Application of methods for modeling hydraulic and non-isothermal steady states of gas transmission systems

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Abstract. In the article, for the first time, an attempt was made to justify the effectiveness of the application of new generalized methods of the theory of hydraulic circuits for modeling stationary modes of operation of gas transmission systems of complex structure and configuration. A classification of the main models of a steady gas flow is given for individual elements of the system, including traditional ones, those implicitly specified in terms of flow rate and dependent on the pressure of the working medium. Generalized flow distribution models are proposed that take into account all these cases. Conditions for the applicability of generalized methods that ensure the existence and uniqueness of the solution to the flow distribution problem are considered. The characteristics of these methods and final algorithms are given, which provide confident convergence and require less computational costs compared to existing methods. Numerical examples illustrating such possibilities are given.

1 Introduction

Modern gas transmission systems (GTS) are large-scale and complex engineering structures that play an important role in the energy, industry, social sphere and the economy of the country as a whole. Improving the efficiency, reliability and quality of the GTS operation requires the widespread use of mathematical and computer modeling methods. The tasks of steady state modeling traditionally occupy a basic position in the processes of analysis and justification of decisions in the design, operation and dispatching control of hydraulic structures.

At the stage of developing schemes for the development of the GTS, these tasks arise when analyzing: 1) the throughput of the system for future loads; 2) functioning of the system in non-design modes (verification calculations); 3) reliability of supply to consumers. During operation: 1) analysis of the admissibility of modes; 2) identification of "bottlenecks" and reasons for violation of restrictions; 3) determination of rational places for installation of automatic regulators and other equipment; 3) analysis of the consequences of accidents, development of scenarios for their localization, etc. In

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dispatcher control - at the stages of planning modes, analysis of the consequences of controls for the period of emergency or repair and restoration work, etc.

This work is devoted to the adaptation to GTS of new generalized methods of the theory of hydraulic circuits [1], which have a rational combination of universality, reliability, and computational efficiency.

2 Problem definition

As shown in [2], the effectiveness (and even applicability) of traditional methods for calculating the flow distribution in pipeline systems depends on the type of models of the steady isothermal flow of the working medium (liquid, gas) through individual elements. As shown below, for GTS all possible flow models take place simultaneously: traditional (explicit), flow-implicit, and pressure-dependent.

In the traditional case $y = p_{\text{H}} - p_{\text{K}} = f(x) = sx\,|\,x\,|$ (where y is pressure head loss in a pipeline; $p_{\text{H}}, p_{\text{K}}$ are pressure on the start and end branch; s – hydraulic resistance, e.g. air cooler), we have explicit equations for the derivative $\partial f / \partial x = 2s\,|\,x\,|$ and inverse to $f(x)$ function $\psi(y) = \sqrt{|\,y\,|/s}\cdot \operatorname{sign}(y)$.

According to [3], the flow rate and pressure at the ends of the gas pipeline (without taking into account the relief of the route) are related by the dependence

$$p_{\text{H}}^2 - p_{\text{K}}^2 = (K_s \lambda\, T\, Z\, L\, \Delta / (E^2 d^5))\cdot x\,|\,x\,|,$$

where λ, L, d, T, Z are a friction factor, pipeline length, inner diameter, average temperature and gas compressibility factor; $p_{\text{H}}, p_{\text{K}}$ are pressure at the beginning and end of the pipeline; Δ is relative density of gas to air; K_s is unit-dependent conversion factor; E is efficiency factor actual throughput of the pipeline. In this case, the model can be reduced to the traditional one, if instead of the nodal pressure, a conditional variable is introduced – the square of the pressure.

Models that are implicitly dependent on flow rate appear when [3]

$$\lambda = 0{,}067\left(158 / \operatorname{Re}(x) + 2k_э / d\right)^{0{,}2},$$

where $k_э$ is equivalent roughness; $\operatorname{Re}(x)$ is the Reynolds number depends on the flow velocity $V(x)$ and kinematic viscosity of the gas, so the resistance of the pipeline in the general case will depend on the flow rate. The function $\psi(y)$ becomes implicit, and the necessary derivatives already need to be determined according to the rules for differentiating such functions [2]: $\dfrac{df}{dx} = (2s + s_x' x)\,|\,x\,|$, where $s_x' = \dfrac{ds}{dx} = \dfrac{ds}{d\lambda}\dfrac{d\lambda}{dV}\dfrac{dV}{dx}$.

Here are examples of pressure-dependent gas flow models that mainly arise from the compressibility property of natural gas. Their characteristic feature is that the pressure drop (or their squares) depends not only on the flow rate, but also on the magnitude of the pressure itself. In [3] a model is given that allows to more adequately take into account the difference in geodetic heights of the ends of the pipeline

$$p_{\text{H}}\,|\,p_{\text{H}}\,| - p_{\text{K}}\,|\,p_{\text{K}}\,|\,(1 + a h_к) = s\left[1 + \frac{a}{2L}\sum_{ii=1}^{b}\left(h_{ii} + h_{ii-1}\right)\cdot l_{ii}\right] x\,|\,x\,|, \qquad (1)$$

where $s = \lambda T_{cp} \Delta Z_{cp} L (3{,}32\cdot 10^{-6}\cdot d^{2{,}5})^{-2}$ is "hydraulic resistance" of a horizontal pipeline; $a = \Delta / (14{,}64 T_{cp} Z_{cp})$; h is difference between the marks of the end and start points of the gas pipeline, m; l_{ii} is length of the *ii*-th section of the pipeline, km; *b* is section count; T_{cp}

is average temperature.

In [4], an alternative model is given

$$p_{\mathrm{H}}^2 - p_{\mathrm{K}}^2 = s\,x\,|\,x\,| + gL\sin(\gamma)\cdot(p_{\mathrm{H}} + p_{\mathrm{K}})^2 / (2ZRT), \tag{2}$$

where γ is pipe angle; R is the gas constant. Similar formulas for taking into account the relief of the gas pipeline route are given in [5].

The dependence of the pressure developed by the compressor on its volume flow at the inlet (q_{H}) is presented in the form compression ratio $\frac{p_{\mathrm{K}}}{p_{\mathrm{H}}} = \varepsilon(q_{\mathrm{H}})$ graphical characteristics.

Functions $\varepsilon(q_{\mathrm{H}})$ are approximated with sufficient accuracy by algebraic polynomials not higher than degree 3 [6, 7] $\varepsilon(q) = a_0 + a_1 q_{\mathrm{H}} + a_2 q_{\mathrm{H}}^2 + a_3 q_{\mathrm{H}}^3$. Taking into account the fact that $q = x/\rho$ and the gas density can be represented as $\rho = p/ZRT$, we obtain

$$\frac{p_{\mathrm{K}}}{p_{\mathrm{H}}} = \alpha_0 + \frac{\alpha_1}{p_{\mathrm{H}}} x + \frac{\alpha_2}{p_{\mathrm{H}}^2} x^2 + \frac{\alpha_3}{p_{\mathrm{H}}^3} x^3 \text{ или } p_{\mathrm{K}} = p_{\mathrm{H}}\alpha_0 + \alpha_1 x + \frac{\alpha_2}{p_{\mathrm{H}}} x^2 + \frac{\alpha_3}{p_{\mathrm{H}}} x^3 . \tag{3}$$

Numerous variations of this model are known [8, 9], which appear as a result of neglecting individual terms of the polynomial or approximation of $\varepsilon^2(q_{\mathrm{H}})$ instead of $\varepsilon(q_{\mathrm{H}})$.

The given examples of flow models (1) – (3) illustrate the impossibility of their reduction to the traditional form $y = f(x)$. Potentially, for the calculation of hydraulic circuits with such flow models, the technique of double iteration loops [1, 11] is applicable. Here, on the inner loop of iterations, the classical problem of flow distribution is solved (using LM or NM) at fixed values $\tilde{s}^*, \tilde{Y}^*$ of pressure-dependent functions $\tilde{s}(p_H, p_K, x)$, $\tilde{Y}(p_{\mathrm{H}}, p_{\mathrm{K}}, x)$ flow models $y = \tilde{s}^* x\,|\,x\,| - \tilde{Y}^*$, and on the outer loop, the values of these functions are refined depending on the obtained values of flow rates and pressures. For example, in (2) we can put $y = p_{\mathrm{H}}^2 - p_{\mathrm{K}}^2$, $\tilde{Y}^* = -\frac{gl\sin(\gamma)(p_{\mathrm{H}}^* + p_{\mathrm{K}}^*)^2}{2ZRT}$. The technique is universal but its application is associated with the ambiguity of the formation of $\tilde{s}, \tilde{Y}$ functions, the need for special justification and convergence, and a multiple increase in computational costs compared to the traditional case.

The analysis of domestic and foreign scientific and methodological literature shows that many authors: 1) pay attention to the fundamental differences between the flow models of an incompressible medium and the case of a compressible one [4, 9, 12–15]; 2) note the impossibility or significant difficulties in applying traditional methods for calculating the flow distribution in these cases [4, 5, 9, 12–16]; 3) state the absence of general, universal methods [4, 9, 12, 14–16]; 4) offer their own methods and algorithms for calculating the flow distribution in complex systems of main gas transport [4, 9, 12, 13–21]. The proposed methods for the general case of GTS with active elements either reduce to the method of nested iteration loops [13–18, 21] or involve an increase in the dimension of the systems of equations being solved compared to the node or loop model [4, 5, 9, 19, 20, 22]. So, for example, at each step of solving the problem of flow distribution by the Newton-Raphson method in [4, 5, 9, 12, 22], it is proposed to solve a system of linearized equations of order *n+m-1* (*n* is the number of bonds, *m* is the number of nodes) to simultaneously search for the direction of the step along flow rates on the branches and nodal pressures, and in [21] orders *n* – in terms of flow rates.

3 Generalized model and methods for calculating flow distribution

All the types of gas flow models listed above can be reduced to a general form $\varphi(\mathbf{p}_{\mathrm{H}},\mathbf{p}_{\mathrm{K}},\mathbf{x})=0$, taking into account which in [2] a generalized model of a steady flow distribution for the system as a whole is proposed

$$\mathbf{Ax}=\mathbf{Q}\,,\mathbf{p}_{\mathrm{H}}=\mathbf{A}_{\mathrm{H}}^{T}\mathbf{P}\,,\ \mathbf{p}_{\mathrm{K}}=-\mathbf{A}_{\mathrm{K}}^{T}\mathbf{P}\,,\ \varphi(\mathbf{p}_{\mathrm{H}},\mathbf{p}_{\mathrm{K}},\mathbf{x})=0\,, \tag{4}$$

where $\mathbf{p}_{\mathrm{H}},\mathbf{p}_{\mathrm{K}}$ are the n-dimensional vectors of pressure on the start and end branch nodes; $\mathbf{A}_{\mathrm{H}},\mathbf{A}_{\mathrm{K}}$ are the $(m-1)\times n$ incidence matrix with elements $(a_{ji})_{\mathrm{H(K)}}=1(-1)$ separately fixing the start and end branch nodes so that $\mathbf{A}_{\mathrm{H}}+\mathbf{A}_{\mathrm{K}}=\mathbf{A}$, $\varphi(\mathbf{p}_{\mathrm{H}},\mathbf{p}_{\mathrm{K}},\mathbf{x})$ is the vector function with the elements $\varphi_i(\mathbf{p}_{\mathrm{H},i},\mathbf{p}_{\mathrm{K},i},\mathbf{x}_i)$, $i=\overline{1,n}$, representing arbitrary flow laws including the pressure-dependent ones.

Let us give a brief description of the generalized (modified) node method (MNM) [2] and loop method (MLM) [1], based on model (4).

MNM is reduced to an iterative process of finding a solution in the nodal pressures space $\mathbf{P}^{k+1}=\mathbf{P}^{k}+\sigma^{k}\Delta\mathbf{P}^{k}$, where the step direction $\Delta\mathbf{P}^{k}$ is determined from the a system solution of order *m*

$$\tilde{\mathbf{M}}^{k}\Delta\mathbf{P}^{k}=-\mathbf{u}_{1}^{k}\,,$$

and the optimal step length σ^{k} is found by one-dimensional minimization condition of vector $\mathbf{u}_1^k=\mathbf{Ax}^k-\mathbf{Q}$ norm. The matrix of coefficients $\tilde{\mathbf{M}}^{k}=[\mathbf{A}(\mathbf{x}'_{\mathrm{PH}})^{k}\mathbf{A}_{\mathrm{H}}^{T}-\mathbf{A}(\mathbf{x}'_{\mathrm{PK}})^{k}\mathbf{A}_{\mathrm{K}}^{T}]$ includes diagonal matrices $\mathbf{x}'_{\mathrm{PH}},\mathbf{x}'_{\mathrm{PK}}$ with elements $(\mathbf{x}'_{\mathrm{PH}})_i=\dfrac{\partial\mathbf{x}_i}{\partial\mathbf{p}_{\mathrm{H}i}}=-\left(\dfrac{\partial\varphi_i}{\partial\mathbf{x}_i}\right)^{-1}\dfrac{\partial\varphi_i}{\partial\mathbf{p}_{\mathrm{H}i}}>0$ and $(\mathbf{x}'_{\mathrm{PK}})_i=\dfrac{\partial\mathbf{x}_i}{\partial\mathbf{p}_{\mathrm{K}i}}=-\left(\dfrac{\partial\varphi_i}{\partial\mathbf{x}_i}\right)^{-1}\dfrac{\partial\varphi_i}{\partial\mathbf{p}_{\mathrm{K}i}}<0,\ i=\overline{1,n}$.

MLM [1] is reduced to an iterative process $\mathbf{x}^{k+1}=\mathbf{x}^{k}+\mathbf{B}^{T}\Delta\mathbf{x}_{\mathrm{C}}^{k}$ of searching for a solution in the space of contour flow rates, when the direction of the step $\Delta\mathbf{x}_{\mathrm{C}}^{k}$ is determined by solving a equation system of order *c=n-m+1* (main loops)

$$\tilde{\mathbf{K}}^{k}\Delta\mathbf{x}_{C}^{k}=-\varphi_{C}^{k}\,,$$

where $\tilde{\mathbf{K}}^{k}=[(\varphi'_x)_C-\mathbf{\Phi}_C^k(\mathbf{\Phi}_T^k)^{-1}(\varphi'_x)_T\mathbf{B}_T^T]$; $\mathbf{\Phi}^{k}=[\varphi'_{\mathrm{PK}}\mathbf{A}_{\mathrm{H}}^{T}+\varphi'_{\mathrm{PH}}\mathbf{A}_{\mathrm{K}}^{T}]=\begin{bmatrix}\mathbf{\Phi}_C\\ \mathbf{\Phi}_T\end{bmatrix}$; $\mathbf{B}_T^T$ is transposed matrix block $\mathbf{B}=\left[\mathbf{B}_C\equiv\mathbf{E}_c \mid \mathbf{B}_T\right]$, $\mathbf{E}_c$ is identity matrix of order *c* [2]; $\varphi'_{\mathrm{PK}}=\dfrac{\partial\varphi}{\partial\mathbf{p}_{\mathrm{K}}}$, $\varphi'_{\mathrm{PH}}=\dfrac{\partial\varphi}{\partial\mathbf{p}_{\mathrm{H}}}$, $\varphi'_x=\dfrac{\partial\varphi}{\partial\mathbf{x}}=\begin{bmatrix}(\varphi'_x)_C\\ (\varphi'_x)_T\end{bmatrix}$ are the diagonal matrices of order *n*. In [1], the derivation of finite formulas and efficient algorithms for calculating only non-zero coefficients of the matrix $\tilde{\mathbf{K}}^{k}$ is given, which greatly simplifies the implementation of the method. The proposed methods completely coincide with the classical NM and LM in the traditional case of flow models [1, 10]

$$\varphi_i(\mathbf{p}_{\mathrm{H},i},\mathbf{p}_{\mathrm{K},i},\mathbf{x}_i)=\mathbf{p}_{\mathrm{H},i}-\mathbf{p}_{\mathrm{K},i}-\mathbf{s}_i\mathbf{x}_i\,|\,\mathbf{x}_i\,|+\mathbf{Y}_i=0,\ \ i=\overline{1,n}\,.$$

4 Accounting for non-isothermal GTS states

The coefficients of hydraulic characteristics of GTS elements depend on the properties of the transported gas (density, viscosity, compressibility) which depend on its temperature and composition. The steady-state temperatures of the GTS for given hydraulic conditions and properties of gas flows can be described by the following system of equations [10], in which the orientation of the branches is brought into line with the direction of flows:

$$\bar{\mathbf{A}}_{\mathrm{H}}\mathbf{X}_{\mathrm{H}}\mathbf{t}_{\mathrm{H}} + \bar{\mathbf{A}}_{\mathrm{K}}\mathbf{X}_{\mathrm{K}}\mathbf{t}_{\mathrm{K}} = \boldsymbol{\theta},\ \ \mathbf{t}_{\mathrm{H}} = \bar{\mathbf{A}}_{\mathrm{H}}^{T}\bar{\mathbf{T}},\ \ g(\mathbf{x},\mathbf{t}_{\mathrm{H}},\mathbf{t}_{\mathrm{K}},\mathbf{P}) = 0, \tag{5}$$

where $\bar{\mathbf{A}}_{\mathrm{H}}, \bar{\mathbf{A}}_{\mathrm{K}}$ are the $m\times n$-incidence matrix separately fixing the start and end branch nodes so that $\bar{\mathbf{A}}_{\mathrm{H}} + \bar{\mathbf{A}}_{\mathrm{K}} = \bar{\mathbf{A}}$; $\mathbf{t}_{\mathrm{H}}, \mathbf{t}_{\mathrm{K}}$ are the *n*-dimensional vectors of temperature on the start and end branch nodes; $\mathbf{X}_{\mathrm{H}}, \mathbf{X}_{\mathrm{K}}$ – $(n\times n)$-are the diagonal matrices with $\mathbf{c}_{\mathrm{H}i}\,\mathbf{x}_i$ and $\mathbf{c}_{\mathrm{K}i}\,\mathbf{x}_i$ on the main diagonal; $\boldsymbol{\theta}$ is the m-dimensional nodal heat vector with elements $\boldsymbol{\theta}_j = \mathbf{c}_j^{+}\mathbf{Q}_j\mathbf{T}_j^{+}$ for inflow gas to the node, $\boldsymbol{\theta}_j = \mathbf{c}_j\mathbf{Q}_j\mathbf{T}_j$ for outflow, $\mathbf{T}_j^{+}$ – temperature inflow, $\mathbf{T}_j$ – temperature outflow in the node j; $\bar{\mathbf{T}}$ is the m-dimensional vector of temperature in nodes; g is the vector function with thermophysical characteristics of branches [3]; $\mathbf{c}_{\mathrm{H}i}$, $\mathbf{c}_{\mathrm{K}i}$, $\mathbf{c}_j^{+}$, $\mathbf{c}_j$ are the heat capacity of the gas, respectively, at the beginning and end of the branch, the external nodal inflow and the mixture of gases in the node, which are functions of pressure and temperature at the corresponding points of the gas flows [3].

In model (5): the first equation reflects the heat balance law at the nodes; the second is the condition for complete mixing of flows; the third is a set of models of temperature changes on the branches. The system of equations (5) is solvable with respect to unknown temperatures ($\mathbf{t}_{\mathrm{H}}, \mathbf{t}_{\mathrm{K}}, \bar{\mathbf{T}}$) if the following are given [10]: temperatures of external inflows, flow distribution ($\mathbf{x}, \bar{\mathbf{Q}} \equiv \bar{\mathbf{A}}\mathbf{x}$), $\bar{\mathbf{P}}$, $\mathbf{c}$, g.

It can be seen from the above relations that the temperature and hydraulic regimes are strictly interdependent, mainly due to the compressibility property of the gas, which relates pressure and temperature through the equation of state. Therefore, for this task, the technique of double iteration cycles [10, 18] is used, which in general terms boils down to the following main steps:

1) assigning an initial approximation for the distribution of flows, their component composition, pressures and temperatures (for example, the same pressures, temperatures and gas component composition for all elements);

2) a gas flows physical parameters (density, viscosity, heat capacity, compressibility, etc.) and GTS elements hydraulic characteristics coefficients calculation;

3) a flow distribution calculation using MNM or MLM (internal cycle of iterations);

4) the gas and its temperatures component composition distribution calculation [23, 24];

5) checking the shutdown conditions (comparison with the predetermined accuracy of flow rates, pressures and temperatures at the last two external iterations), if they are not observed, go to step 2.

5 Numerical example

Let us give an example of the application of the proposed methods for a high-pressure GTS fragment [9] (Fig. 1) and their convergence (Fig. 2). All sections of pipelines are horizontal and are modeled by equation $|\mathbf{p}_{\mathrm{H},i}|\mathbf{p}_{\mathrm{H},i} - |\mathbf{p}_{\mathrm{K},i}|\mathbf{p}_{\mathrm{K},i} = \mathbf{s}_i\mathbf{x}_i\,|\mathbf{x}_i|$, $i = 2,\overline{6,10}$. The equation for compressors is

$$\varphi(\mathbf{p}_{\mathrm{H},i},\mathbf{p}_{\mathrm{K},i},\mathbf{x}_i) = \left(\beta_{0,i} + \frac{\beta_{1,i}^2}{4\beta_{2,i}}\right)\mathbf{p}_{\mathrm{H},i}\,|\mathbf{p}_{\mathrm{H},i}| - \mathbf{p}_{\mathrm{K},i}\,|\mathbf{p}_{\mathrm{K},i}| - \beta_{2,i}\left(\mathbf{x}_i - \frac{\beta_{1,i}}{2\beta_{2,i}}\mathbf{p}_{\mathrm{H},i}\right)\left|x_i - \frac{\beta_{1,i}}{2\beta_{2,i}}\mathbf{p}_{\mathrm{H},i}\right| = 0,$$

where $\beta_{0,1}$ =1,040975262; $\beta_{1,1}$ =0,4520492230; $\beta_{2,1}$ =0,1660378943;

$\beta_{0,3}$ =1,040975262; $\beta_{1,3}$ =0,4520492230; $\beta_{2,3}$ =0,1660378943; $\beta_{0,4}$ =1,056105913;
$\beta_{1,4}$ =0,4352722015; $\beta_{2,4}$ =0,2396158372; $\beta_{0,5}$ =1,049124727; $\beta_{1,5}$ =0,3668417249;
$\beta_{2,5}$ =0,1867004063.

$\mathbf{s}_2$ =0,006 $\mathbf{s}_8$ =0,436 $\mathbf{Q}_1$ =19,1 $\mathbf{Q}_4$ =14,8
$\mathbf{s}_6$ =0,253 $\mathbf{s}_9$ =1,332 $\mathbf{Q}_5$ =0,632 $\mathbf{Q}_6$ =0,32
$\mathbf{s}_7$ =4,757 $\mathbf{s}_{10}$ =0,349 $\mathbf{P}_9$ =33,778 at.

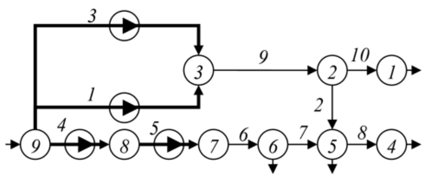

Fig. 1. Scheme of the gas transmission system (thick lines – compressors, thin lines – pipeline sections) with initial data (s_i is hydraulic resistance, Q_j is gas consumers, million m^3/day).

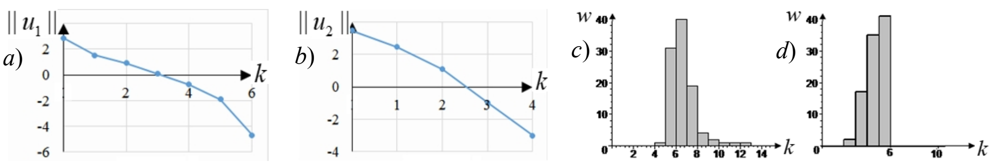

Fig. 2. Convergence process: a) MNM, b) MLM; the number of design conditions w (%) under which the problem is solved in k iterations: c) MNM, d) MLM.

It can be seen that: 1) both methods demonstrate confident convergence; 2) the number of iterations weakly depends on the proximity of the initial approximation to the solution; 3) MLM requires on average fewer iterations than MNM. Such indicators of convergence are explained by the fact that the matrices $\tilde{M}$ and $\tilde{K}$ have a diagonal (modulo) predominance, symmetrical in structure, although not in the values of the elements. The number of iterations weakly depends on the dimension of the scheme, however, the computational costs for each iteration depend on its topology, and for the same dimension of the matrices $\tilde{M}$ and $\tilde{K}$, these costs are higher for the MLM than for the MNM.

6 Conclusion

The article proposes new methods of the theory of hydraulic circuits for calculating the isothermal and non-isothermal GTS steady states. The proposed methodology, methods and algorithms are universal with respect to the type of using flow models involved for different elements of the GTS design scheme, have high speed compared to existing ones, providing confident convergence with a fairly arbitrary initial approximation.

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