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Probabilistic modeling of functioning of district-distributed heating systems for the reliability analysis

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Abstract. The study is devoted to modeling a random process of functioning of district-distributed heating systems, aimed at determining the probabilities of states corresponding to failures of their components. The probabilities of emergency states, along with the physical assessment of the consequences of failures, are used to reliability analysis and synthesis for studied systems. The basic stochastic model of the system functioning corresponds to a markov random stationary process under the conditions of the simplest events flow. Modifications of random process models are proposed, accounting the non-ordinarity events (simultaneous failures of components) and the dependence between some events (mainly, failures), if the corresponding technological possibility is established. A number of computational experiments is provided to assess the degree of influence of considered factors on the results of probabilistic modeling of studied systems.

1 Introduction

One of the directions in the development of modern district heating systems (DHS) is associated with their transformation into systems of a district-distributed type, integrating various energy technologies (including renewable energy) to achieve maximum efficiency and reliability of heat supply to consumers with an optimal combination of district and distributed generation of thermal energy [1–3]. The sector of distributed generation in the district-distributed heating systems (DDHS) is formed primarily at the level of prosumers [4–6]. The implementation of prosumers with their own heating sources (HS) causes new functional properties, structure and parameters of studied systems. Consequently, the transition to DDHS with prosumers significantly complicates their technological structure and component diagram, and, accordingly, the simulated random process describing the states evolution in these systems. Within the framework of reliability problem, the simulation of a random process of the system operating is aimed at defining probabilities of emergency states corresponding to component failures. For restorable systems, including heating systems, *markov random processes* are widely applied, which make it possible to obtain solutions using reliability parameters of components (failure and restoration rates) in

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a large range of initial conditions. Some studies on the reliability of heating systems (incl. sources and networks) using markov random processes are considered in [7–14]. The probabilities of emergency states, along with the physical assessment of the consequences of failures, are used to reliability analysis and synthesis for studied systems.

A number of initial conditions corresponding to traditional heating systems make it possible to use the markov process model to describe their functioning under constrains of the *simplest event flow* with the assumptions of stationarity, ordinarity and independence of events (failures and restorations of components) [7]. With the transition to DDHS, these assumptions significantly reduce the adequacy of the models, which is due to the parallel operation of district and distributed sources. We propose modifications of the basic model of the random process of the system functioning to take into account additional factors associated with the *non-ordinarity* events (simultaneous failures of components) and the *dependence* between some events (mainly, failures), if the corresponding technological possibility is established.

2 Methodological approaches and models

2.1 Basic model of random process

As noted earlier, the initial basic model of the evolution of states of the studied system corresponds to the simplest event flow with corresponding constrains. Taking these conditions into account, the stationary markov random process of the system functioning is described by the following system of linear equations [8]:

$$p_s \sum_{m \in M_s} (\lambda_m + \mu_m) = \sum_{z \in E_s} p_z \sum_{m \in M_z} (\lambda_m + \mu_m), \quad s \in E, \tag{1}$$

where p_s, p_z – probabilities of some states of the system; λ_m, μ_m – transition probabilities of the ransom process: failure and restoration rates of some component m respectively, 1/h; E – set of system states; E_s – subset of system states from which the direct transition (without intermediate states) to the state s is possible; M_s – subset of system components whose failure or restoration corresponds to the direct transition from the state s to some other state z; M_z – subset of system components whose failure or restoration corresponds to the direct transition from the state z to some state s.

It should be noted that the stationarity condition of reliability parameters of components corresponds to the real-life systems within the calculation period (heating season), during which these parameters practically do not change, and consideration of wearing of equipment proves significant in a longer period of time and is considered at the level of formation of initial reliability data [7]. In addition, wear of equipment should be partially compensated by the maintenance and timely replacement.

2.2 Non-ordinarity events model

When operating such complex systems as DDHS, many different technological processes occur in parallel, and failures of several components at the same time seem to be more probable events than for traditional systems. With mutual independence of events, the probability of their combination is the product of their probabilities [15]. Accordingly, the intensity of *non-ordinarity transition* between states, interpreted in this case as a transition probability, is determined by the product of the failure and restoration rates:

$$\nu_{sw} = \prod_{m \in M_w} \lambda_m \prod_{m \in M_s} \mu_m, \ s \in E, w \in E_1^*, \tag{2}$$

where ν_{sw} – intensity of a non-ordinarity transition from the state s to the state w, to which several failed components correspond, 1/h; E_1^* – subset of states into which the system can transition from the state s due to simultaneous of several events (failures and/or restorations); M_w – subset of failed components in the state w; M_s – subset of failed components in the state s.

Depending on the number of simultaneous events taken into account, several levels of implementation of non-ordinarity events can be considered. Taking these conditions into account, the model of the random process is represented in general form as follows:

$$p_s \left(\sum_{z \in E_1} \nu_{sz} + \sum_{u \in U} \sum_{w \in E_1^*} \nu_{swu} \right) = \sum_{z \in E_2} p_z \nu_{zs} + \sum_{u \in U} \sum_{w \in E_2^*} p_w \nu_{wsu}, \quad s \in E, \tag{3}$$

where ν_{swu} and ν_{wsu} – intensities of non-ordinarity transitions (transition probabilities) from some state s to some state w and back for the u-th level of non-ordinarity events, 1/h; U – set of considered levels of non-ordinarity events; p_w and E_2^* – probability and subset of states from which the system can transition to the state s due to concurrent realization of several events.

2.3 Dependent events model

When the functioning of DDHS, along with the simultaneous implementation of several events, dependent events in subsystems can also occur, if the corresponding technological possibility is establish. In this case, the formation of a set of system states is carried out using conditional probabilities [8, 15]. Let us consider the component m with the failure rate λ_m, given the failure of which the conditional probability of failure of the component k is equal to $\zeta_{k/m}$. Obviously, the conditional probability of failure varies within the limits from 0 to 1 depending on the relationship of considered components.

Dependent failure of the component k given the failed component m will occur with the rate $\lambda_{k/m}$ determined by the value of the conditional probability $\zeta_{k/m}$:

$$\begin{cases} \lambda_{k/m} = \lambda_k (1 + \zeta_{k/m}) \quad given \quad 0 \le \zeta_{k/m} < 1, \\ \lambda_{k/m} = \lambda_m \quad given \quad \zeta_{k/m} = 1. \end{cases} \tag{4}$$

The comprehensive use of the model for non-ordinarity events (3) with the expressions for determining the dependent events rate (4) allows us to present a model of the random process of the system operation accounting both of these factors:

$$\begin{gathered} p_s \left(\sum_{z \in E_1} \nu_{sz} (2 \pm \zeta_{z/s}) + \sum_{u \in U} \sum_{w \in E_1^*} \nu_{swu} (2 \pm \zeta_{w/s}) \right) = \sum_{z \in E_2} p_z \nu_{zs} (2 \pm \zeta_{s/z}) + \\ + \sum_{u \in U} \sum_{w \in E_2^*} p_w \nu_{wsu} (2 \pm \zeta_{s/w}), \quad s \in E \end{gathered} \tag{5}$$

where $\zeta_{z/s}$ and $\zeta_{s/z}$ – conventional probabilities of ordinary transitions of a set from states z to s and back, accordingly; $\zeta_{w/s}$ and $\zeta_{s/w}$ – conventional probabilities of non-ordinarity transitions of a set from states w into s and back, accordingly; $\nu_{sz/s}$ and $\nu_{zs/z}$ – rates of dependent transitions from state s into state z and back; $\nu_{swu/s}$, $\nu_{wsu/w}$ – similar indicators for non-ordinary transitions. Each value of $\zeta_{z/s}$ is associated either with a failure or restoration.

3 Numerical study

A number of computational experiments were carried out using developed probabilistic models to describe the random process of functioning of some simplified aggregated diagram of heating system. Some characteristics are obtained that make it possible to determine modeling conditions and the change in the results of reliability analysis accounting the considered factors (non-ordinarity and dependent events). So, in Fig. 1 shows one of the generalized results of numerical study of the test diagram accounting dependent events (failures of components).

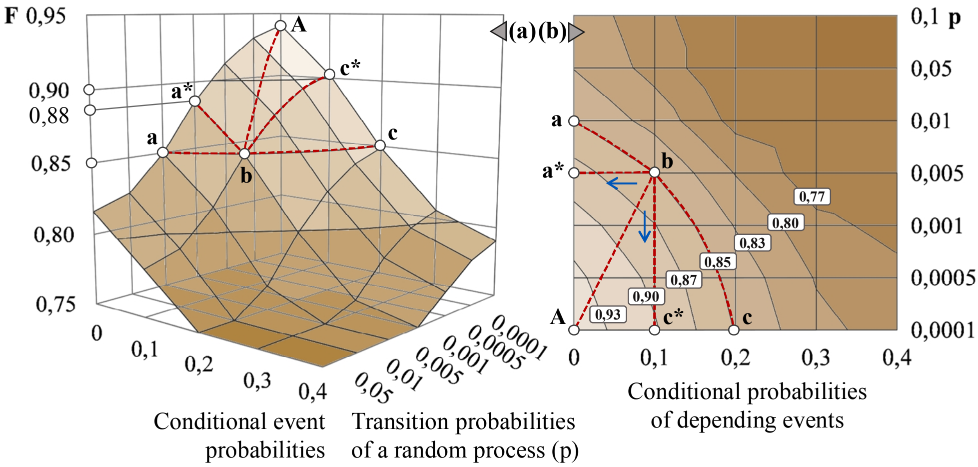

Fig. 1. Results of numerical study based on the test calculation diagram of the heating system: (a) dependence of the reliability function F on the values of conditional probabilities of dependent events for different values of transition probabilities (failure rates); (b) gradient of the reliability function F (projection of the diagram shown in Fig. 1-a)

The diagram shown in Fig. 1-a, is the dependence of the reliability function F of the system on the values of the conditional probabilities of dependent events at different values of the transition probabilities of a random process describing the state evolution of the system (in the considered case, the failure rates of components were used as transition probabilities). This diagram reflects the degree of possible decrease in the reliability of the system (function F) from some calculated initial level (point A) under the influence of the dependency factor between failures of a group of system components. On this surface, it is possible to single out the range of F values, limited by some specified minimum allowable level, for example, F = 0.85 (line a–b–c in Fig. 1-a). The figure cut off by this curve, projected onto the horizontal plane of the diagram, contains the ratios of the initial values of parameters, under which the specified reliability requirements are met. The corresponding projection of the diagram shown in Fig. 1-b is the gradient of the index F. A similar diagram, created for some real-life system, can be used to determine the initial parameters necessary to ensure a certain level of reliability, as well as to evaluate the necessary reserves that provides compensation for possible emergencies in case of dependent failures, including the forced outage of components.

4 Conclusions

Increasing the accuracy of reliability analysis of district-distributed heating systems can be achieved by modeling non-ordinarity and dependent events. The paper proposes methodological approaches to formalization the random processes of the studied system with the simultaneous implementation of several events based on the use of the rule of their combination with mutual independence. Accounting for the relationship of transitions between some events (if there is such a relationship) is carried out by introducing conditional probabilities. Based on the results of probabilistic modeling, accounting dependent events, characteristics were obtained that make it possible to quantify the degree of possible (expected) decrease in the reliability of heating to consumers under the influence of the considered dependence factor.

The considered approaches to the probabilistic modeling of the heating system can also be adapted for other energy systems with a complex multi-level structure, including hybrid (integrated) energy systems.

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