On the issue of dynamics of two-tier mechanical hydroponic installations with rigid load-bearing elements

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Abstract. Hydroponic green forage (HGF) is a high-quality ecological product containing a full range of required nutrients and vitamins. A two-tier hydroponic installation with rigid load-bearing elements has been developed at Kuban State Agrarian University named after I. T. Trubilin. The purpose of the research is to determine the strength and mass complex ratios that ensure stable dynamics of movement of all elements of the system during the operation of a two-tier hydroponic installation. The derivation and solution of differential equations of the dynamics of the elements of the installation is given. As a result of experimental studies, it is proved that the maximum deviation of the center of mass of the system, tray-crop-guides does not exceed 15 cm. Based on theoretical studies, the ratio of the parameters of the system under consideration was determined, in which there is no beating phenomenon. The lengths of the load-bearing elements should not be different, and the ratio of the weights of the loaded and free trays should vary within 0.15–0.25.

1 Introduction

Hydroponic green forage (HGF) is a high-quality ecological product containing a full range of required nutrients and vitamins. A two-tier hydroponic installation with rigid load-bearing elements has been developed at Kuban State Agrarian University named after I. T. Trubilin [1-6].

The purpose of the research is to determine the strength and mass complex ratios that ensure stable dynamics of movement of all elements of the system during operation of a two–tier hydroponic installation.

The experience of operating two-tier mechanical hydroponic installations with rigid load-bearing elements and trays, the vegetative surface of which exceeds 1.5 m^2, allows suggesting that these systems have a significant disadvantage.

In the process of unloading the lower tray, the entire installation system is set in oscillatory motion. Flat hinged joints of rigid bearing elements with trays and a suspension point of the installation exclude the pliability of the entire system in the horizontal and one of the vertical planes. The lack of compliance leads to bending of rigid load-bearing rods

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(reinforcing bars with diameters of 5-6 mm). The latter causes additional repair work with preliminary dismantling of the installation before the next laying of the source material for germination process.

In hydroponic installations with a growing area of no more than 1.5 m^2, these imperfections are practically absent due to the small masses and moments of inertia of the trays. However, the dynamics of such systems has not been studied, despite the fact that these installations are simple to manufacture and are most suitable for the equipment of hydroponic workshops in farms. Moreover, two-tier installations with rigid load-bearing elements are much cheaper than similar ones with elastic load-bearing ropes of the same diameter.

2 Results of research

Consider the mechanical model shown in Figure 1 to study the dynamics of a two-tier installation with trays of a special design.

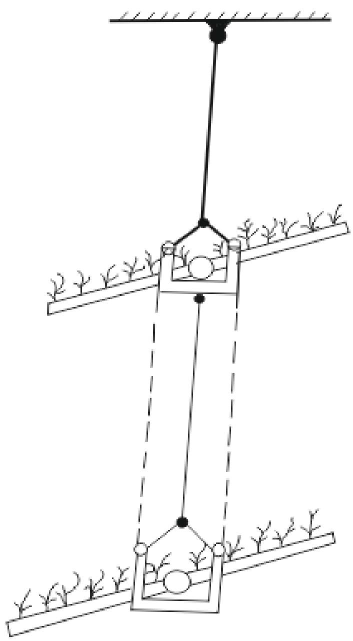

Fig. 1. Mechanical model of a two-tier hydroponic installation.

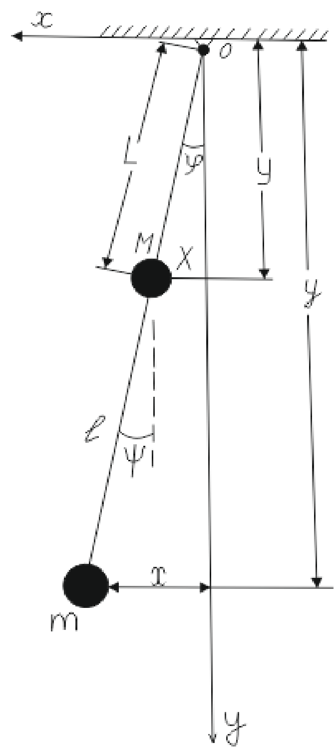

Fig. 2. The design scheme of a two-tier hydroponic installation.

The design scheme of the installation when unloading the lower tray is shown in Figure 2, where L and l, respectively, are the lengths of the supporting elements, M is the mass of the upper tray with the crop and guides, m is the mass of the lower tray with guides. Let's place the origin of the coordinate axes at the suspension point of system O. The direction of the axes (x y) is shown in Figure 2.

We use the Lagrange equations of the second kind in the form [7-8] to derive the differential equations of motion of the system,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \quad (1)$$

where $L = T - P$ – the function the Lagrange equation,
T and P – accordingly, the kinetic and potential energies of the system,
q_i and $\dot{q}_i$ – generalized coordinates and generalized velocities.

The angles of deviation from the vertical axis φ and Ψ are taken as generalized coordinates.

Then the following relations will be valid:

$$\begin{cases} X = L\sin\varphi, & x = L\sin\varphi + l\sin\Psi, \\ Y = L\cos\varphi, & y = L\cos\varphi + l\sin\Psi. \end{cases} \quad (2)$$

The kinetic and potential energy of the system under consideration are respectively equal

$$\begin{cases} T = \frac{M}{2}\left(\dot{X}^2 + \dot{Y}^2\right) + \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) = \frac{m}{2} l^2\dot{\Psi}^2 + mlL\cos(\varphi - \Psi)\,\dot{\varphi}\dot{\Psi} + \frac{M+m}{2} L^2\dot{\varphi}^2, \\ P = -mgy - MgY = -mgl\cos\Psi - (M+m)\,gL\cos\varphi = 0. \end{cases} \quad (3)$$

Substituting (2,3) into (1) we obtain a nonlinear system of differential equations of motion of the installation in question in the form of the Lagrange equation of the second kind

$$\begin{cases} \ddot{\varphi} + \frac{ml}{(M+m)L}\ddot{\Psi}\cos(\varphi - \Psi) - \frac{ml}{M+m}\dot{\Psi}^2\sin(\varphi - \Psi) + \frac{g}{L}\sin\varphi = 0, \\ \ddot{\Psi} + \frac{L}{l}\ddot{\varphi}\cos(\varphi - \Psi) - \frac{L}{l}\dot{\varphi}^2\sin(\varphi - \Psi) + \frac{l}{g}\sin\Psi = 0. \end{cases} \quad (4)$$

Experimental studies conducted in the hydroponic laboratory of Kuban State Agrarian University named after I. T. Trubilin showed that if the technology of laying soaked grain in trays of 5.5 kg/ m^2 (vegetation area of trays of 1.5 m2) is observed, the yield of green mass is 55 kg per square meter [1-3]. Unloading of the crop was carried out on the eighth day. In the course of the experiments, the deviation of the center of mass of the lower tray from the vertical was established by direct measurements. Это отклонение изменялось в пределах 14 – 16 см при длине жестких несущих стержней 0,85 м. This deviation varied in the range of 14 – 16 cm with a length of rigid bearing rods of 0.85 m. The upper loaded tier with a bearing element length of 0.70 m deviated by less than 14 cm. The experimental results obtained give grounds to assume the deflection angles φ and Ψ small. In this regard, the system of equations (4) takes the form:

$$\begin{cases} \ddot{\varphi} + \frac{ml}{(M+m)l}\ddot{\varphi} + \frac{g}{L}\varphi = 0 \\ \ddot{\Psi} + \frac{L}{l}\ddot{\varphi} + \frac{g}{l}\Psi = 0. \end{cases} \quad (5)$$

In the resulting system of linear differential equations (5) we put

$$\varphi = \frac{X}{L}, \qquad \Psi = \frac{x - X}{l}, \quad (6)$$

then substituting (6) into (5), the following system of equations with respect to linear coordinates X and x is obtained in the form

$$\begin{cases} \ddot{X} + K_1^2 X - \alpha x = 0 \\ \ddot{x} + K_2^2 x - K_2^2 X = 0, \end{cases} \quad (7)$$

where $K_1^2 = \frac{g}{L} + \left(1 + \frac{m}{M}\right) + \frac{mg}{Ml}, \; \gamma = \frac{mg}{Ml}, \; K_2^2 = \frac{g}{l}.$

The latter transformation will be valid when $\frac{m}{M} > 1$ and $L < l$ which corresponds to real two-tier installations when unloading the crop. Assuming that $L < l$ and introducing the notation $\frac{g}{L} = \omega_0^2$, система уравнений (7) the system of equations (7) will take the form примут вид,

$$\begin{cases} \ddot{X} + \left(2\frac{m}{M} + 1\right)\omega_0^2 X - \frac{m}{M}\omega_0^2 x = 0 \\ \ddot{x} + \omega_0^2 x = \omega_0^2 x. \end{cases} \tag{8}$$

The solution of the resulting system of equations will be in the form [7-8]

$$X = Be^{i\lambda t},\ x = Ae^{i\lambda t},\ \text{где } i = \sqrt{-1}. \tag{9}$$

Substituting equations (9) into (8) makes it possible to obtain the following system of algebraic equations for determining λ

$$\begin{cases} (\omega_0^2 - \lambda^2)A - \omega_0^2 B = 0 \\ -\frac{m}{M}\omega_0^2 A + \left[\left(1 + 2\frac{m}{M}\right)\omega_0^2 - \lambda^2\right] = 0. \end{cases} \tag{10}$$

We transform the resulting system (10) into a quadratic equation with respect to λ^2, revealing its determinant

$$(\lambda^2 - \omega_0^2)^2 + 2\frac{m}{M}\omega_0^2(\omega_0^2 - \lambda^2) - \frac{m}{M}\omega_0^4 = 0. \tag{11}$$

The roots of this equation with sufficient accuracy for practice will be equal to

$$\omega = \lambda = \omega_0\left(1 + \sqrt{\frac{m}{4M}}\right), \tag{12}$$

$$\omega_1 = \lambda_1 = \omega_0\left(1 - \sqrt{\frac{m}{4M}}\right). \tag{13}$$

The general solution of the original system (7) can be written in the following form

$$x = C_1\cos\omega t + C_2\sin\omega t + C_1'\cos\omega_1 t + C_2'\sin\omega_1 t,$$

$$X = \beta C_1\cos\omega t + \beta C_2\sin\omega t + \beta' C_1'\cos\omega_1 t + \beta' C_2'\ \sin\omega_1 t.$$

Values β and β' are the values of the relation $\frac{\mathrm{B}}{\mathrm{A}}$, which are obtained from the equations (12, 13), after substituting in them $\lambda^2 = \omega^2$ and $\lambda^2 = \omega_1^2$. Based on the above, we get:

$$\beta = -\sqrt{\frac{m}{M}},\ \ \beta' = \sqrt{\frac{m}{M}}.$$

Initial conditions should be entered to define arbitrary constants C_1, C_2, C_1' and C_2'. At $t = 0$ $x = x_0$. $X = 0$, $\dot{x} = 0$, $\dot{X} = 0$, which corresponds to the real system, i.e. at the initial moment of time, the center of mass of the lower tray with the harvest shifted to x_0 in the direction of overturning, the unloading of the crop took place. The center of mass of the upper tray at t =0 remained stationary. Considering the above, the following relations are obtained to determine arbitrary constants

The obtained ratios allow determining unknown quantities

$$C_1 = -\frac{x_0\beta'}{\beta-\beta'},\qquad C_1' = \frac{x_0\beta}{\beta-\beta'},\qquad C_2 = C_2' = 0.$$

Finally, the solution of the original system of equations (7) can be written as

$$\begin{cases} x = \frac{x_0\beta'}{\beta-\beta'}\cos\omega t + \frac{x_0\beta}{\beta-\beta'}\cos\omega_1 t \\ X = \frac{\beta\beta' x_0}{\beta-\beta'}\cos\omega t + \frac{\beta\beta' x_0}{\beta-\beta'}\cos\omega_1 t\ , \end{cases} \tag{14}$$

$$\text{or} \begin{cases} x = \frac{x_0}{\beta-\beta'}(\beta \cos \omega_1 t - \beta' \cos \omega t) \\ X = \frac{\beta\beta' x_0}{\beta-\beta'}(\cos \omega_1 t - \cos \omega t) \ . \end{cases} \quad (15)$$

It should be noted that when $\frac{m}{M} \gg 1$ value ω near to ω_1, what excites the "beating" character in the system [9-10].

The law of movement of the upper tray with a crop and guides $X = X(t)$ imagine in the form

$$X = \frac{\beta\beta' x_0}{\beta-\beta'}(\cos \omega_1 t - \cos \omega t) \ = 2\frac{\beta\beta' x_0}{\beta-\beta'} - \sin\frac{\omega_1-\omega}{2}t \cdot \sin\frac{\omega_1+\omega}{2}t. \quad (16)$$

Values $\sin\frac{\omega_1-\omega}{2}t$ will be equal 0 in the event that the equality is met

$$\frac{\omega_1-\omega}{2}t = \pi n, \ \ n = 1,2,3,... \quad (17)$$

or at a moment in time

$$t = \frac{2hn}{\omega_1-\omega}. \quad (18)$$

The lower tray will return to the equilibrium position, while the upper one tends to the maximum deviation. Thus, when the values of ω_1 and ω are close, there is a mutual exchange of the energies of the moving masses of the system [9-10]. Figure 3 shows the nature of the change in the movement of the masses of the upper and lower tiers over time.

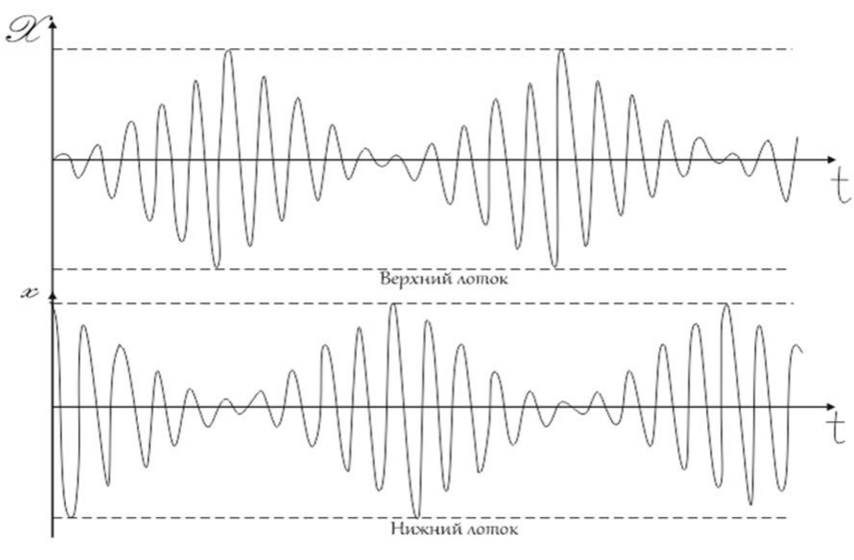

Fig. 3. Changing the law of movement of the upper and lower trays over time.

This phenomenon can be avoided by varying the mass and geometric parameters of the installation, i.e. the lengths of the load–bearing elements should not be different, and the ratio of the masses of loaded and free trays should vary within 0.15-0.25. The initial conditions play an essential role. The graphs shown in Fig.3 are obtained with the following geometric and mass characteristics:$L = \ell = 1m, \ \ \frac{m}{M} = 0{,}5$.

The determination of the forces in the lower and upper rods is carried out according to known methods [7-8]. The force in the lower bearing rod S can be determined for small deviations of the masses, according to the following formula

$$S \approx mg\cos\psi. \tag{19}$$

The forces in the upper bearing element are determined based on the differential equations of the unfree motion of a material point or the Dalembert principle. Consider the calculation scheme shown in Figure 4.

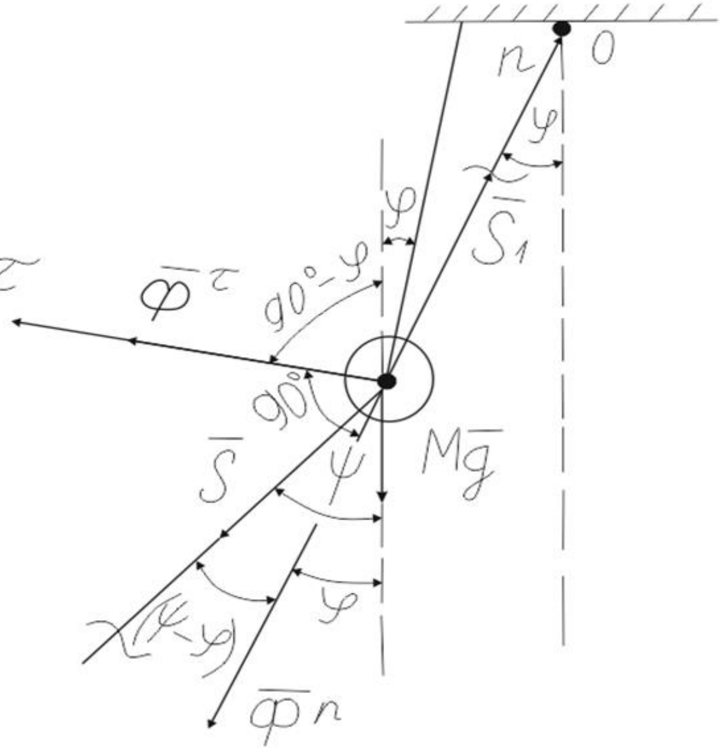

Fig.4. Calculation scheme for determining the forces in the rods.

We associate with the moving point of the movable coordinate axis τ and n, denoting all the given forces g , as well as an unknown tension (force in the rod) S *and* S_1.

According to the Dalembert principle, the tangent is introduced Φ^τ and normal Φ^n as the inertia force. These forces are directed as shown in Figure 4, that is, to the opposite point to the vectors of the true accelerations of the point $\bar{a}^r$ and $\bar{a}^n$ at this point in time. The values of the normal and tangential inertia forces will be equal, respectively:

$$\Phi^n = M\frac{V^2}{L}, \quad \Phi^\tau = M\frac{dV}{dt}, \tag{20}$$

where V – the speed of movement of the mass center of the tray with the crop and guides. For this speed, you can take the speed of movement of the axis of the tray sector. Projecting all forces onto the n axis we get

$$S_1 = Mg\cos\varphi + S\cos(\Psi - \varphi) + M\frac{V^2}{L}. \tag{21}$$

The speed of movement V, with sufficient accuracy for practice, can be defined as $V = \frac{dX}{dt}$ or make a differential equation of the motion of the mass center in the projection on the axis τ, which gives

$$\Phi^{\tau} + S\sin(\Psi - \varphi) - \mathrm{Mg}\sin\varphi = 0,$$

or

$$M\frac{dV}{dt} = \mathrm{Mg}\sin\varphi + S\sin(\Psi - \varphi). \quad (22)$$

Given the fact that the angles of mass deflection are small, the definition of force S_1, there are no fundamental difficulties in the upper bearing rod. Thus, having determined the magnitude of S and the velocity V, finds S_1.

Based on the above theoretical studies with real mass and geometric parameters

$$M = 96\,kg, m = 15{,}2\,kg, l = 0{,}85\,m, L = 0{,}75\,m$$

dynamic forces in rigid load–bearing elements surpassing static ones by 12 - 15%. If we take into account the fact that the installation operates in a quasi-static mode, then the safety margin of the load-bearing elements should not exceed n =2-2.5 [11]. Reinforcing bars with a diameter of 4-5 mm used as bearing elements have a margin of safety for installations with a seeding area of 1.5 m^2 n = 12-15. Such ratios of strength reserves are acceptable only in cases where it is not possible to replace the existing positions of the bearing elements with others.

3 Conclusions

1. Differential equations of dynamics of elements of the two-tier hydroponic installation with rigid load-bearing rods are derived and solved. Based on theoretical studies, the ratio of the parameters of the system under consideration was determined, in which there is no beating phenomenon.

2. It has been experimentally proved that during the operation of a two-tier hydroponic installation with rigid load-bearing rods, the maximum deviation of the mass center of the system, tray-crop-guides does not exceed 15 cm.

3. The phenomenon of runout can be avoided by varying the mass and geometric parameters of the installation, i.e. the lengths of the bearing elements should not be different, and the ratio of the masses of loaded and free trays should vary within 0.15–0.25.

4. Based on the above theoretical studies, with real mass and geometric parameters (M=96 kg, m=15.2 kg, l=0.85 m, L= 0.75 m), the dynamic forces in rigid load–bearing elements exceed static ones by 12-15%, and the safety margin of the load-bearing elements should not exceed n = 2-2.5.

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