Development of a mathematical model of the process of sorting municipal solid waste using elements of similarity of technical systems

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Abstract. The article studies the similarity of ensuring the technical condition of the solid municipal waste sorting line system. The functional dependence of the productivity of the waste sorting line is given, partial expressions and numerical values of the criteria for similarity of the functioning of the sorting process are obtained. A generalized criterion of similarity of the state of the solid municipal waste sorting process is also obtained, the nature of the influence of the parameters of functional dependence on the value of the output characteristic is investigated.

1 Introduction

One of the main methods of waste management is their preliminary sorting. MSW sorting involves separating waste into different categories such as paper, plastic, glass, metal, etc. This process helps increase the amount of recyclable materials that can be recovered from the waste stream, reducing the amount of waste that ends up in landfills. In addition, it can help reduce the environmental impact of waste by reducing the amount of resources needed to produce new products from virgin materials, as well as by reducing the amount of greenhouse gas emissions that are generated during waste management [1-2].

The aim of the work is to determine the rational configuration of the sorting line by using elements of the theory of similarity of technical systems, depending on the conditions of local sorting of MSW.

To achieve this goal, it is necessary to solve the following tasks:

- Determine the functional dependencies on the output characteristics.
- To obtain partial expressions and numerical values of similarity criteria for the functioning of the MSW sorting process.
- To obtain a generalized similarity criterion for the state of the MSW sorting process.
- Obtain a generalized criterion expression for the output characteristic.

Similar to real processes and objects are their mathematical models (more precisely, models of their state or functioning), which are a system of mathematical relations describing the relationship between parameters, or functional dependencies between them under given conditions, which determine the values of these parameters as a result of

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calculation [3-5]. It should be noted that no model can fully correspond to the described object, and the task of the researcher is to choose a reasonable compromise between the accuracy of the model and the simplicity of calculating the desired parameters or functions [6].

For the correct formulation and processing of experiments, the results of which would allow us to establish general patterns and could be applied to cases in which the experiment was not carried out directly, it is necessary:

- To delve into the essence of the issue under study.
- Give a general qualitative analysis of the subject area.
- Represent the quantities characterizing the process in a dimensionless form. When setting up experiments and in general for practice, it is very important to choose the right dimensionless parameters, the number of which should be minimal [7].

The possibility of such a preliminary qualitative-theoretical analysis and the choice of a system for determining dimensionless parameters is provided by the theory of dimension and similarity.

Traditionally, dimensionless numbers and similarity criteria are denoted by the uppercase or lowercase Greek letter π. In the classical representation of the similarity number, it can be obtained in two ways:

- From differential equations describing a process or phenomenon.
- From the analysis of the dimensions of the parameters of the problem [8].

2 Materials and methods

To study the similarity of ensuring the technical condition of the solid municipal waste sorting line system, we will use the dimensional analysis method, taking into account its versatility. The solution of problems by this method is based on the second, or Pi-theorem [9].

The study of the similarity of ensuring the technical condition of the system of the municipal solid waste sorting line (MSW) is carried out in the following sequence:

1) The functional dependence on the output characteristic of Psl. (productivity of the MSW sorting line) has the form:

$$P_{sl} = f\,(t_s,\ m_w,\ v_c,\ l_c,\ b_c,\ P_{w.e},\ m_p,\ m_{pl},\ m_g,\ m_a,\ m_s,\ \rho_p,\ \rho_{pl},\ \rho_g,\ \rho_a,\ \rho_s) \tag{1}$$

Where t_s – working shift time; m_w – maximum mass of waste on the conveyor belt; v_c – conveyor belt speed; l_c – conveyor belt length; b_c – conveyor belt width; $P_{w.e}$ – productivity of one working element; m_p – paper weight; m_{pl} – plastic weight; m_g – glass weight; m_a – weight of aluminum; m_s – weight of steel; ρ_p – paper density; ρ_{pl} – plastic density; ρ_g – glass density; ρ_a – aluminum density; ρ_s – steel density.

2) Obtaining private expressions and numerical values of similarity criteria for the functioning of the MSW sorting process. We accept as independent parameters: t_s, m_w, $l_{c.}$

The resulting expressions for the similarity criteria have the form:

$$\pi_{P_{pwe.}} = \frac{P_{w.e.}}{m_{w.} \cdot t_s^{-1}};\ \ \pi_{vc} = \frac{v_c}{l_c \cdot t_s^{-1}};\ \ \pi_{P_{sl.}} = \frac{P_{sl}}{m_w \cdot t_s^{-1}};\ \ \pi_{bc} = \frac{b_c}{l_c};\ \ \pi_{mp} = \frac{m_p}{m_w};$$
$$\pi_{m_g} = \frac{m_g}{m_w};\ \ \pi_{m_p} = \frac{m_p}{m_w};\ \ \pi_{m_a} = \frac{m_a}{m_w};\ \ \pi_{ms} = \frac{m_s}{m_w};\ \ \pi_{\rho_p} = \frac{\rho_p}{m_w \cdot l^{-3}{}_c};\ \ \pi_{\rho_g} = \frac{\rho_g}{m_{w.} \cdot l^{-3}{}_c}; \tag{2}$$

$$\pi_{\rho_p} = \frac{\rho_p}{m_w \cdot l^{-3}{}_c}; \ \pi_{\rho_a} = \frac{\rho_a}{m_{w.} \cdot l^{-3}{}_c}; \ \pi_{\rho_s} = \frac{\rho_s}{m_s \cdot l^{-3}{}_c}$$

Based on the data given in [10], the predominant composition of MSW generated on average in Russia is presented in Table 1.

Table 1. Morphological composition of municipal solid waste.

Component	% by weight
Paper	41
Plastic	6.2
Glass	4.4
Aluminium	1.7
Steel	2.8
Other	43.9

Using expressions for particular similarity criteria and tabular conditions for the uniqueness of the MSW sorting process in the analogue system "Waste Sorting Complex in Novoronezh" (Table 2), we calculate the numerical values of these criteria.

Table 2. Conditions for the uniqueness of the MSW sorting process of the analogue system.

Parameter	Dimension	Analog value
$P_{sl.}$	t/h	15
t_s	h	8
m_w	kg	20
v_c	km/h	2.16
l_c	m	25
b_c	m	1
P_{we}	t/h	0.35
m_p	t	8.2
m_{pl}	t	1.24
m_g	t	0.88
m_a	t	0.34
m_s	t	0.56
ρ_p	kg/m^3	0.0009
ρ_{pl}	kg/m^3	1050
ρ_g	kg/m^3	2200
ρ_a	kg/m^3	2712
ρ_s	kg/m^3	7874

These values are:

$$\pi_{P_{we.}} = 0.14; \ \pi_{v_c} = 691.2; \ \pi_{P_{sl}} = 6; \ \pi_{bc} = 0.04; \ \pi_{mp} = 0.41;$$
$$\pi_{m_g} = 0.044; \ \pi_{m_{pl}} = 0.062; \ \pi_{m_a} = 0.17; \ \pi_{m_s} = 0.028; \ \pi_{\rho_p} = 0.0007; \ \pi_{\rho_g} = 1718.75; \quad (3)$$
$$\pi_{\rho_{pl}} = 820.31; \ \pi_{\rho_a} = 2118.75; \ \pi_{\rho_s} = 6151.56$$

We take the values of π-criteria constant for the process under study and the process of the analogue.

3) To obtain a generalized similarity criterion for the state of the MSW sorting process, we determine the nature of the influence of the parameters of the functional dependence on the value of the output characteristic $P_{sl.}$ The results of the nature are presented in Table 3.

Table 3. The nature of the influence of the parameters of the functional dependence on the value of the output characteristic.

Parameter	$P_{p.o}$	$v_{кон}$	$b_{кон}$	$m_б$	$m_п$	m_c	m_a	m_{cm}	$\rho_б$	$\rho_п$	ρ_c	ρ_a	ρ_{cm}
	+1	+1	+1										
				-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Taking into account the above-presented influence of the parameters of the functional dependence, the generalized criterion for the similarity of the technical state of the MSW sorting process has the form:

$$\pi_{\prod_{i=1}^{k} P_{sl.}} = \frac{\pi_{P_{sl.}} \cdot \pi_{P_{we}} \cdot \pi_{v_c} \cdot \pi_{b_c}}{\pi_{m_p} \cdot \pi_{m_g} \cdot \pi_{m_p} \cdot \pi_{m_a} \cdot \pi_{m_s} \cdot \pi_{\rho_p} \cdot \pi_{\rho_g} \cdot \pi_{\rho_{pl}} \cdot \pi_{\rho_a} \cdot \pi_{\rho_s}} \tag{4}$$

3 Results and Discussion

To obtain a generalized criterion expression for the output characteristic of P_{sl}, we use the expression of a generalized similarity criterion. In this case, the right side of the expression is revealed through the expressions of the corresponding partial similarity criteria:

$$\pi_{\prod_{i=1}^{k} P_{sl}} = \frac{P_{sl.} \cdot t_s}{m_{w.}} \cdot \frac{P_{we} \cdot t_s}{m_{w.}} \cdot \frac{v_c \cdot t_s}{l_c} \cdot \frac{b_c}{l_c} \cdot \frac{m_{w.}}{m_p} \cdot \frac{m_{w.}}{m_{pl}} \cdot \frac{m_{w.}}{m_g} \cdot \frac{m_{w.}}{m_a} \cdot \frac{m_{w.}}{m_s} \cdot$$
$$\cdot \frac{m_{w.}}{\rho_p \cdot l^3{}_c} \cdot \frac{m_{w.}}{\rho_{pl} \cdot l^3{}_c} \cdot \frac{m_{w.}}{\rho_g \cdot l^3{}_c} \cdot \frac{m_{w.}}{\rho_a \cdot l^3{}_c} \cdot \frac{m_{w.}}{\rho_s \cdot l^3{}_c} = \frac{P_{sl} \cdot t_s^3 \cdot P_{we} \cdot v_c \cdot b_c \cdot m^8{}_w}{l^{17}{}_c \cdot m_p \cdot m_{pl} \cdot m_g \cdot m_a \cdot m_s \cdot \rho_p \cdot \rho_{pl} \cdot \rho_g \cdot \rho_a \cdot \rho_s} \tag{5}$$

Substituting into numerical values for particular similarity criteria, we get:

$$\pi_{\prod_{i=1}^{k} P_{sl}} = 3.4 \cdot 10^{-3} \tag{6}$$

From expression (5) we obtain a generalized criterion expression for the output characteristic:

$$P_{sl} = 3.4 \cdot 10^{-3} \cdot \frac{l^{17}{}_c \cdot m_p \cdot m_{pl} \cdot m_g \cdot m_a \cdot m_s \cdot \rho_p \cdot \rho_{pl} \cdot \rho_g \cdot \rho_a \cdot \rho_s}{t_s^3 \cdot P_{we} \cdot v_c \cdot b_c \cdot m^8{}_w} \tag{7}$$

4 Conclusions

Thus, in this work, functional dependencies on output characteristics are defined, particular expressions and numerical values of similarity criteria are obtained, and a generalized similarity criterion and a generalized criterion expression on the output characteristic are obtained. As a result, a mathematical model of the MSW sorting process was created, including partial models with (1) - (7), with the help of which it is possible to calculate the rational parameters of the MSW sorting line.

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