Hydraulic parameters of well of Pskom hydroelectric power plant in Uzbekistan

Sharifjon Sharopov[1], *Abdulaziz* Abduraimov[1], *Jumaniyoz* Donoboyev[1] and *Aleksandra* Ostyakova[2, 3*]

[1]"Tashkent Institute of Irrigation and Agricultural Mechanization Engineers", National Research University, Tashkent, Uzbekistan
[2]National Research University "Moscow Power Engineering Institute", Moscow, Russia
[3]Water Problem Institute RAS, Moscow, Russia

Abstract. In arid regions, the construction of hydraulic structures is relevant due to the shortage of water resources. Using the example of the Pskov hydroelectric power station, the results of the hydraulic calculation of the end structure of the energy spillway, made in the form of a water well of a unique design, are presented. A culvert wall, made with an initial expanding part and a sloping spillway part forming an oblique spillway, forms the well. The depths and speeds in each part of the water well and the parameters of the hydraulic jump are calculated. It is shown how significant damping of the water flow energy occurs in the selected water well design. The water velocity at the outlet of the water well becomes less eroded, decreasing several times. The critical depth is calculated when the width of the stream changes, the depth of water in the waterhole, the pressure on the waterhole wall, the height of the waterhole wall located along the spillway face of variable width, the interface of the water flows behind the waterhole wall.

1 Introduction

Measures to eliminate the shortage of water resources have always been relevant due to the aridity of the climate of Uzbekistan, the growing population, and the requirements for environmental protection. There is a severe water shortage in this region for agriculture, industrial water supply, household needs of the population, and other purposes. The load level on power facilities is estimated as high due to the lack of hydraulic power plants and insufficient rational regulation in the presence of the hydraulic potential of the rivers of the Republic of Uzbekistan. In 2020, Uzbekistan's electricity generating capacity was 12.9 thousand megawatts, while hydroelectric power generation was about 14.3%, which is not enough.

After completion of construction, the Pskom hydroelectric power station will be one of the largest hydroelectric power stations in Uzbekistan after the Charvak hydroelectric

*Corresponding author: alex-ost2006@yandex.ru

power station [1]. Pskom is located in a large area of the river, located in the Charvak Reservoir, which is located in the territories of Uzbekistan and allows to partially exclude imports. The area of the Pskov River basin is 2840 km^2, and the length is 70 km; the supply of the Pskom River is glacial, and the river has more than 40 tributaries that are located in the mountainous part of Uzbekistan, the flow of which is formed by meltwater and rainwater and is 82.2 m^3/s [2]. The largest tributaries of the Pskom River are the Maidantal River (Sarybashsay) (17 % of the flow of the river Pskom) and the Oygaing River (34 % of the runoff of the Pskom).

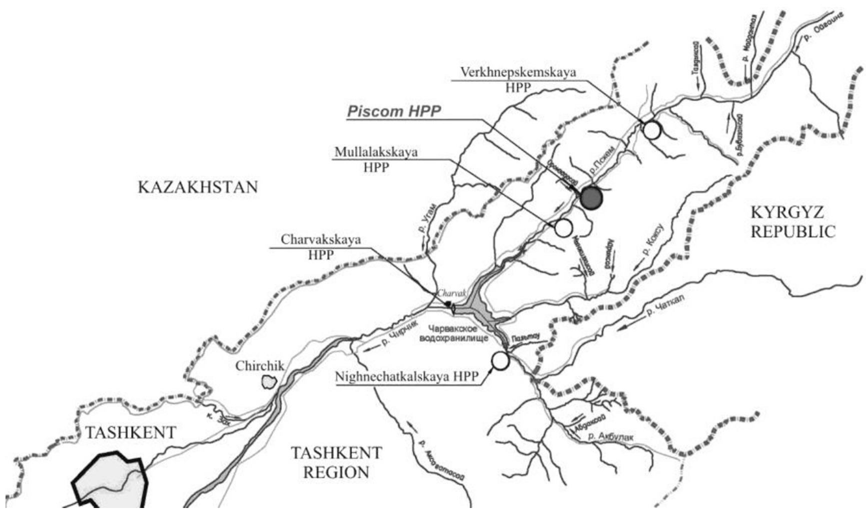

Fig. 1. Diagram of cascade of hydroelectric power plants of Republic of Uzbekistan

The highest water levels and expenditures in the Pskom River are observed most often in June. The average annual flow rate of the Pskom River in the dam of the Pskom hydroelectric power station is one-third of the flow to the Charvak hydroelectric power station. According to several observations for more than 80 years, the maximum water consumption was 558 m^3/s (1959, 1981, and 1992 years) and 514 m^3/s (1969 year). The estimated maximum water consumption of the Pskom River in the alignment of the Pskom hydroelectric dam is determined for the provision of Class I structures (0.01, 0.1, 0.5, 1, 3, 5 and 10%) for the observation period 1932-2015 years, for which data are available, and are given in table 1.

Table 1. Estimated maximum water consumption of the Pskom River - the Pskom hydroelectric power station dam.

Observation period	$Q_{cp max}$, m^3/s	Maximum water consumption of various security, %						
		0.01	0.1	0.5	1	3	5	10
1932-1943 years 1945-2015 years	271	1003	673	564	518	449	417	373

According to the project, the structure of the hydroelectric complex includes a dam with a height of 195 m, spillway structures, an energy spillway, a hydroelectric power station building, and a switchgear. After construction, the Pskom hydroelectric power plant will be one of the largest in the region in terms of capacity and will provide its water supply.

This article aims to provide a calculated justification of the hydraulic modes of operation of the Pskom hydroelectric well for a further selection of a coupling structure that would meet the reliability and necessary water throughput [3].

The design mark of the normal retaining level ∇NRL = 1166.0 m. This mark corresponds to a water flow rate of 612.0 m^3/s. On the right bank of the Pskem River in the area of the Pskov hydroelectric power station under construction in the Bostanlyk district of the Tashkent region, the end construction of a tunnel spillway - a water well formed by a water wall has been designed. A tunnel-type spillway gate chamber is located in front of the culvert. The threshold mark of the shutter chamber is 982.0 m; the chamber has two spans at the same level, separated by a 2 m thick support. The end section of the tunnel is located on the section from the junction of the 2nd tier of the energy spillway with the gate chamber and has a variable slope and a diameter of 6.5 m (figure 2).

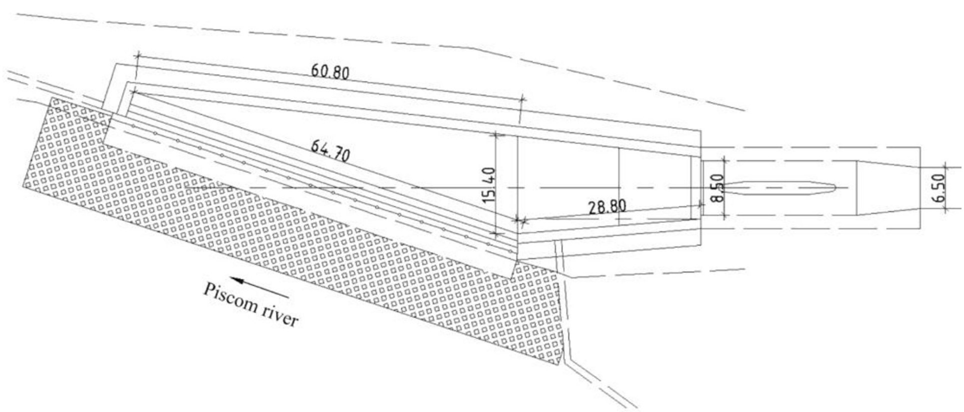

Fig. 2. Plan of terminal structure

When the reservoir formed by the Pskom hydroelectric power station is fully filled and the specified value of the normal retaining level, the operating pressure and energy reserve is a significant amount, therefore it was necessary to design a water well so that the water energy is extinguished in it as much as possible. To reduce the specific flow rate in the design of the water well, an extension is provided in its initial part with a length along the axis of 28.8 m, then in the expanding part of the water well, with a width of 15.4 m, a planned part of the water well is designed with a side spillway and a water wall along its entire width. The device of the water wall is necessary to exclude the discharge of the hydraulic jump and the damping of the flow energy in the water well formed by the wall. The length of the entire water well along the axis is 46 m, and the width of the side spillway is 64.7 m. The width of the water well at the initial expansion site, according to the available materials, varies asymmetrically from the left and right sides – the angle of expansion of the water well is 7° on the right along the flow path and 5° on the left. To exclude the washing of the end structure, a 2 m high springboard sock was designed (figure 3). After the water flow exits the water well, its speed should be less than the speed eroding the bottom of the river.

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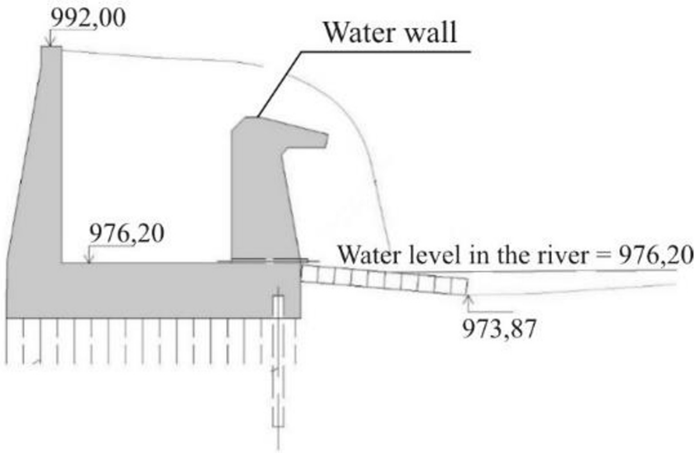

Fig. 3. Cross-section of water well.

Well-known scientific works on the topic of hydraulic calculation of waterworks related to the calculation of hydraulic jump parameters in prismatic wells [2-6]. The calculation of the hydraulic parameters of the lateral spillway is given in the works [7-13]. The results of calculating the differential waterworks [14-17] and the expanding part of the fast-flowing [18-22] are known. The design of the water well described in this article is unique. The previously obtained results of the calculation of combined water wells, such as in the Pskov hydroelectric power station project, are unknown and, therefore, new.
Since the hydraulic calculation is performed for the unique design of the water well, the task was set to calculate the hydraulic characteristics for optimal operation of the water well of the described design, which ensures flooding of the hydraulic jump in it.

2 Methods

The initial data for hydraulic calculations of the water well are the long-term results of monitoring the flow of the Pskov River, as well as design solutions.

Calculation of hydraulic modes of operation of the considered water well is carried out based on materials of extensive laboratory studies and generalization of all available materials suitable for the case under consideration.

The end section of the tunnel spillway is arranged in the form of a combined culvert formed by a culvert wall, with expansion in its initial part and a further lateral discharge of water downstream. The tunnel with a diameter of 6.5 m changes its shape in the gate chamber from round to rectangular; its exit section has a rectangular shape with a width of 8.5 m, a height of 5 m, and is the beginning of a combined water well. The lower mark of the tunnel of the energy spillway in front of the gate chamber, the mark of the bottom of the well, and the mark of the water surface in the Pskov River in the lower reaches are equal to 976.2 m. When passing through the gate chamber, the water flow loses some of its energy due to the change in the shape of the tunnel, the separation of the flow into two parts, and losses in the gate chamber. The water wall and the water well formed by it are arranged to exclude the discharge of the hydraulic jump, dampen the energy of the water flow, and reduce the average flow velocity before the water is discharged into the lower reaches of the river. When the water flow is rotated, and the water is subsequently discharged through the culvert, the energy is additionally extinguished since the culvert works as an unflooded spillway.

With an estimated water flow rate Q_{calc} = 612 m3/s and a normal retaining level

∇NRL = 1166.0 m, the effective pressure is equal to the difference in the elevation of the water at the normal retaining level and the downstream level:

$$z = \nabla NRL - \nabla DSL = 199.8 \text{ m} \quad (1)$$

Along the stream on the left, after the expansion section, the water is discharged downstream, while the flow begins to change its direction as it approaches the water wall, moving along the axis of the water well. Both in the initial and the final part, the water well is asymmetrical: in the expanding part, the angles of deviation from the axis of the well on the left and right are not the same (5° and 7°); in the spillway part, the right wall of the well is rectilinear, the angle of deviation from the axis is the same as in the expanding part, and the left part of the flow reduces its width; additionally, the left side of the spillway part of the well is a spillway, preventing the speed from increasing when the section narrows and maintaining a constant depth along the spillway face.

Figures 4 and 5 show photographs of the operation of a model water well of this design; tests were carried out at the All-Russian Research Institute of Hydraulic Engineering, named after B.E. Vedeneyev. The photo shows that with a large flow rate, the water flow exits the tunnel in a turbulent state, forming a hydraulic jump. Unfortunately, the authors of this article do not have the data of the results of laboratory measurements to compare them with the performed hydraulic calculations. Nevertheless, the results of the calculations given in this article are consistent and adequate.

Fig. 4. Laboratory model of water well

Fig. 5. Photos of laboratory studies of hydraulic modes of water well

Thus, the energy of the water flow coming out of the tunnel is extinguished in the water well. The calculation is carried out under the assumption that the tunnel works in pressure mode with an estimated flow rate Q_{calc} = 612 m^3/s.

The area of the circular section of the tunnel is equal to

$$\omega_{round} = \frac{\pi d^2}{4} \quad (2)$$

The flow velocity at the exit from the tunnel into the gate chamber is equal to

$$V_{round} = \frac{Q_{calc}}{\omega_{round}} \quad (3)$$

The exit section after the gate chamber becomes rectangular, width B = 8.5, height H = 5.0 m. After the gate chamber, the water flow exits into the water well. The area of the exit section at the end of the gate chamber, which is the area of the initial section of the water well, as well as the velocity in it, are equal

$$\omega_{square} = B_k H \quad (4)$$

$$V_{square} = \frac{Q_{calc}}{\omega_{square}} \quad (5)$$

The width at the end of the expanding part of the water well B = 15.4; at the calculated flow rate, the average water flow rate depends on the depth of the hydraulic jump in the water well. The average speed in this section will be calculated further.

2.1 Determination of the specific flow rate and critical depth at the site of the expansion of the water well

The specific flow rate Q_{calc} under the assumption of constancy of the width of the water well is equal to

$$q_{square} = \frac{Q_{calc}}{B} \quad (6)$$

then the critical depth of water in a water well of rectangular cross-section is equal to

$$h_k = \sqrt[3]{\frac{\alpha q_{square}^2}{g}} \quad (7)$$

where α - Coriolis coefficient; $g = 9.81$ m/s^2 - earth acceleration.

For the part of the well expanding in a plan, the critical depth is specified according to the graph of the dependence of the specific energy of the section on the depth $E = f(h)$:

$$E = h + \frac{\alpha Q_{calc}^2}{2g\omega^2} \quad (8)$$

where $\omega = BH$ is cross-sectional area, m^2, $Q_{calc} = 612$ m^3/s is estimated consumption.

The minimum of the function $E(h)$ gives the value of the critical depth h_k. The value of the critical depth h_k calculated for a rectangular section and the value h_k determined by the graph of the function (7) differ. The graph is based on the values given in Table 2. The graph shows the values of the minimum specific energy of the section $E(h)$ at the exit of the flow from the tunnel and in the section at the end of the expanding section of the water well (figure 6).

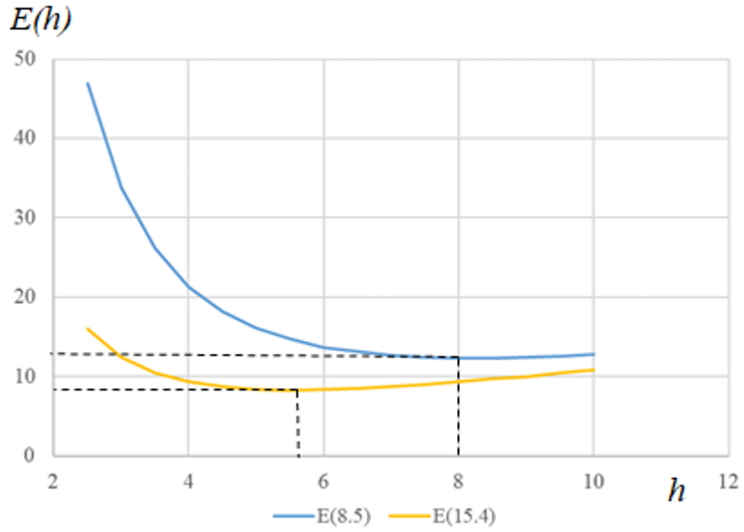

Fig. 6. Graph of dependence of specific energy of the cross-section on depth of flow in water well.

The minimum values of the specific energy of the section corresponding to the critical depths in these sections: the critical depth at and B = 8.5 m is equal to h_k = 8.0 m in the exit section from the gate chamber to the water well. At the expansion section of the water well from B = 8.5 m to B = 15.4 m, the critical depth decreases to h_k = 5.5 m. The greater the width of the flow, the lower the depths, and the more energy is extinguished – this is one of the design features of this water well.

2.2 Determination of the separate depth and length of the hydraulic jump

On the planed section of the water well, due to a decrease in the width of the flow and the specific flow rate along the bevel, the average speed along the length of the water well changes slightly, and the depths increase significantly due to the coupling of the depths in the hydraulic jump. The following is a calculation of the parameters of the hydraulic jump for the considered water well. The water flow enters the water well in a turbulent state (Fr > 1). Its depth increases downstream, so the depth at the well outlet is equal to the compressed depth h_c = 5.0 m. The conjugate depth is the split depth of the h_{sep}, which is determined by the formula

$$h_{sep} = \frac{h_c}{2}\left[\sqrt{1+8\left(\frac{h_{cr}}{h_c}\right)^3}-1\right] \tag{9}$$

The length of the hydraulic jump at flow rate according to the formula of M.D. Chertousov at a critical depth h_{cr} = 8.0 m is equal to

$$L_{jump} = 10.3h_c\left[\sqrt{\left(\frac{h_{cr}}{h_c}\right)^3}-1\right]^{0.81} \tag{10}$$

At a lower critical depth, the L_{jump} is smaller, but here we consider the case when the length of the water well can be longer. Then the depth of water in the culvert in front of the wall at Q_{calc} = 612 m3/s is equal to

$$t_{culv} = 1.05 h_{sep} \tag{11}$$

The length of the culvert along the axis to the culvert wall

$$L_{culv} = 0.8 L_{jump} \tag{12}$$

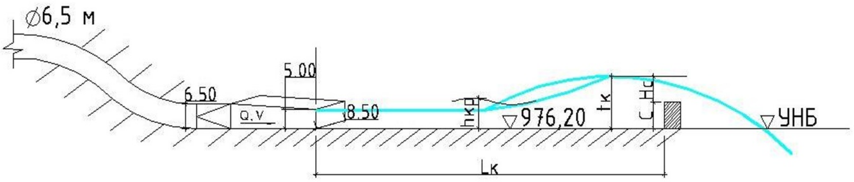

Fig. 7. Hydraulic jump calculation scheme.

The maximum length of the water well L_{culv} = 42 m, obtained from the hydraulic calculation at a critical depth h_{cr} = 8.0 m, almost coincides with the length of the well along the axis (figure 2), which is 46.0 m.

The water depth in the well, equal to the second conjugate depth t_{culv}, will be located at a distance L_{culv} from the beginning of the waterhole at the calculated flow rate. The expansion of the water flow in the initial part of the water well leads to a decrease in critical and separate depths, depth t_{culv}, the length of the hydraulic jump L_{jump} and the length of the water well L_{culv}, as well as to the quenching of energy in this area.

The average velocity of water flow in the water well in the extension section with a length of 28.8 m decreases due to an increase in the width of the flow. Further, in the section of the lateral discharge of water through the culvert wall, due to a decrease in the width of the flow, the specific water flow decreases, with approximately the same speed along the length of the well bevel, and the depth also remains almost unchanged. Therefore, the average flow velocity at the beginning of the planned part of the water well can be found in the expression

$$V_{culv} = \frac{Q_{calc}}{t_{culv} B} = 6.31 \text{ m} \tag{13}$$

After the energy is extinguished using a hydraulic jump in the water well, the Froude and Reynolds numbers will change, as shown in Table 2.

Table 2. Average velocity V, numbers Fr and Re in characteristic sections of water well.

№	Section shape	Characteristic size	V, m/s	$\mathrm{Fr} = \frac{V^2}{gh}$	$\mathrm{Re} = \frac{V \cdot 4R}{\nu}$
1	In a circular tunnel	d = 6.5 m	18.45	5.33	$120 \cdot 10^6$
2	In the output rectangular section	B = 8.5 m	14.40	4.23	$90 \cdot 10^6$
3	At the end of the expanding part of a rectangular water well	B = 15.4 m	3.15	0.08	$60 \cdot 10^6$
4	Along the spillway face	B = 64.7 m	1.51	0.02	$32 \cdot 10^6$

The average water speed in a water well is less than non-eroding (permissible) flow velocities for siltstones that form the riverbed behind the water well. At the depth of the riverbed in the lower reaches 2.3 non-eroding (permissible), siltstone velocities are 6…6.5 m/s. This is more than the water flow rate in the water well; therefore, the condition of the indissolubility of the channel behind the water well is provided.

The hydraulic jump begins to increase the depth from the beginning of the flow exit into the water well from the tunnel, while the compressed depth hc = 5.0 m, the critical depth hcr = 8.0 m; the water depth in the well t_k = 12.6 m. The depth ratio shows that the hydraulic jump in the water well is flooded.

2.3 Calculation of the height of the water wall

In the hydraulic calculation of the water wall, its height c and the length of the well formed by the wall l_{culv} are determined. The length of the well is defined above; it is equal to 42 m. The height of the water wall (figure 5) is calculated by the formula:

$$c = t_{culv} - H_{wall}, \tag{14}$$

where $H_{ст}$ is the pressure on the wall, which is calculated by the formula

$$H_{wall} = \sqrt[3]{\frac{Q_{calc}^2}{m_{wall}^2 \sigma_{wall}^2 B^2 2g}} - \frac{\alpha V_{culv}^2}{2g}, \tag{15}$$

where V_{culv} is average flow rate in the well; m_{wall} is the flow coefficient, its value as for a spillway with a wide threshold, m_{wall} = 0.38; σ_{wall} = 1.0 are the coefficient of flooding of the wall, the water wall is not flooded.

Checking the waterhole wall for flooding is not required since the mark of the bottom of the waterhole coincides with the mark of the downstream level; therefore, the presence of water in the well ensures that the waterhole wall is not flooded. This calculation is given under the condition of passing the maximum calculated flow rate at a normal retaining level. With lower costs, the pressure on the wall will be less, the speeds in the water well will also decrease

To establish the flow interface mode behind the water wall, it is necessary to calculate the compressed depth behind the wall h_{cw}, the separate depth after the wall $h_{sep.w,}$ and compare them with the depth of water in the lower reaches of the riverbed, equal to h_0 = 2.93 m. The depth of h_{cw} is determined by the approximation method; after the second approximation, we get:

$$h_{cw} = \frac{Q_{calc}}{\varphi_{wall} B \sqrt{2g\left(t_{0culv} - h_{cw}\right)}}, \tag{16}$$

where t_{0culv} is full reserve of specific energy in front of the wall:

$$t_{0culv} = t_{culv} + \frac{\alpha V_{culv}^2}{2g}, \tag{17}$$

φ_{wall} is coefficient of velocity from the accepted outline of the water wall, $\varphi_{wall} = 0.95$.
To determine the critical depth of the h_{cr} behind the culvert wall in the riverbed, you need to know the cross-sectional area of the riverbed behind the wall. Water is discharged into the riverbed through a side spillway 64.7 m wide, then the specific flow rate on the spillway wall is $q = \frac{612}{64.7} = 9.46$ m²/s. Considering the riverbed behind a rectangular water well, we calculate the critical depth in the riverbed according to the formula (7); we get $h_{cr} = 2.12$ m.
Using the obtained values, we determine the value of the separate depth after the wall:

$$h_{sep.w} = \frac{h_c}{2}\left[\sqrt{1 + 8\left(\frac{h_{cr}}{h_c}\right)^3} - 1\right] \tag{18}$$

A comparison of the separate depth behind the water wall and the depth in the riverbed h_0 = 2.93 m shows that after the wall in the riverbed, the hydraulic jump is in the driven state: $h_{sep.w} > h_0$. It is necessary to install additional dampers for the energy of the water flow.

2.4 Energy losses

Energy losses along the length of a combined water well are calculated using the Darcy-Weisbach formula to estimate the velocity of the water flow behind the wall

$$h_l = \lambda \frac{l}{4R} \frac{V^2}{2g} \tag{19}$$

The coefficient of hydraulic friction along the length is calculated according to the Shifrinson formula since the resistance mode in the water well is quadratic:

$$\lambda = 0.11\left(\frac{k_e}{4R}\right)^{0.25} \tag{20}$$

where R *is* hydraulic radius, k_e is equivalent roughness coefficient, for concrete $k_e = 0.05$ mm.

The hydraulic radius at $B = 8.5$ m is equal to $R = 1.57$ m, then the coefficient $\lambda = 0.01$; at $B = 15.4$ m, the value $R = 3.47$ m and the coefficient $\lambda = 0.008$. Losses along the length of the initial section of the water well according to the Darcy-Weisbach formula are calculated on the expansion section as the average value of losses at $B = 8.5$ m ($h_l = 0.49$ m) and at $B = 15.4$ m ($h_l = 0.03$ m) and are 0.26 m. On the sloping section of the water well,

the flow changes direction by about 37°, which creates the value of local losses for turning the flow, for its expansion (12°), and losses on the water wall working as a spillway.
Losses on local resistances are calculated by the Weisbach formula:

$$h_м = \sum \zeta \frac{V^2}{2g}, \tag{21}$$

but due to the incomparable smallness of losses in length, they are not considered.

3 Results and Discussion

The results of calculating the hydraulic parameters of the well of the Pskov hydroelectric power station according to the formulas (2)-(20) are given in Table 3.

Table 3. Result of hydraulic calculation of parameters of well of Pskom hydroelectric power station

ω_{round}, m^2	ω_{square}, m^2	V_{round}, m/s	V_{Streyt}, m/s	h_k (formula), m	h_k (graph), m
33.16	42.5	18.45	14.4	8.21	8.0 at B = 8.5 m 5.5 at B = 15.4 m
H_{razd}, m	T_k, m	L_{kol}, m	V_{kol}, m/s	c, m	H_{wall}, m
6.33	12.6	42.0	3.15 (at the beginning of the slanting part)	4.9	7.7

At the right side of the water well, the width of the flow is reduced to a minimum (to zero), and the length of the flow is reduced to a maximum. The length of the spillway face of the oblique spillway is l = 60.8 m. Since the depth in the well increases over a short length using a hydraulic jump, we assume at the end of the well a depth equal to $H = t_k$. Then it is possible to calculate the h_l losses along the length depending on the width B; when calculating, we take the value of the hydraulic resistance coefficient λ = 0.03 (Table 4).

Table 4. Loss of flow energy along length of beveled part of well.

№	B, m	Q, m^3/s	q, m^2/s	V, m/s	R, m	h_l, m
1	3.3	612	185.5	14.7	1.5	3.4
2	4.4		139.1	11.0	1.9	1.5
3	5.5		111.3	8.8	2.3	0.8
4	6.6		92.3	7.3	2.6	0.5
5	7.7		79.5	6.3	2.95	0.3
6	8.8		69.6	5.5	3.3	0.2
7	9.9		61.8	4.9	3.6	0.2
8	11.0		55.6	4.4	3.8	0.1
9	12.1		50.6	4.0	4.1	0.1
10	13.2		46.4	3.7	4.3	0.1
11	14.3		42.8	3.4	4.6	0.1
12	15.4		39.7	3.2	4.8	0.1

Thus, a hydraulic calculation of the parameters of a unique design water well has been performed, which will be used as the end structure of the tunnel spillway of the Pskov hydroelectric power station. An innovative explanation of the calculation results lies in the uniqueness of the design of the water well since this structure differs in shape from conventional rectangular water wells. Well-known calculations of waterworks were used for rectangular water wells. The hydraulic calculation in this article is performed to construct a water well with an initial symmetrically expanding part. And then, immediately, the well begins to work like a spillway with a beveled in terms of the spillway face. Calculations of the flow and quenching of energy in such water wells were still unknown.

4 Conclusions

The considered design of the water well is combined and quite complex for design and the production of hydraulic calculations. The results obtained are based on the available design solutions. At the initial site, the well is expanding; flow rates and depths at the beginning and end of the water well are calculated here, considering the hydraulic jump. The hydraulic parameters of the well and the height of the water wall are calculated.
The parameters of the water well obtained by hydraulic calculations will help to clarify the design values of the hydroelectric facility under construction in Uzbekistan.

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