The function of coefficients of added mass of water

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Abstract. The hydrodynamic effect of the water mass of the reservoir on the pressure face of the dam can be determined from the function of the coefficient of the added mass of water obtained in the Westergard solution. For an inclined pressure face, the design standards introduce a correction factor, which is recommended to be taken into account until the slope of the pressure face does not exceed 15^0; there are no such recommendations for earth dams, however, in the calculations of seismic resistance of earth dams, taking into account hydrodynamic water pressure is also important. Theoretically, the values of the coefficients of the added mass of water can be determined based on the numerical solution of the Laplace equation in the computational domain with given boundary conditions and the graphical-analytical construction of a hydrodynamic grid. Thus, in the MATLAB environment, computational schemes were built for different laying of the pressure head of the dam in the range from vertical to flat with laying 1:3, and orthogonal hydrodynamic grids were built, reflecting the solution of the Laplace equations concerning the functions of pressure and velocity potentials. For all problems, diagrams of the coefficients of the added mass of water are constructed, and they are compared with the Westergaard analytical solution. For practical use in the work, all solutions for constructing diagrams of the coefficients of the added mass of water are presented in the form of a nomogram.

1 Introduction

The development of theoretical foundations for taking into account the interaction of structures with the aquatic environment during vibrations began in the 30s of the last century; however, even today, it is being actively developed by domestic [1, 2] and foreign scientists [3-10], both finite element methods and analytical methods are used for solving individual problems. When solving the problem of the hydrodynamic interaction of a solid body with a liquid, some prerequisites are taken into account: about the properties of the liquid (viscosity, compressibility, uniformity); about the state of the free surface of the

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water (wave, waveless); - about the properties of the host bed of the reservoir (elastic, rigid); about the properties of a solid body (elastic, rigid). In solving problems, the provisions of continuum mechanics and the theory of small deformations are also adopted. The movement of water during oscillations is considered to be irrotational; therefore, the value of the velocity at each point is described by some potential function $\varphi(x, y, t)$ for which we can write:

$$\frac{\partial\varphi}{\partial x} = -\frac{\partial u}{\partial t}; \frac{\partial\varphi}{\partial y} = -\frac{\partial v}{\partial t} \tag{1}$$

where $u(x, y, t)$ and $v(x, y, t)$ are the displacement functions in the direction of the coordinate axis X and Y.

The velocity potential function $\varphi(x, y, t)$ satisfies the wave equation known from hydromechanics:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{C_0^2}\frac{\partial^2\varphi}{\partial t^2} \tag{2}$$

For the hydrodynamic pressure function, one can also write:

$$p(x, y, t) == \rho_0 \frac{\partial\varphi}{\partial t} \tag{3}$$

The simplest theoretical model that makes it possible to obtain a solution about water's hydrodynamic pressure on a structure's pressure face is the model of an ideal incompressible fluid. The Westergard solution obtained for the seismic water pressure on the rigid vertical pressure face of the dam for an incompressible fluid and a structure located in a rectangular canyon is a reference in all calculations of hydraulic structures and underlies the methods used.

2 Methods

The hydrodynamic impact of water on structures during an earthquake can be modeled using an "attached" mass of water. At present, calculations of hydraulic structures during earthquakes are carried out using the recommended Russian State Standards: SP 14.13330.2018 Construction in seismic areas and SP 358.1325800.2017 Hydraulic structures. Rules for design and construction in seismic regions; calculation formulas for determining the value of the added mass of water attributable to the estimated area of the dam pressure face: $(S_{p.\mathrm{f}})$

$$m_k = \rho_0 H \mu_k \psi S_{p.f} \tag{4}$$

where ρ_0 is the density of water;

H - water depth at the base of the structure;

$S_{p.f}$ - the surface area of the pressure face of the kth element;

μ_k - the coefficient of the added mass of water in the kth element of the pressure face;

ψ - the coefficient taking into account the limited length of the reservoir, is determined according to [11].

The main issue of calculating the added mass of water remains the determination of the value of the coefficient μ_k , which depends on the location of the pressure face of the dam.

As part of the consideration of horizontal vibrations, we will assume that earthquakes cause harmonic vibrations, and the dam can be represented as a rigidly fixed wall that moves as a whole along with the foundation. In such a case, the displacements can be determined from the equation:

$$\varepsilon_0 = \frac{AT^2}{4\pi^2}\cos\left[\frac{2\pi t}{T}\right] \tag{5}$$

where A is the accepted acceleration of the earth's surface during an earthquake.

Assuming that the displacements of the dam elements are small, then the relationships between hydrodynamic pressure, time, and orthogonal displacements (u, v, s) will be expressed by the following differential equations:

$$\begin{aligned} \frac{\partial P}{\partial x} &= \frac{\gamma_w}{g}\frac{\partial^2 u}{\partial t^2} \\ \frac{\partial P}{\partial y} &= \frac{\gamma_w}{g}\frac{\partial^2 v}{\partial t^2} \\ \frac{\partial P}{\partial z} &= \frac{\gamma_w}{g}\frac{\partial^2 s}{\partial t^2} \end{aligned} \tag{6}$$

Taking into account the assumption of the incompressibility of water [12], the continuity equation has the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial s}{\partial z} = 0 \tag{7}$$

Using equations (3) and (4), to determine the hydrodynamic pressure, we obtain:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0 \tag{8}$$

The resulting elliptic differential equation is called the Laplace equation, which also describes the behavior of a direct electric current in a conducting medium. The identity of differential equations makes it possible to obtain solutions to a hydrodynamic problem by solving an electrodynamic problem, on which the method of electrohydrodynamic analogy is based, which is widely used to solve problems of laminar motion of a fluid in a porous medium (filtration) [13]. It was this method that K. Zangar used to determine the hydrodynamic pressure on the pressure faces of dams in his work using electrohydrodynamic analogy devices.

As part of solving the problem of determining the coefficient μ_k, consider a flat area describing the inclination of the pressure head and some area of the reservoir behind it (Fig. 1).

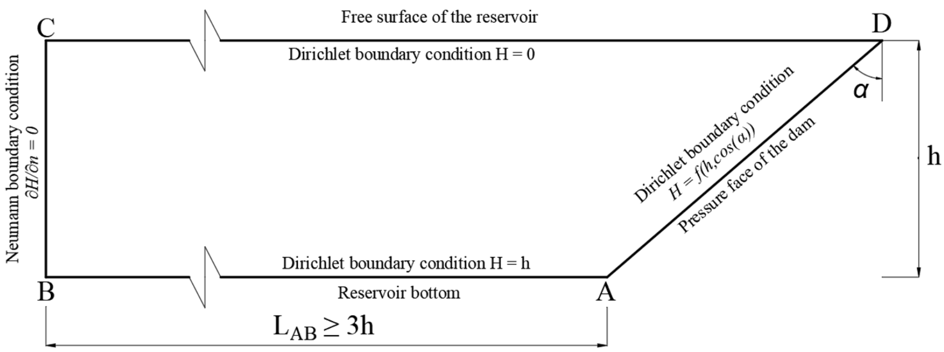

Fig. 1. Problem solution area

It is convenient to represent the solution of the Laplace equation for a given region in the form of a hydrodynamic grid composed of the intersection of a family of equipotentials (H = const) and streamlines (P = const).

First, to construct a hydrodynamic grid, it is necessary to set the boundary conditions that describe the behavior of the above functions on the area's boundaries under consideration (lines AB, BC, CD, DA). Following [14], boundary conditions will be satisfied at the region's boundaries, which in mathematical physics are called the Dirichlet and Neumann conditions.

The Dirichlet condition (or condition of the 1st kind) is the method of specifying the boundary condition when the distribution of some desired quantity is given directly by the function:

$$F_b(x) = H(x) \tag{6}$$

The Neumann condition (or condition of the 2nd kind) is the method of specifying the boundary condition when the distribution of some desired quantity is given not by a direct function but by its normal derivative:

$$F_b(x) = \frac{\partial H(x)}{\partial n} \tag{7}$$

In the area studied in the framework of our problem, the following conditions will be satisfied at the boundaries:

1. AB is at the bottom of the reservoir; the hydrodynamic head is constant

$$H = h = const \text{ (Dirichlet condition)} \tag{8}$$

2. BC is the conditional watertight boundary of the reservoir, which is a streamline

$$\frac{\partial H}{\partial n} = 0 \text{ (Neumann condition)} \tag{9}$$

3. CD is the free surface of the reservoir; the hydrodynamic head is equal to zero

$$H = 0 = const \text{ (Dirichlet condition)} \tag{10}$$

4. DA is the pressure face of the dam, the pressure at each point of the boundary is determined as a function of the height and the angle α between the pressure face and the vertical

$$H = f(h, \cos(\alpha)) \text{ (Dirichlet condition)} \tag{11}$$

Note that to eliminate the influence of the length of the reservoir on the solution of the Laplace equation, the following condition was fulfilled in the calculations [11]:

$$L_{res} \geq 3h \tag{12}$$

The Laplace equation with these boundary conditions will be solved by the finite element method in the MATLAB programming language for slopes of the dam pressure head with an angle $\alpha = 0°, 5°, 10°, 15°, 20°, 40°, 60°, 75°$. The solution algorithm is considered in detail in [15,16]. Note that the construction of streamlines was carried out graphically (by hand), ensuring that the hydrodynamic grid was orthogonal and quadratic.

The calculation results and coefficient diagrams μ_k for some calculation cases are presented in the figures below; equipotentials and streamlines are shown by solid and dotted lines, respectively.

3 Results

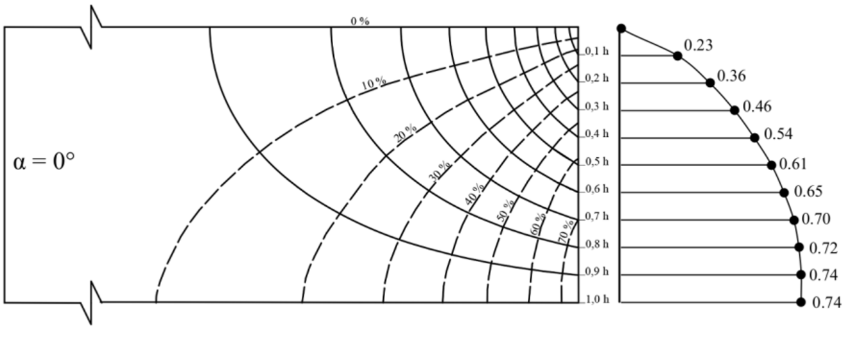

Fig. 2. Hydrodynamic grid and coefficient distribution diagram μ_k for $\alpha = 0°$

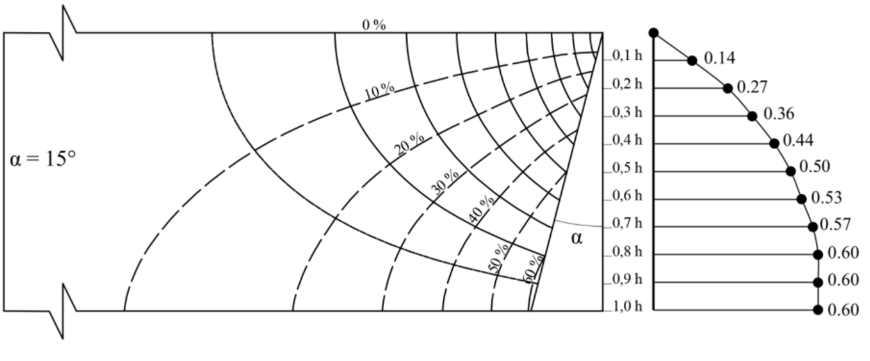

Fig. 3. Hydrodynamic grid and coefficient distribution diagram μ_k for $\alpha = 15°$

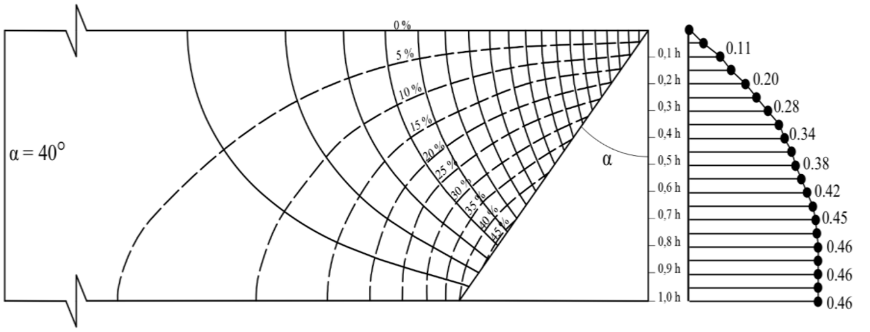

Fig. 4. Hydrodynamic grid and coefficient distribution diagram μ_k for $\alpha = 40°$

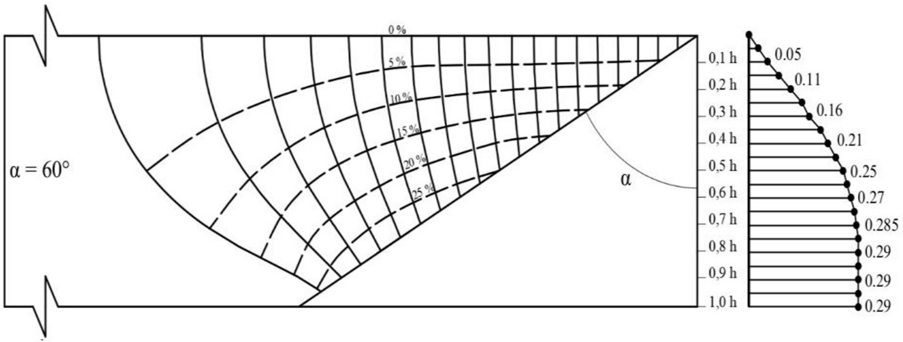

Fig. 5. Hydrodynamic grid and coefficient distribution diagram μ_k for $\alpha = 60°$

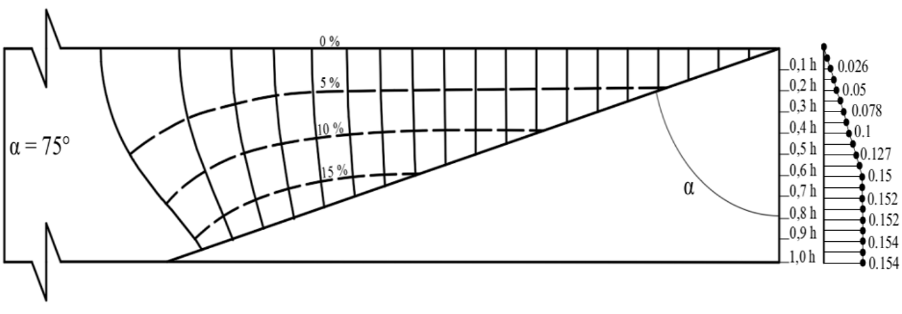

Fig. 6. Hydrodynamic grid and coefficient distribution diagram μ_k for $\alpha = 75°$

Figure 6 shows a generalization of the results of constructing coefficient distribution diagrams μ_k for all considered vertical angles α :

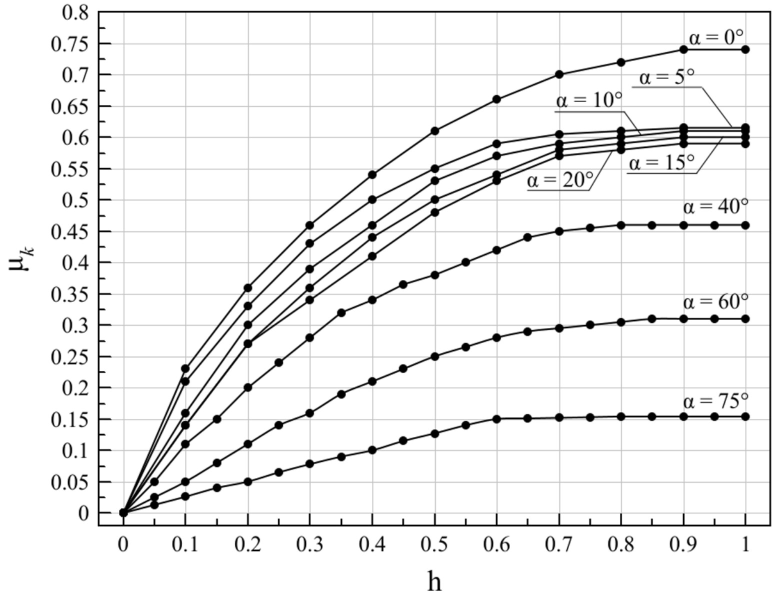

Fig. 7. Dependence of coefficient μ_k on relative depth of water at various values of vertical angle α

The figure below shows an alternative generalization of the obtained results in a nomogram for the practical construction of diagrams for plotting the coefficient μ_k for vertical angle values within 0 ... 75°.

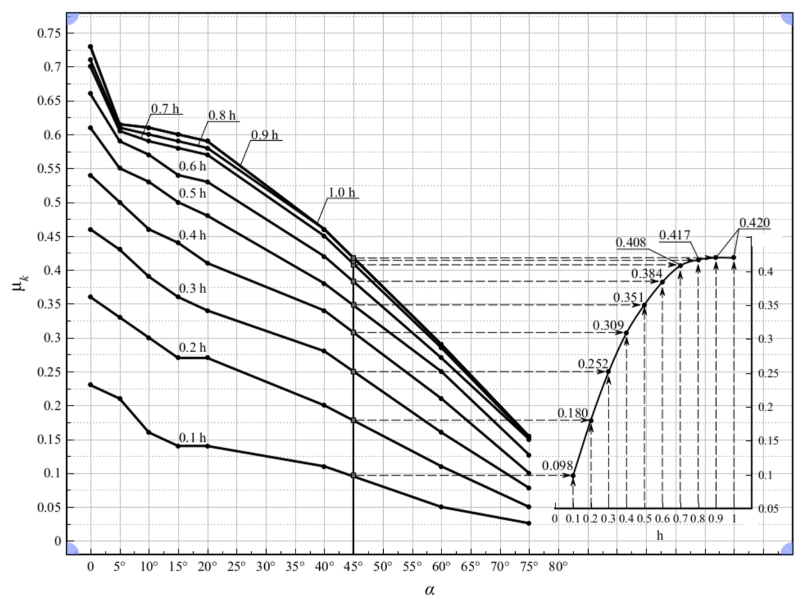

Fig. 8. Nomogram for plotting coefficient μ_k distribution diagram

4 Discussion

Comparing the obtained coefficient distribution diagram for the vertical pressure face with the diagram constructed according to the results of the Westergaard solution and fixed in the Russian State Standards:

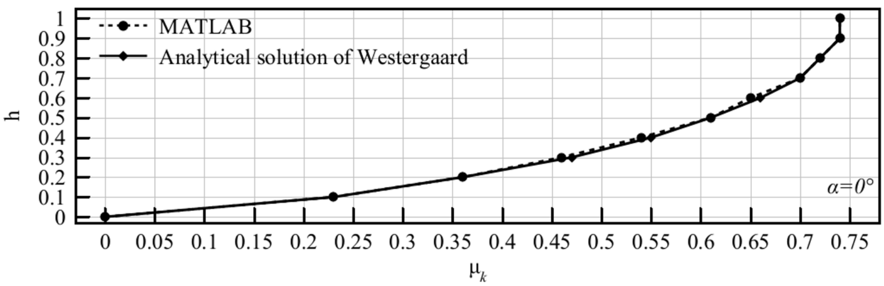

Fig. 9. Comparison of diagrams μ_k obtained by various calculation methods

As seen from Figure 8, the solutions coincided with a very high accuracy, which indicates the correctness and adequacy of the methodology for the solution of the Laplace equation.

5 Conclusions

1. Hydrodynamic grids and coefficient distribution diagrams μ_k for pressure faces of dams with a vertical laying angle $\alpha = 0°, 5°, 10°, 15°, 20°, 40°, 60°, 75°$ are constructed.
2. The results of the obtained solutions are generalized, and a nomogram is constructed for the practical construction of coefficient distribution diagrams μ_k for the pressure faces of dams within 0…75°
3. The obtained result is compared with the well-known Westergaard solution, and the correctness and adequacy of the chosen method for determining the values of the coefficient μ_k is proved.

References

1. R. A. Abdikarimov, Kh. Eshmatov, Sh. P. Bobanazarov, Mathematical modeling and calculation of hydraulic structures such as a dam-plate, taking into account seismic load and hydrodynamic water pressure, Engineering and Construction Journal, No. 3 (21), (2011)
2. I.D. Muzaev, N.I Muzaev, B.A Dzeboev, Mathematical modeling of the seismic resistance of a dam with regard to the influence of the aquatic environment, Computational technologies, V. 17, No. 1 (2012)
3. V. Lotfi, Dynamic Analysis of Concrete Gravity Dams by Decoupled Modal Approach in Time Domain, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, Paper No. 467 (2004)
4. Mansouri, R. Rezaei, Considering Dynamic Analysis Results of Interactions between Concrete Dams and Reservoirs in Time Domain and Frequency Domain for Choosing the Optimimal Model, European Journal of Scientific Research, Vol.46 (4), (2010)

5. S. Sica, A. D. Russo, Seismic response of large earth dams in near-source areas, Computers and Geotechnics, Volume 132 (2021)
6. X. Yang, S. Chi, Seismic stability of earth-rock dams using finite element limit analysis, Soil Dynamics and Earthquake Engineering,Volume 64 (2014)
7. J. Ghosh, J. E. Padgett, L. Dueñas-Osorio, Surrogate modelling and failure surface visualization for efficient seismic vulnerability assessment of highway bridges, Probabilistic Engineering Mechanics, Volume 34 (2013)
8. M. Ali, M. R. Alam, M. Haque, M. J. Alam, Comparison of Design and Analysis of Concrete Gravity Dam, Natural Resources, Vol. 3, No. 1 (2012)
9. M. Raja, B. Maheshwari, Behaviour of Earth Dam under Seismic Load Considering Nonlinearity of the Soil, Open Journal of Civil Engineering, 6 (2016)
10. Ciancimino, R. M. Cosentini, F. Figura, S. Foti, Simplified assessment of the seismic vulnerability of small earth dams, Procedia Structural Integrity, Volume 44 (2023)
11. A.S. Bestuzheva, Calculation of seismic resistance of structures, MISI-MGSU (2020)
12. B.T. Yemtsev, Technical hydromechanics, Mashinostroenie (1987)
13. A.D. Girgidov, Fluid and Gas Mechanics, Infra-M (2014)
14. I.V. Grebennikova, Equations of Mathematical Physics, Ural Federal University (2016)
15. P.P. Chernorus, Petr. P. Chernorus, Numerical methods and their application in MATLAB, BSTU (2018)
16. C.P. Lopez, MATLAB Differential Equations, Madrid: Springer-Apress (2014)