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Estimating the coordinate error for leak location in oil pipelines

Anton Chionov[1,*], and *Sergei* Korshunov[1]

[1]Transneft R&D, Moscow, Russia

Abstract. This paper discusses the method of hydraulic location for leak detection. A statistical criterion for leakage incident detection has been developed. A series of simulation experiments conducted has confirmed that the developed criteria are adequate. An algorithm has been developed to evaluate the sensitivity of a leak detection algorithm based on the availability of pressure instruments at the process section.
Key words: leak detection, decision-making algorithms, oil pipeline, condition monitoring system, equipment sufficiency assessment.

Introduction

The subject of leak detection in oil and product pipelines currently remains topical [1-5]. One of the control tasks is oil and product leakage monitoring and prevention. Oil transport process control algorithms continuously monitor multiple measurements. The basic task for a leak detection system (LDS) is to establish a set of admissible values for the monitored parameters, so that the algorithm would detect system upset when the actual values fall outside this set. It is not enough to propose one or more methods for leak detection, so an algorithm has to be developed to calculate the criteria to be used for decision making when a leak is detected at a particular process section (PS), taking into account the actual availability of instruments.

Hydraulic location method

The method of hydraulic location was described in [6]. It is based on a known analytical solution for steady-state hydraulic equations: a hydraulic gradient is proportional to the friction loss, which is in turn directly related to the liquid velocity. In [7,8] this method was generalized for the use on pipes equipped with routine pressure instruments. The following diagram shows the essence of the method.

* Corresponding author: ChionovAM@niitnn.transneft.ru

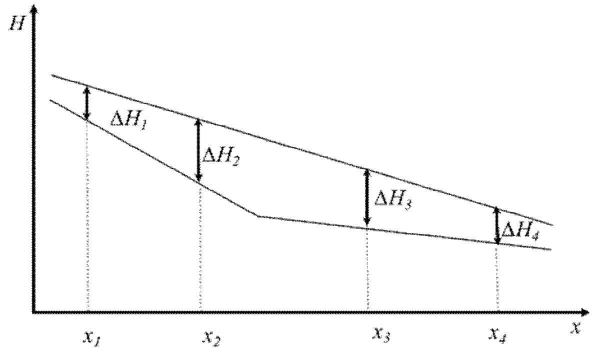

Fig 1. Diagram of the hydraulic location method.

Head/pressure is measured along the pipeline at different points in time—presumably before and after the onset of a leak. A characteristic V-shaped broken line (Fig. 1) is drawn based on the head measurements. The leak is located where the V-line has the break.

Problem definition

Let there be measurement results for head loss H_i in points with coordinates x_i ($i = 1 \div N$, where N is the number of measurement points). If the oil density ρ is assumed to be constant, then the change in head $H=\square+p/\rho g$ can be calculated from the change in pressure, as suggested in [7]; here, □ is the pressure detector's elevation.
The set of measurements taken shall be used to plot a relationship in the known form [9] using the method of least squares (MLS). It should be noted that MLS has a strict probabilistic substantiation, subject to a normal distribution of the measurement error [9].
$H(x)$ is a random head function. The form $H(x)$ is defined by an analytical solution to the hydraulic equations [10] and is a broken line formed by two straight line sections. Let $H(x)$ be represented as:

$$H(x) = \begin{cases} H^r(x) = A_0^r + A_1^r x, \; x \le x_0 \\ H^l(x) = A_0^l + A_1^l x, \; x \ge x_0 \end{cases} \tag{1}$$

where $A_i^{l,r}$ are random variables. The expectation $H(x)$ is in turn a non-random function in the following form:

$$\phi(x) = \begin{cases} h^r(x) = a_0^r + a_1^r x, \; x \le x_0 \\ h^l(x) = a_0^l + a_1^l x, \; x \ge x_0 \end{cases} \tag{2}$$

where $a_i^{l,r}$ are expectations for the corresponding random variables $A_i^{l,r}$. The indices l and r designate accordingly the left and right shoulders of the V-curve. The slopes of the left a_1^l and right a_1^r shoulders (2) are proportional to the change in squared flow rate in the pipe across the point of leakage, therefore the difference between them can be correlated with the leakage value.

Written in non-dimensional form, the condition of minimum squared deviations is:

$$\sum_{i=1}^{N} \frac{\left(H_i - \phi(x_i)\right)^2}{\sigma_i^2} \to \min \tag{3}$$

Here σ_i are standard deviations (SDs) for every measured head H_i. The value σ_i is characteristic of the random measurement error value. If the number of experiments is small, the error of the instrument may be taken as a reasonable estimate for σ_i.

The condition (3) in general leads to a non-linear system of equations, as the leak coordinate x_0 is related to the parameters $a_i^{l,r}$, which are in turn determined from (3), which is in turn related to x_0.

The paper [11] suggests to reduce the problem (3) to 2 independent MLSs separately for the left and right shoulders. Assume that we know the instruments, between which x_0 is located. Let N_l pressure detectors be located to the left of the leak and N_r detectors to the right. The problem (3) then divides into two independent ones:

$$\sum_{i=1}^{N_l} \frac{\left(H_i - h^l\left(x_i\right)\right)^2}{\sigma_i^2} \to \min \tag{4}$$

$$\sum_{i=1}^{N_r} \frac{\left(H_i - h^r\left(x_i\right)\right)^2}{\sigma_i^2} \to \min \tag{5}$$

The minimum conditions (4) and (5) lead to conventional systems of linear equations (SLEs):

$$\begin{pmatrix} \sum_{i=1}^{N_l} \frac{1}{\sigma_i^2} & \sum_{i=1}^{N_l} \frac{x_i}{\sigma_i^2} \\ \sum_{i=1}^{N_l} \frac{x_i}{\sigma_i^2} & \sum_{i=1}^{N_l} \frac{x_i^2}{\sigma_i^2} \end{pmatrix} \begin{pmatrix} a_0^l \\ a_1^l \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N_l} \frac{H_i}{\sigma_i^2} \\ \sum_{i=1}^{N_l} \frac{H_i x_i}{\sigma_i^2} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} \sum_{i=1}^{N_r} \frac{1}{\sigma_i^2} & \sum_{i=1}^{N_r} \frac{x_i}{\sigma_i^2} \\ \sum_{i=1}^{N_r} \frac{x_i}{\sigma_i^2} & \sum_{i=1}^{N_r} \frac{x_i^2}{\sigma_i^2} \end{pmatrix} \begin{pmatrix} a_0^r \\ a_1^r \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N_r} \frac{H_i}{\sigma_i^2} \\ \sum_{i=1}^{N_r} \frac{H_i x_i}{\sigma_i^2} \end{pmatrix} \tag{7}$$

The systems (6) and (7) are to be solved analytically. The problem (3) is to be solved by direct search among the pairs of detectors, between which the suspected point of leakage is located [11].

The leak coordinate is determined from the equation $h^l\left(x_0\right) = h^r\left(x_0\right)$:

$$x_0 = -\frac{a_0^l - a_0^r}{a_1^l - a_1^r} \tag{8}$$

As seen from the algorithm described above, there will always exist some solution in the form (2); therefore, the coordinate (8) can also be calculated almost always, regardless of the existence of a leak (subject to the denominator (8) being other than 0). Therefore, a rule has to be defined to make a decision about a leak, which cannot be done without taking into account the stochastic nature of the measurements.

Decision-making criteria

Basic criterion. A leak is nothing but a difference in flow rate across its location. Since the squared flow rate is proportional to the slope (for a quadratic friction mode), the value of

$$q_0 = a_1^l - a_1^r \tag{9}$$

is proportional to the squared leakage flow rate. The absence of leakage is equivalent to the condition of $q_0 = 0$. The three-sigma rule will be used to compare q_0 with zero. If

$q_0 > 3\sigma_{q_0}$, then the leakage flow rate significantly differs from zero and, therefore, a leak exists. Here the value $\sigma_{q_0}^2 = D(q_0)$ is the variance of q_0. In other words, if the difference between the V-curve drawn by the MLS and a straight line is statistically significant, then there are sufficient grounds to conclude there is a leak.

For the convenience of subsequent derivations, the solution for the SLEs (6) and (7) is to be represented as follows:

$$a_0^l = \sum_i^{N_l} \alpha_i^0 H_i \quad a_1^l = \sum_i^{N_l} \alpha_i^1 H_i \tag{10}$$

where $\alpha_i^0 = \left(\frac{\gamma_{11}}{\sigma_i^2} + \frac{\gamma_{12} x_i}{\sigma_i^2}\right)$, $\alpha_i^1 = \left(\frac{\gamma_{21}}{\sigma_i^2} + \frac{\gamma_{22} x_i}{\sigma_i^2}\right)$, γ_{ij} are coefficients of the inverse matrix for (6).

$$a_0^r = \sum_i^{N_r} \beta_i^0 H_i \quad a_1^r = \sum_{i=N_1}^{N_r} \beta_i^1 H_i \tag{11}$$

Here $\beta_i^0 = \left(\frac{\kappa_{11}}{\sigma_i^2} + \frac{\kappa_{12} x_i}{\sigma_i^2}\right)$, $\beta_i^1 = \left(\frac{\kappa_{21}}{\sigma_i^2} + \frac{\kappa_{22} x_i}{\sigma_i^2}\right)$, and κ_{ij} are coefficients of the inverse matrix for (7). Since H_i are independent, then, in accordance with (10) и (11), the following is valid for the variance of q_0 [8]:

$$D(q_0) = \sum_i^{N_r} \left(\beta_i^1\right)^2 D(H_i) + \sum_i^{N_l} \left(\alpha_i^1\right)^2 D(H_i). \tag{12}$$

Confidence interval of the leak coordinate

Taking into account (10) and (11), the leak coordinate x_0 may be represented as follows [8]:

$$x_0 = -\frac{\sum_i^{N_l} \alpha_i^0 H_i - \sum_i^{N_r} \beta_i^0 H_i}{\sum_i^{N_l} \alpha_i^1 H_i - \sum_{i=N_1}^{N_r} \beta_i^1 H_i} \quad . \tag{13}$$

Having (13) expanded in a series up to the linear terms around x_0 and using (10) and (11), the following may be written after the transformations for the coordinate variance [8]:

$$D(x_0) = \sum_i^{N_l} \left(\frac{x_0 \alpha_i^1}{q_0} + \frac{\alpha_i^0}{q_0}\right)^2 D(H_i) + \sum_i^{N_r} \left(\frac{x_0 \beta_i^1}{q_0} + \frac{\beta_i^0}{q_0}\right)^2 D(H_i) \quad . \tag{14}$$

SD for $\sigma_{x_0} = \sqrt{D(x_0)}$ defines the confidence interval $x_0 \pm \sigma_{x_0}$, in which the coordinate of the suspected leak is located with a high probability. This parameter is used to analyze the outputs of several algorithms running simultaneously to find intersections between the intervals $x_0 \pm \sigma_{x_0}$ and the outputs of other independent methods.

Head drop in the suspected leak location

It is commonly known that sections with reduced flow area, called gravity-flow sections, may occur during oil and oil product transportation. Properties of such sections, such as the length and the actual flow area, depend on the actual flow properties and may vary. In particular, pressure control downstream a gravity-flow section may cause oil to fill the gravity-flow cavity. This obviously creates a difference in liquid flow rate across the gravity-flow cavity, which the model unambiguously interprets as a leak. In physical terms, it may be called a leak but not to the environment but to the internal source of oil.
However, no head (or pressure) drop h_0 is observed in the point of leakage in such a process. The following is valid for the h_0 value:

$$h_0 = \frac{a_1^l a_0^r - a_0^l a_1^r}{a_1^l - a_1^r} \tag{15}$$

Having (15) expanded in a series up to the linear terms and taking (10) and (11) into account, the following may be written after the transformations for the h_0 variance [8]:

$$D(h_0) = \sum_i^{N_l} \left(\frac{a_0^r}{q_0}\alpha_i^1 - \frac{a_1^r}{q_0}\alpha_i^0 - \frac{h_0}{q_0}\alpha_i^1 \right)^2 D(H_i) + \sum_i^{N_r} \left(\frac{a_1^l}{q_0}\beta_i^0 - \frac{a_0^l}{q_0}\beta_i^1 + \frac{h_0}{q_0}\beta_i^1 \right)^2 D(H_i) \tag{16}$$

Relation between coordinate variance and head drop value

It is intuitive that the better a leak is visible and the more a V-curve is different from a straight line, the more accurately the leak coordinate can be determined, which is confirmed analytically. In addition, it is intuitive that the more pressure instruments are installed on a monitored section, the more reliable and accurate will be the leak coordinate calculation result.

A series of simulation experiments has been carried out, where leaks at various rates in different points of test pipelines were simulated. Parameters of the problem, such as the pipe length and size, the leakage rate and location, and the number of pressure instruments, varied in a wide range. To simplify the analysis, it was assumed that the pressure instruments are spaced evenly. Also, all the variances of inputs (*i.e.*, errors of all the pressure instruments) were taken to be equal: $\sigma_i=\sigma_p$ for all i. The results of the simulation experiments are shown in the diagram below.

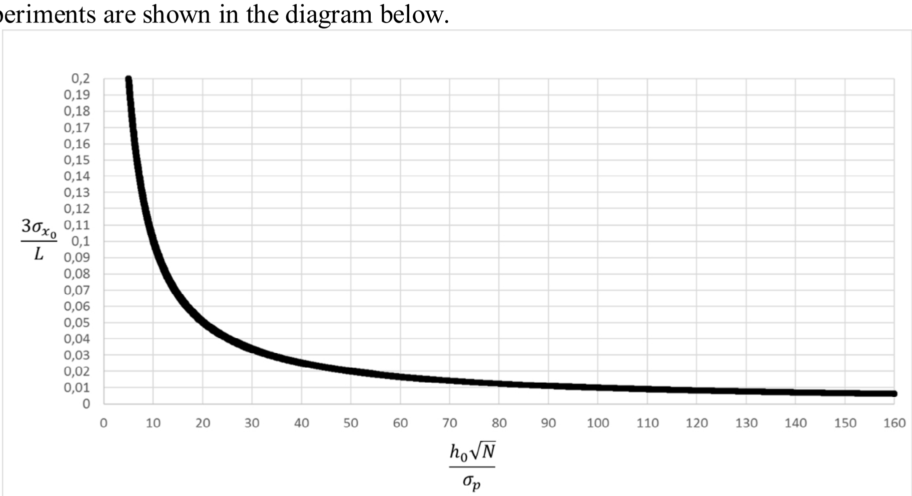

Fig. 2. Dimensionless relation for the error in the calculated leak coordinate.

Discussion

As seen from the results of the simulation experiments, a quite simple relationship between the pressure drop at the point of leakage, the availability of pressure instruments at the pipeline section, and the expected error in the calculated leak coordinate can be obtained.

The y-axis in Figure 2 represents the dimensionless confidence interval for a leak coordinate by three-sigma rule $3\sigma_{x0}/L$, L is the section length. The x-axis represents a dimensionless ratio for the pressure drop at the leak location, corrected to the number of pressure instruments at the section, $(h_0 \sqrt{N})/\sigma_p$.

It follows from the obtained results that there are two ways to increase the accuracy of a coordinate calculation: increase the number of pressure instruments N, or reduce the pressure instrument error σ_p, with the accuracy growing as $\sqrt{N}$ in one case or as $1/\sigma_p$ in the other case. Thus, there is a limit to the possible increase in the accuracy of leak coordinate determination by increasing the number of pressure instruments.

In addition, the obtained results allow setting methodologically substantiated accuracy requirements for LDS field tests or project-specific engineering. It is obvious that, without taking into account the results obtained above, LDS accuracy requirements may be either understated or overstated or unachievable in principle.

Conclusion

Quantitative criteria have been developed for the hydraulic location method, allowing substantiated decision-making concerning a leak. The developed criteria allow to establish the relation between the sensitivity of the hydraulic location method and the pipeline section equipment parameters. The obtained criteria allow to determine the pressure instrument requirements for a process section given the set LDS sensitivity level.

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