Damage detection in structural elements: using adaptive Mamdani model

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Abstract In real life all the structural and machine elements work under dynamic or variable loading. Application of dynamic loading leads to fluctuating stress. Due to fluctuating stress fatigue cracks initiates. These fatigue cracks are the main reason of failures. So, it is very important to detect the crack and predict the crack life. There are different types of damages but crack is one of the most encountered damage. There are different conventional methods to detect the damage but these methods are time taking and requires removal from the machines. Therefore, researchers are giving more importance to the unconventional methods to find the damage. In the present work a method has been introduced to find the damage site using Fuzzy Logic System and Regression Analysis. In particular, this paper focuses on applying statistical process control methods. A data pool has been created from the dynamic analysis of the cracked cantilever beam and then the data pool is trained in the proposed methodology to find the crack location. It has been noticed that the proposed methodology gives result within the tolerable range.

Key Words: Crack, Mamdani fuzzy logic, Regression analysis

1. Introduction

Nowadays structural health monitors have become very important aspect in both structural and Aerospace industries [1,2]. In this regard Artificial Intelligence and the soft computing methods are taken as a substitute to traditional methods in many industries. From the last two decades vibration analysis method is gaining importance for damage detection in structural and machine parts [3,4]. A very minute hairline crack can lead to a catastrophic failure. In Aerospace applications minute damage leads to dangerous human casualties. So, crack detection and localization in Aerospace is becoming a topic for constant research. Due to the above-mentioned reasons, the materials used in Aerospace applications need constant health monitoring [5,6]. There are different conventional structural health maintenance methods. But these types of SHM need time and money. But particularly Aerospace applications need quick and effective detection of damage.

In the current research work one such health monitoring method for damage detection has been proposed. Here an adaptive fuzzy logic system has been proposed. In

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large buildings, bridges or in aircrafts, it is really very difficult to get the damage location and predict its severity [7,8]. But a hair line crack may play a very dangerous role to initiate a very serious failure, which could be dangerous to mankind. It has been observed from a thorough literature survey that though many research works have been done, FLS has some loopholes to be applied. As most of the data from the field are given by a person where there is a chance that the data mining process may not be proper. It is known to all that due to the data mining process, there may be some unclarified data, which may create problem to get a proper solution. So, to minimize the error data, Regression Analysis has been incorporated to reduce the percentage of error [9,10,11].

In this research work, the data has been collected from Finite Element Analysis (FEA). Due to the changes in the physical parameters for the initiation of crack, the stiffness of the material is changed. The changes in the stiffness of the material lead to changes in the natural frequencies and the mode shapes. To keep the proposed methodology simple, only the natural frequencies of the first three mode shapes are taken as the input variables and the crack depth and crack location are taken as the output variables to the system. First a data pool is generated using Finite Element Analysis. Then values of all the input and output parameters are normalized by dividing with the respective values of the undamaged beam.

2. Application of Finite Element Analysis for data generation

The beam to be assumed in this analysis is Euler-Bernoulli type. Here a hairline crack is taken and the crack is assumed to be open. The vibration type is free vibration for a rectangular cross-section. The governing equation for the above-mentioned beam type is given below.

$$EI\frac{d^4y}{dx^4}-m\omega_i^2y=0 \qquad (1)$$

Where 'm' is the mass of the beam per unit length (kg/m), 'ω_i' is the natural frequency of the ith mode (rad/sec), 'E' is the modulus of elasticity (N/m^2) and 'I' is the moment of inertia (m^4). By characterizing $\lambda^4=\dfrac{m\omega_i^2}{EI}$ mathematical statement is adjusted as a fourth-arrange differential equation as follows:

$$\frac{d^4y}{dx^4}-\lambda^4y=0 \qquad (2)$$

The general solution to equation is

$$y=A\cos\lambda_i x+B\sin\lambda_i x+C\cosh\lambda_i x+D\sinh\lambda_i x \qquad (3)$$

Where A, B, C, D are constants and 'λ_i' is a frequency parameter.
The governing differential equation for the system is given as:

$$[M]\ddot{x}+[C]\dot{x}+[K]x=F\sin(\omega t) \qquad (4)$$

But it is assumed that there is no damping and there is no external force applied on the system. So the governing equation becomes

$$[M]\ddot{x}+[K]x=0 \qquad (5)$$

The equation of motion for natural frequencies for undamped free vibration is given in equation (4.5). To solve equation (4.5) it is assumed that

$$\{x\}=\{\phi\}\sin\omega t \qquad (6)$$

Where

ϕ - The Eigen vector or mode shape

ω - Circular natural frequency

3. Fuzzy mechanism for crack detection

For several years, fuzzy control has emerged as most active areas of reserch in the application of the fuzzy set theory. This logic based theory is much closer to human thinking and natural language than the traditional logical system. Fuzzy logic provides a mean of using approximate, in exact nature of the real world problems. The important part of the FLC is a set of linguistic control rules connected by the concept of fuzzy implication and inference fuzzy associative rules. The fuzzy logic system provides a method to convert the linguistic control strategy to an expert knowledge automatic control strategy. From the experience of other researchers it can be observed that FLC provides. Superior results from those obtained by conventional control algorithms. Fuzzy logic control system becomes useful when the available sources of informations are inexact and uncertain. This logic is a step forward of conventional precise mathematical control and human like decision making. The fuzzy controller has been developed where there are 3 inputs and 2 outputs parameter. The natural linguistic representations for the input are as follows

First natural frequency in normalized form= 'nfnf'

Second natural frequency in normalized form = 'nsnf'

Third natural frequency in normalized form = 'ntnf'

The natural linguistic term used for the outputs are

Crack depth in normalized form = 'ncd'

Crack length in normalized form = 'ncl'

Based on the above fuzzy subset the fuzzy rules are defined in a general form asfollows:

If (nfnf is $nfnf_i$ and nsnf is $nsnf_j$ and ntnf is $ntnf_k$) then (ncd is ncd_{ijk} and ncl is ncl_{ijk}) (7)

Where i= 1to 9, j=1 to 9, k=1 to 9

Because of 'nfnf', 'nsnf', 'ntnf' have 9 membership functions each.

From the above expression, two set of rules can be written

$$\left. \begin{array}{l} \text{If (fnf is } fnf_i \text{ and snf is } snf_j \text{ and tnf is } tnf_k) \text{ then cd is } cd_{ijk} \\ \text{If (fnf is } fnf_i \text{ and snf is } snf_j \text{ and tnf is } tnf_k) \text{ then cl is } cl_{ijk} \end{array} \right\} \qquad (8)$$

According to the usual Fuzzy logic control method (Parhi, 2005), a factor W_{ijk} is defined for the rules as follows:

$$w_{ijk} = \mu_{fnf_i}(freq_i) \wedge \mu_{snf_j}(freq_j) \wedge \mu_{tnf_k}(freq_k)$$

Where $freq_i$, $freq_j$ and $freq_k$ are the first, second and third natural frequency of the cantilever beam with crack respectively; by Appling composition rule of interference (Parhi, 2005) the membership values of the relative crack location and relative crack depth (location).

$$\left. \begin{array}{l} \mu_{ncl_{ijk}}(location) = w_{ijk} \wedge \mu_{ncl_{ijk}}(location) \qquad \forall_{length} \in ncl \\ \\ \mu_{ncd_{ijk}}(depth) = w_{ijk} \wedge \mu_{ncd_{ijk}}(depth) \qquad \forall_{depth} \in ncd \end{array} \right\} \qquad (9)$$

The overall conclusion by combining the output of all the fuzzy can be written as follows:

$$\mu_{ncl}(location) = \mu_{ncl_{111}}(location) \vee \dots \vee \mu_{ncl_{ijk}}(location) \vee \dots \vee \mu_{ncl_{191919}}(location)$$

$$\mu_{ncd}(depth) = \mu_{ncd_{111}}(depth) \vee \dots \vee \mu_{ncd_{ijk}}(depth) \vee \dots \vee \mu_{ncd_{191919}}(depth)$$

$$(10)$$

The crisp values of relative crack location and relative crack depth are computed using the center of gravity method (Parhi, 2005) as:

$$\text{Normalized crack location} = ncl = \frac{\int location.\mu_{ncl}(location).d(location)}{\int \mu_{ncl}(location).d(location)}$$

$$\text{Normalized crack depth} = ncd = \frac{\int depth.\mu_{ncd}(depth).d(depth)}{\int \mu_{ncd}(depth).d(depth)}$$

$$(11)$$

4. Application of Regression Analysis to make Mamdani FLS Adaptive

Regression Analysis comes under the category of supervised learning based on the statistical modeling of the problem. Regression Analysis is used to find the relationship between the two variables. This method is mainly a statistics-based method. It can be described in the form of Cause (independent variable) and Effect (dependent variable).
The fundamental equation for theRegression Analysis is

$$Y = a + bX + c \qquad (12)$$

where,
Y= dependent variable
X=independent variable
a=constant
b= slope of the regression line
c= error term or residual factor
Before doingRegression Analysis, some assumptions must be considered. For the current Regression Analysis method following are the assumptions considered by the author.

1. The expected values for the errors are zero, or we can say there is no residual factor. So, the equation (12) becomes $Y = a + bX$
2. The values of the independent variables are fixed and they are non-random in nature.
3. The dependency between the dependent and independent variables are linear in nature.

For the present analysis, Y=nfnf, nsnf, ntnf and X=ncd, ncl.

This analysis mainly related to the dynamic responses of the damaged structural element. This has been observed that the responses from any damaged element are scalar values. Due to which univariate stastical tests have been proposed to predict the possible alterations in the parameters related to a particular location.The application of Regression Analysis (RA) to increase the adaptiveness of Fuzzy Logic System (FLS) is explained in the schematic diagram given in the Figure 1.
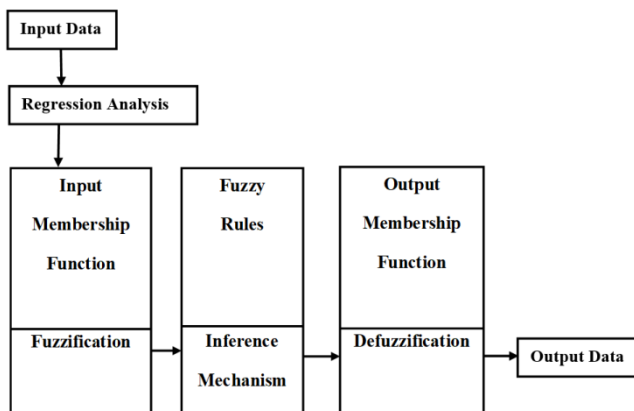
Fig.1. Schematic diagram of the proposed method

5. Result and Discussions

The following tables give the comparison of results from different methods. The first three columns of the table give the dimensionless values of first natural frequency, second natural frequency and third natural frequency. Next two columns present the dimensionless values of crack depth and crack location. Then the percentage of error is calculated using the following formulae. Table 1and 2 gives the comparison of results from FEA with various methods used in the proposed methods. So that a clear comparison can be done.

((result from FEA – result from the proposed technique) / (result from FEA)) ×100   (13)

((result from Exp. – result from the proposed technique) /(result from Exp.))×100   (14)

Total error in %= (% error in rcd + % error in rcl)/2   (15)

Table 1. Comparison of results for FLS

Sl. No	nfnf	nsnf	ntnf	ncd	ncl	rcd using the FLS technique	rcl using the FLS technique	percentage error ncd	percentage error ncl
1	0.9923	0.9912	0.9966	0.322	0.2185	0.3077	0.2086	4.43	4.52
2	0.9931	0.9926	0.9978	0.3	0.206	0.2866	0.1967	4.44	4.5
3	0.9946	0.9942	0.9972	0.2872	0.231	0.2742	0.2206	4.51	4.48
4	0.9959	0.99772	0.999	0.122	0.2185	0.1165	0.2086	4.5	4.51
5	0.9974	0.9977	0.9965	0.274	0.3622	0.2617	0.346	4.46	4.47

Table 2. Comparison of results for Adaptive FLS

Sl. No	nfnf	nsnf	ntnf	ncd	ncl	rcd using the FLS technique	rcl using the FLS technique	percentage error rcd	percentage error rcl
1	0.9923	0.9912	0.9966	0.322	0.2185	0.3122	0.2118	3.02	3.05
2	0.9931	0.9926	0.9978	0.3	0.206	0.291	0.1998	2.99	2.99
3	0.9946	0.9942	0.9972	0.2872	0.231	0.2785	0.224	3	3.01
4	0.9959	0.99772	0.999	0.122	0.2185	0.1183	0.212	3.03	2.97
5	0.9974	0.9977	0.9965	0.274	0.3622	0.2658	0.3514	2.98	2.98

6. Conclusion

In the current research work an online damage detection method has been proposed. The method comprises of Fuzzy Logic System (FLS) and Regression Analysis (RA). As human errors are involved inFLS in generating data and making the rules. So, regression analysis is used to clear the errors in the data generation.The percentage error in case of FLS is around 4.5%. But the percentage error sharply decreases to 3.5% when the proposed method is applied for damage detection. So, from the results it can be observed that the proposed method gives a better result as compared to Fuzzy Logic System and can be used as an online tool for damage detection.

References

1. K.Dziedziech, L.Pieczonka, P.Kijanka, and W. J. Staszewski, Enhanced nonlinear crack-wave interactions for structural damage detection based on guided ultrasonic waves. Struct. Control Health Monit., **23**: 1108– 1120. doi: 10.1002/stc.1828. (2016)
2. D.Feng, MQ.Feng: Output-only damage detection using vehicle-induced displacement response and mode shape curvature index, Struct. Control Health Monit, (2016).
3. Z.Sun, T.Nagayama, D.Su, Y.Fujino: A Damage Detection Algorithm Utilizing Dynamic Displacement of Bridge under Moving Vehicle, Shock and Vibration, (2016).
4. A.Ranjbaran, M.Ranjbaran,: New Finite-Element Formulation for Buckling Analysis of Cracked Structures, Journal of Engineering Mechanics, (2014).
5. M.Quila,, Sarkar., S.C.S.: Free Vibration Analysis of an Un-cracked and Cracked Fixed Beam, Journal of Mechanical and Civil Engineering, **11**(3),76-83(2014).
6. P.R Baviskar,, V.B Tungikar,.: Multiple Cracks Assessment using Natural Frequency Measurement and Prediction of Crack Properties by Artificial Neural Network, International Journal of Advanced Science and Technology, **54**, 23-37(2013).
7. N.B Hui, V Mahendar., D.K Pratihar.: Time-optimal, collision-free navigation of a car-like mobile robot using neuro-fuzzy approaches, Fuzzy Sets and systems, **157**(16), 2171-2204(2006).
8. M Eftekhari,.M.,Majidi.: Securing interpretability of fuzzy models for modeling nonlinear MIMO systems using a hybrid of evolutionary algorithms, Iranian Journal of Fuzzy Systems, **9**(1), 61-77(2012).

9. P.G Kumar., D Devaraj,.: Fuzzy classifier design using modified genetic algorithm, International Journal of Computational Intelligence Systems, **3**(3), 334-342(2010).
10. S.S Golnaz,, M.B. Nigro, S.N Pakzad.,Pan., Yuchen.: Structural damage detection and localisation using multivariate regression models and two-sample control statistics. *Structure and Infrastructure Engineering ahead-of-print*, **1**-17(2014).
11. D.N.K Worden,, E.J Cross,.:On robust regression analysis as a means of exploring environmental and operational conditions for SHM data. *Journal of Sound and Vibration*, **347**, 279-296(2015).
12. L Aleksandar., R Kanapady. C,Kamath,.: Effective localized regression for damage detection in large complex mechanical structures. *In Proceedings of the tenth ACM SIGKDD international conference on Knowledge discovery and data mining*, 450-459(2004).