial P}{\partial x}(v_m+v_t)\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right),\\
\frac{\partial \vec{V}_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}+V_z\frac{\partial V_y}{\partial z}=-\frac{1}{p}\frac{\partial P}{\partial x}(v_m+v_t)\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right)-\vec{g}\beta T,\\
\frac{\partial \vec{V}_z}{\partial t}+V_x\frac{\partial V_z}{\partial x}+V_y\frac{\partial V_z}{\partial y}+V_z\frac{\partial V_z}{\partial z}=-\frac{1}{p}\frac{\partial P}{\partial x}(v_m+v_t)\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right)
\end{gathered} \tag{11}$$

2. Equation of continuity (continuity):

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0 \tag{12}$$

3. Heat equation:

$$\frac{\partial T}{\partial t}+V_x\frac{\partial T}{\partial x}+V_y\frac{\partial T}{\partial y}+V_z\frac{\partial T}{\partial z}=c(v_m+v_t)\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right) \tag{13}$$

4. Equality of thermal conductivity of the wall of the plate baffle of the air filter:

$$\frac{\partial T}{\partial t}=a^2\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{14}$$

The developed analytical model showing the movement of dusty flows in the air filter of the hydraulic system of a mining excavator consists of difference equations of the first and second orders. The model combines its kinematic and dynamic parameters, as well as the saturation of the dust flow along the oil layer. By changing the technical parameters of the air filter, it is possible to optimize the same geometric dimensions.

We also consider the difference scheme of the equation of system (11):

– stage 1:

$$V_{xi,j,k}^{n+\frac{1}{4}} = V_{xi,j,k}^{n} + \frac{\tau}{4}\left(-\frac{V_{xi,j,k}^{n} + \left|V_{xi,j,k}^{n}\right|\left(V_{xi,j,k}^{n+\frac{1}{4}} - V_{xi-1,j,k}^{n+\frac{1}{4}}\right)}{2h} - \frac{V_{xi,j,k}^{n} - \left|V_{xi,j,k}^{n}\right|\left(V_{xi+1,j,k}^{n+\frac{1}{4}} - V_{xi,j,k}^{n+\frac{1}{4}}\right)}{2h} + \right. \tag{15}$$

$$\left. + (v_m + v_t)\left(\frac{V_{xi+1,j,k}^{n+\frac{1}{4}} - 2V_{xi,j,k}^{n+\frac{1}{4}} + V_{xi-1,j,k}^{n+\frac{1}{4}}}{h^2}\right)\right)$$

– stage 2:

$$V_{xi,j,k}^{n+\frac{2}{4}} = V_{xi,j,k}^{n} + \frac{\tau}{4}\left(-\frac{V_{xi,j,k}^{n+\frac{1}{4}} + \left|V_{xi,j,k}^{n+\frac{1}{4}}\right|\left(V_{xi,j,k}^{n+\frac{2}{4}} - V_{xi,j-1,k}^{n+\frac{2}{4}}\right)}{2h} - \frac{V_{xi,j,k}^{n+\frac{1}{4}} - \left|V_{xi,j,k}^{n+\frac{1}{4}}\right|\left(V_{xi,j+1,k}^{n+\frac{2}{4}} - V_{xi,j,k}^{n+\frac{2}{4}}\right)}{2h} + \right. \tag{16}$$

$$\left. + (v_m + v_t)\left(\frac{V_{xi,j+1,k}^{n+\frac{2}{4}} - 2V_{xi,j,k}^{n+\frac{1}{4}} + V_{xi,j-1,k}^{n+\frac{2}{4}}}{h^2}\right)\right)$$

– stage 3:

$$V_{xi,j,k}^{n+\frac{3}{4}} = V_{xi,j,k}^{n+\frac{2}{4}} + \frac{\tau}{4}\left(-\frac{V_{xi,j,k}^{n+\frac{2}{4}} + \left|V_{xi,j,k}^{n+\frac{2}{4}}\right|\left(V_{xi,j,k}^{n+\frac{3}{4}} - V_{xi,j,k-1}^{n+\frac{3}{4}}\right)}{2h} - \frac{V_{xi,j,k}^{n+\frac{2}{4}} - \left|V_{xi,j,k}^{n+\frac{2}{4}}\right|\left(V_{xi,j,k}^{n+\frac{3}{4}} - V_{xi,j,k-1}^{n+\frac{3}{4}}\right)}{2h} + \right. \tag{17}$$

$$\left. + (v_m + v_t)\left(\frac{V_{xi,j,k+1}^{n+\frac{3}{4}} - 2V_{xi,j,k}^{n+\frac{3}{4}} + V_{xi,j,k-1}^{n+\frac{3}{4}}}{h^2}\right)\right)$$

– stage 4:

$$V_{xi,j,k}^{n+1} = V_{xi,j,k}^{n+\frac{3}{4}} + \frac{\tau}{4p}\frac{P_{i+1,j+1,k+1}^{n+1} + P_{i+1,j+1,k-1}^{n+1} + P_{i+1,j-1,k+1}^{n+1} + P_{i+1,j-1,k-1}^{n+1}}{8h} \times \tag{18}$$

$$\frac{-P_{i-1,j+1,k+1}^{n+1} - P_{i-1,j+1,k-1}^{n+1} - P_{i-1,j-1,k+1}^{n+1} - P_{i-1,j-1,k-1}^{n+1}}{8h}$$

From the foregoing, one can find the roots of equations (15)-(18) using methods for solving boundary value problems for solving partial differential equations [11-19]. In order to start the calculations, it is necessary to set the initial boundary requirements. The air velocity on a solid wall is equal to zero, here the boundary condition of the first kind will be used. The boundary contains inlet and outlet openings for which boundary conditions of the first kind are specified, in this case the velocity projection, which is parallel to the boundary, is equal to zero, and the perpendicular one is not equal. The following steps are performed in the calculations [11].

The pressure in each cell is calculated by the weak compressibility method according to the equation:

$$\frac{\partial P}{\partial t} = -c^2 divV \tag{19}$$

where c – is a constant.

Turbulent viscosity is calculated according to the Sekundov model, and molar viscosity is a constant and is determined by equation (5). This model is accurate and stable in calculations.

$$\frac{\partial v_{nurb}}{\partial t}+V_i\sum_{i=1}^{3}\frac{\partial v_{nurb}}{\partial x_i}=\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\left(v_{mol}+\aleph v_{nurb}\right)\frac{\partial}{\partial x_j}v_{nurb}\right)+ \\ +v_{nurb}f\left(\frac{v_{nurb}}{8v_{mol}}\right)D-\gamma L_{\min}^{-2}\left(v_{mol}-\beta v_{nurb}\right)v_{nurb} \quad (20)$$

$$f(z)=0,2\frac{z^2+1,47z+0,2}{z^2+1,47z+1} \quad (21)$$

where $\aleph,\beta,\gamma$ – are constants $(\aleph=2;\ \beta=0,06;\ \gamma=50)$

4 Conclusion

During the settling of dust on the surface of the oil layer, the saturation of the dusty air flow decreases. It can be seen from the studies that the dust capacity of the filter depends on the speed of propagation and the trajectory of the movement of dusty air through the air filter and on the thickness of the oil layer, therefore, the analytical description of this process depends on the coefficient of filtration.

The density of the dust and gas flow in the continuity equation when determining the mathematical model changes, respectively, to the volume of air passing through the filter. To calculate these equations, it is also necessary to know the initial dust saturation at the air filter inlet. This system of equations is solved numerically using software packages of mathematical algorithms. Ultimately, there are aerodynamic characteristics in all coordinates of the aerosol flow in relation to the geometric parameters of the filter.

Thus, the problem has been solved using the least squares, sweep and local modification methods. Predetermining the geometric and technical parameters of the dust and gas flow in the mathematical model, we obtain its aerodynamic characteristics. The adequacy of the mathematical model was verified based on the results of the mathematical description of the zone before filtration. Based on the proposed mathematical apparatus, it is possible to set the appropriate parameters of air filters to ensure efficient air filtration due to the ability to retain fine particles.

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