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Simulation of fluid outflow from an axisymmetric channel

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Abstract. The subject of the study is the outflow of fluid from a channel consisting of segments with different geometries used to improve thermal and hydraulic characteristics. Research methods are based on Newton's rheological law; the continuity equation and the Navier-Stokes equation, which are the basic equations of fluid flow; the method of mathematical modeling and the analytical method of its solutions. In the article, generalized formulas for determining the hydrodynamic characteristics of a fluid flow in axisymmetric channels are obtained. As an example, analytical expressions for the pressure and average flow velocity are obtained when fluid flows out of a cylindrical channel of a constant cross section and from a channel of a hyperbolic form. In perspective, by introducing an arbitrary arithmetic expression of the axisymmetric section of the channel, it is possible to determine the hydrodynamic characteristics of the flow.

1 Introduction

Channels with different segments and geometries improve the thermal and hydraulic flow characteristics. Determination of hydrodynamic parameters in pipeline segments of different geometries is an urgent task of modeling hydrodynamic processes. Studies of fluid motion in channels with segments of circular and elliptical cross section [1], with square twisted pipes [2] were conducted. Fluid outflow from a channel of complex geometry was studied for a channel with parabolic inlet section, cylindrical middle section, and hyperbolic outlet section [3].

The determination of the flow parameters depending on the geometry [1,2,3], external factors [4], and internal factors [5] are solved using numerical [2] and analytical [3, 6] methods. Often, such studies were carried out by numerical methods with the involvement of modern information and communication systems. For this purpose, new methods of numerical modeling and numerical calculations were developed. The involvement of these new mathematical and numerical models in various problems is a laborious job.

The article considers the outflow of fluid from a reservoir into an axisymmetric channel. Assuming that the movement is a Poiseuille flow, analytical expressions for the pressure and the average flow rate of fluid during its outflow from an axisymmetric channel are obtained.

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2 Methods

Research methods are based on classical laws of fluid and gas mechanics; methods of mathematical and numerical modeling.

On modern level of development of laws of fluid and gas mechanics the applied models are becoming more and more complex, since they take into account a number of features of the environment, for example. her electric conductivity, ionization, polarization and magnetization, as well as those occurring in the chemical environment. reactions and phase transitions. When describing the conditions at the flow boundaries, the surface tension, as well as heat and mass transfer, are taken into account. If the mean free path of gas particles exceeds the characteristic size of the problem (for example, when considering gas in vacuum devices or the upper atmosphere), it is necessary to use the methods of rarefied gas theory.

3 Materials

A nozzle is installed to the reservoir with a flat wall. It has the form of an asymmetric channel formed by rotation around the axis of the curve, which does not intersect the axis of rotation. The axis of rotation of the channel is taken as the x-axis, and the radial coordinate r is taken perpendicular to it.

The channel starts in section $x=L\_$ and ends in section L_+. Its inner surface in the axial section is described by the curve $R= R(x)$. From the reservoir, where the pressure $P\_$ is maintained, the fluid flows into an asymmetric channel and, separating away in the cross section $x=x_{om}$, exits in the form of a free jet into the air-filled space. It is required to determine the parameters of flow and jet.

The flow in the channel can be described by two-dimensional non-stationary Navier-Stokes equations for an incompressible fluid in cylindrical coordinates [7]:

$$\frac{1}{r}\frac{\partial rv}{\partial r}+\frac{\partial u}{\partial x}=0\,, \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+u\frac{\partial u}{\partial x}=\frac{1}{\rho}\frac{\partial P}{\partial r}+v\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{\partial^2 u}{\partial x^2}-\frac{v}{r^2}\right), \tag{2}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial x}=\frac{1}{\rho}\frac{\partial P}{\partial x}+v\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial x^2}\right). \tag{3}$$

Here u, v are the longitudinal and radial components of the velocity vector, $P$is the pressure, ρ, υ are the density and kinematic viscosity of a fluid.

The boundary value problem can be formulated as follows:

$$\text{for } x=x\_ \text{ and } 0<r<R\_ \quad \begin{aligned} u(0,r,t)&=u\_(t),\\ v(0,r,t)&=0\\ P(0,r,t)&=0 \end{aligned} \tag{4}$$

$$\text{for } x>x\_ \text{ and } r=0{:}\quad \frac{\partial u}{\partial r}=0, v=0\,. \tag{5}$$

$$\text{for } x>x\_ \text{ and } r=0{:}\quad \frac{\partial P}{\partial r}=\frac{3\mu}{2}\frac{\partial^2 v}{\partial r^2}\,. \tag{6}$$

for $r= R_T$: $u=0$, $v=0$.

$$\text{for } x{=}x_{+}: \frac{\partial u}{\partial x}=0, \frac{\partial v}{\partial x}=0 . \tag{7}$$

Let us turn to the continuity equation. We rewrite it in the following form:

$$\frac{\partial rv}{\partial r}+\frac{\partial ur}{\partial x}=0 . \tag{8}$$

Integrating the latter over *r* from *0* to *R(x*) we get:

$$\frac{\partial}{\partial r}\int_0^R urdr = f_1(x,t) . \tag{9}$$

From this follows the dependence for the flow rate:

$$\frac{\partial Q}{\partial x}=f(x,t), \tag{10}$$

where

$$f(x,t)=2\pi\rho f_1(x,t) . \tag{11}$$

Based on the geometric parameters of the channel, when solving the problem, it is possible to use the “narrow channel” approximation, a kind of similarity to the theory of the boundary layer [7]. (In contrast to the theory of the boundary layer, the integral of conservation of mass and total head along the channel length is realized here).

Equation (3) is rewritten in divergent form:

$$\frac{\partial u}{\partial t}=\frac{1}{r}\frac{\partial uvr}{\partial r}+\frac{\partial u^2}{\partial x}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})+\frac{\partial^2 u}{\partial x^2}) . \tag{12}$$

We multiply it by *2πρr* and integrate over *r* from *0* to *R(x).*

$$\begin{aligned}&\frac{\partial}{\partial t}\int_0^R 2\pi\rho urdr+2\pi\rho\int_0^R\frac{\partial uvr}{\partial r}dr+2\pi\rho\frac{\partial}{\partial x}\int_0^R u^2rdr=\\&=-\frac{\partial}{\partial x}\int_0^R P2\pi rdr+2\pi\mu\int_0^R\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})dr+\nu\frac{\partial^2}{\partial x^2}\int_0^R 2\pi\rho urdr\end{aligned} \tag{13}$$

The first integral on the right side of the equation and the last integral on the right side of the equation represent the fluid flow rate *Q(x, t).* The second integral on the left side of the equation, due to the boundary conditions on the axis *(r=0*) and the solid boundary (*r= R*) of the flow, is zero. The second term on the right side of the equation gives the force of viscous resistance of the unit length of the channel to the fluid flow:

$$2\pi\mu\int_0^R\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})dr=2\pi r\mu\frac{\partial u}{\partial r}\Big|_0^{R(x)}=2\pi R\tau_R . \tag{14}$$

In a similar way, the integral equation takes the following form:

$$2\pi\frac{\partial}{\partial x}\int_0^R(\rho u^2+P)rdr=\frac{\partial Q}{\partial t}+\nu\frac{\partial^2 Q}{\partial x^2}+2\pi R\tau_R . \tag{15}$$

The only integral in this equation can be eliminated by replacing the longitudinal velocity component *u* with its average flow rate:

$$\mathrm{Q}=2\pi\rho\int_0^R urdr=\pi R^2U\rho . \tag{16}$$

Then equation (12) takes a simplified form:

$$\frac{\partial}{\partial x}[(\rho u^2+P)R^2]=-\rho\frac{\partial R^2U}{\partial t}+\frac{\partial^2 R^2U^2}{\partial x^2}+2\pi R\tau_R\,. \tag{17}$$

which is

$$\frac{\partial}{\partial x}[(\rho u^2+P)\,\mathrm{R}^2]=-\rho R^2\frac{\partial R^2U}{\partial t}+\frac{\partial^2 R^2U^2}{\partial x^2}+2\pi R\tau_R\,. \tag{18}$$

Within the framework of this study, we confine ourselves to considering a stationary problem, when in equations (17) and (18) the first two terms from the right-hand sides are zero.

The shear stress $\tau_R$on the wall will be determined approximately. We assume that in each section the velocity profile has the form of a quadratic parabola [8], which corresponds to the Poiseuille flow

$$u(r)=\frac{\Delta P}{4\mu l}(R^2-r^2)\,,\ \mathrm{Q}=\frac{\pi\Delta P}{8\nu l}R^4\,. \tag{19}$$

Excluding the longitudinal pressure gradient from the latter, we obtain:

$$u=\frac{2Q}{\pi\rho R^4}(R^2-r^2)\,. \tag{20}$$

Hence $\tau_R=-\dfrac{4\nu Q}{\pi R^3}$. Moreover, considering $\mathrm{Q}=\pi\rho\,\mathrm{R}^2U=\pi\rho R_{\_}^2U\_$, from (17) we obtain the relation for pressure

$$\frac{\partial\rho R^2}{\partial x}=-\frac{Q}{\pi}\frac{\partial U}{\partial x}-\frac{8\nu Q}{\pi R^2}\,. \tag{21}$$

We integrate the latter from the original section $x\_$ to x:

$$P=-\frac{Q}{\pi R^2}U\Big|_{x\_}^{x}-\frac{8\nu Q}{R^2}\int_{x\_}^{x}\frac{dx}{R^2}\,, \tag{22}$$

considering $\mathrm{Q}=\pi\,\mathrm{R}^2\,\rho U=const$:

$$P=-\frac{Q}{\pi R^2}(U-U\_)-\frac{8\nu Q}{\pi R^2}\int_{x\_}^{x}\frac{dx}{R^2}=\rho U(U\_-U-\int_{x\_}^{x}\frac{dx}{R})\,. \tag{23}$$

In a partial case, when a cylindrical channel of a constant cross-section ($R=R_{ж}=const$) is considered, the average flow rate remains constant down the channel:

$$U(x_{æ})=(\frac{R\_}{R_{æ}})^2U\_\,, \tag{24}$$

and the pressure drop in this section is:

$$P(x)-P(x_{æ})=-8\nu\rho U_{æ}\frac{x-x_{æ}}{R_{æ}^2}\,. \tag{25}$$

where $x_{ж}$ are the coordinates of the beginning of the cylindrical section.

Let the channel have a hyperbolic form: $R^2=\dfrac{b^2}{a^2}(x^2+a^2)\,.$

Let us calculate the integral:

$$\int_{x\_}^{x}\frac{dx}{b^2/a^2(x^2+a^2)}=\frac{a^2}{b^2}\int_{x\_}^{x}\frac{dx}{x^2+a^2}=\frac{a^2}{b^2}\int_{arctg\frac{x}{x\_}}^{arctg\frac{x}{a}}\frac{\frac{a}{\cos^2 t}}{\frac{a^2}{\cos^2 t}}dt=\frac{a}{b^2}\int_{arctg\frac{x}{x\_}}^{arctg\frac{x}{a}}dt=\frac{a}{b^2}(arctg\frac{x}{a}-arctg\frac{x\_}{a}) \cdot \quad (26)$$

Here a replacement is performed: $x = atgt, x^2+a^2=\frac{a^2}{\cos^2 t}$.

Thus, for the hyperbolic channel we get:

$$P(x)-P(x\_)=\rho U_{\_}^{2}[\frac{x^2-L^2}{x^2+a^2}-8\nu\frac{a}{b^2}(\mathrm{arctg}\frac{x}{a}-\mathrm{arctg}\frac{x\_}{a})] \quad . \quad (27)$$

Where $U\_$ is the speed at the inlet; $L\_$ is the distance from the inlet to the cut with the least area; b is the smallest radius of the channel corresponding to the origin of the $x=0$-axis; $R\_$ is the radius of the inlet section; U is the average flow rate.

4 Results and discussion

Calculations were conducted based on the analytical solutions obtained. Figure 1 shows the profiles of the average flow rate for *R_=0,75 mm, L_=7 mm, b=0,5 mm: 1- U_=0,2; 2- U_=0,3; 3- U_=0,3; 4- U_=0,4 m/s*. As can be seen from the figure, at such input velocity in the middle of the channel, its value increases by thirty times. When the input velocity is increased by ten times, this tendency is not observed.

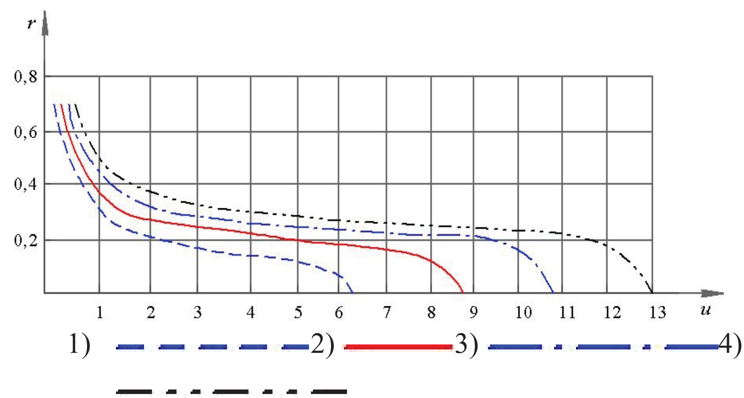

Fig. 1. Velocity profiles for u_=0,1(1);0,2(2);0,3(3);0,4(4) m/s.

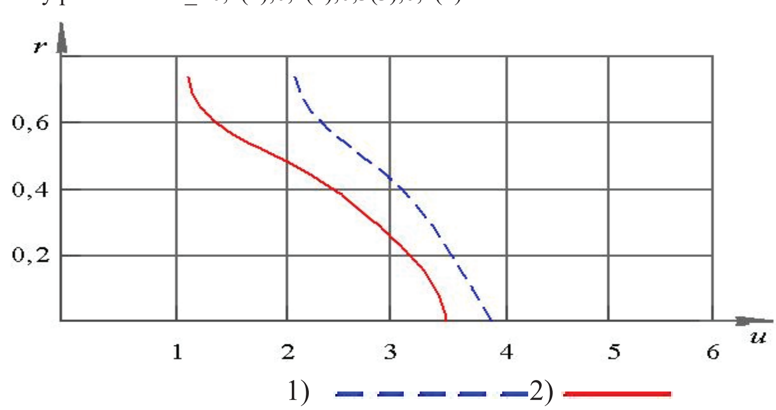

Fig. 2. Velocity profiles for *u_=1*(1)*;2*(2) *m/s.*

Figure 3 shows the velocity profiles: *1 - U=1m/s; 2 -U=2 m/s*. Here an almost uniform distribution of velocity begins. This is explained by the law of conservation of mass.

5 Conclusion

Thus, a mathematical model of fluid outflow from a channel of arbitrary shape was developed. Analytical solutions were obtained for the problems of fluid outflow from nozzles of a cylindrical channel with a form of constant cross section and a hyperbola. A numerical analysis is presented of the velocities of fluid flow in the channel depending on the size of the channel.

References

1. S. Dogan, International Journal of Hear and Mass Transfer **145**, 118731 (2019)
2. S. M. A. Navqi, Energy Procedia **158**, 5770-5775 (2019)
3. K. Karimov et al, E3S Web of Conferences **224**, 02003 (2020). https://doi.org/10/1051/e3conf/202022402003-87
4. S. K. Selvi, A. N. S. Srinivas, Thermal Science and Engineering Progress **8**, 439-447 (2018). https://doi.org/10.1016/j.tsep.2018.08.019
5. S. Jasmeen, S. Asghar, Results in Physics **7**, 3307-3314 (2017). https://doi.org/10.1016/j.rint.2017.08.44
6. M. Khudjaev, Journal of Physics: Conf. Ser. **1614**, 012093 (2020). https://doi.org/10.1088/1742-6596/1614/1/012093
7. L. G. Loitsyansky, *Mechanics of fluid and gas* (Drofa, M., 2003)