Pulsating flow of stationary elastic-viscous fluids in flat-wall channel

A. Sh. Begjanov[1], *Z. K.* Shukurov[2], and *B. Sh.* Yuldoshev[3*]

[1]Urgench State University, Urgench, Uzbekistan
[2]Kattakurgan branch of Samarkand State University, Samarkand, Uzbekistan
[3]"Tashkent Institute of Irrigation and Agricultural Mechanization Engineers" National Research University, Tashkent, Uzbekistan

Abstract. In this article, a specific problem of pulsating flow of visco-elastic fluids in a flat channel in a stationary state was solved. The main goal is to study the movement of visco-elastic fluids based on simplified mathematical models and to determine the existing hydrodynamic laws and hydrodynamic effects in the pulsating flow of a Newtonian fluid based on the obtained results.

1 Introduction

Stationary oscillating (pulsating) flows, in which transient processes occur in the flow of liquids, are particularly interesting in science, technical and technological processes. In such processes, even if the fluid movement occurs in a stationary mode due to the presence of oscillatory motion, the considered process consists of a periodic function of time. In this case, it is considered that the fluid fluctuations occur in the same state in each period. Therefore, when solving fluid flow problems, periodic functions of time can be used, which makes it much easier to solve a system of differential equations. Many scientific and practical studies have been devoted to the pulsating flows of Newtonian fluids in flat channels and cylindrical pipes by domestic and foreign scientists. In particular, in research studies [1-7], the stationary oscillation flows of pulsating viscous fluids in channels and pipes have been sufficiently studied. Scientific research on the pulsating flow of viscous fluids was applied to the circulatory system of biomechanics [8-12]. In this area, the research works conducted by applying the generalized topological models of Shulman-Khusid [13-20] to the pulsating flow of visco-elastic fluids are insufficient. It is known that the generalized topological models of Shulman-Husid play an important role in characterizing the pulsating behavior of polymer fluids, turbid water mixtures, and other similar fluids [21-24].

2 Methods

In formulating the problem, the distance between the walls of a flat channel is defined as $2h$, and the length of the channel is defined as *L*. Here *L* is large enough that h/L=1, the

*Corresponding author: baxtiyor_yuldashev68@mail.ru

condition is satisfied. The flow is stabilized in such cases, and the transverse velocity value is found using the continuity equation. And channel axes are defined as follows: Axis x is directed along the middle of the channel in a horizontal direction and is called the longitudinal axis, and the axis y is taken in a vertical direction perpendicular to the axis x and is referred to as the vertical axis. The mathematical model of the problem is as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial \tau}{\partial y}, \quad \dfrac{\partial u}{\partial x} + \dfrac{\partial \vartheta}{\partial y} = 0, \\ \tau = \sum_{k=1}^{\infty} \tau_k, \quad \dfrac{\partial \tau_k}{\partial t} + \dfrac{g_k}{\lambda_k}\tau_k = 2 p_k \mathrm{D}_{21}, \\ \dfrac{\partial p_k}{\partial t} + \dfrac{g_k}{\lambda_k} p_k = \dfrac{\eta_k}{\lambda^2{}_k} f_k, \quad \mathrm{D}_{21} = \dfrac{1}{2}\dfrac{\partial u}{\partial y}. \end{cases} \tag{1}$$

Since the pulsating flows of visco-elastic liquids are considered at small deformations of liquid particles, functions f_k and g_k can be taken as invariant $f_k = 1$, $g_k = 1$ based on the work [10-12]. Then equations (1) can be expressed in form (2) by performing mathematical operations:

$$\begin{cases} i\omega u_1 = -\dfrac{1}{\rho}\dfrac{\partial p_1}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial \tau_1}{\partial y}, \\ \dfrac{\partial u_1}{\partial x} + \dfrac{\partial \vartheta_1}{\partial y} = 0, \\ \tau_1 = \sum_{k=1}^{\infty} \tau_{1k}, \; i\omega\tau_{1k} + \dfrac{1}{\lambda_k}\tau_{1k} = p_k \dfrac{\partial u_1}{\partial y}, \\ i\omega p_k + \dfrac{1}{\lambda_k} p_k = \dfrac{\eta_k}{\lambda_k^2}. \end{cases} \tag{2}$$

Since the flow is symmetrical about the longitudinal axis of the flat channel, the boundary conditions are formulated as follows:

$$\begin{aligned} &y = h \text{ да } u_1 = 0, \upsilon_1 = \frac{h\gamma^*}{\eta}(p_1 - p_c), \\ &y = 0 \text{ да } \frac{\partial u_1}{\partial y} = 0, \; \upsilon_1 = 0, \\ &x = 0 \text{ да } \mathrm{p}_1 = p_1^0, \\ &x = L \text{ да } \mathrm{p}_1 = p_1^L. \end{aligned} \tag{3}$$

Solving the resulting system of equations (2) using boundary conditions (3), we express the solution by changing the form as follows:

$$p_1(x) = p_1^0 \frac{sh\sqrt{\bar{k}\, z_0}L\left(1-\dfrac{x}{L}\right)}{sh\sqrt{\bar{k}\, z_0}L} + p_1^L \frac{sh\sqrt{\bar{k}\, z_0}L\dfrac{x}{L}}{sh\sqrt{\bar{k}\, z_0}L}\Bigg], \tag{4}$$

$$< u_1(x) >= p_1^0 \frac{ch\sqrt{\bar{k}\, z_0}L\left(1-\dfrac{x}{L}\right)}{sh\sqrt{\bar{k}\, z_0}L} - p_1^L \frac{sh\sqrt{\bar{k}\, z_0}L\dfrac{x}{L}}{sh\sqrt{\bar{k}\, z_0}L}\sqrt{\frac{\bar{k}}{\bar{z}}}. \tag{5}$$

The found formulas (4) and (5) represent the formulas for finding the pressure and averaged velocities of the fluid along the longitudinal axis in a flat channel with a permeable wall, respectively. By analyzing the characteristics of the magnitude in the argument of the hyperbolic sine and hyperbolic cosine functions in these formulas, it is possible to determine the propagation speed of pulse waves and their attenuation along the longitudinal axis. From the found solutions (4) and (5), the formula defining the distribution law of the longitudinal velocity profile in the pulsating flow of a Newtonian fluid is derived. For this, it is sufficient to set the relaxation coefficient in a visco-elastic fluid equal to zero.

To obtain calculation results using the found formulas (4) and (5), one begins by analyzing the characteristics of the quantities in the argument of the hyperbolic sine and hyperbolic cosine functions. It is known that these quantities are one of the main factors of wave propagation. With the help of these arguments, it was noted above that it is possible to determine the propagation speed of pulse waves and its attenuation along the longitudinal axis. Taking this into account, below, we present the analysis results of these quantities for a Newtonian fluid. The found formulas (4) and (5) express the change of pressure and velocity along the longitudinal axis, and since these formulas mainly depend on the complex parameter $\sqrt{\bar{k}\, z_0}L$, expressing it in this form:

$$\sqrt{\bar{k}\, z_0}L = \bar{\chi} + \bar{\beta} i. \tag{6}$$

We distinguish the real and abstract parts of z_0 as follows:

$$z_0 = \left[\frac{3}{i\alpha_0^2}\left(1 - \frac{\sin\left(i^{\frac{3}{2}}\alpha_0\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)\cos\left(i^{\frac{3}{2}}\alpha_0\right)}\right)\right]^{-1} = \frac{\bar{R}}{3} + \frac{\bar{L}}{3}i,$$

$$\bar{R} = \frac{\alpha_0^2\left(A_1^2 + B_1^2\right)}{(A_2^2 + B_2^2)} B_2,$$

$$\bar{L} = \frac{\left(A_1^2 + B_1^2\right)\alpha_0^2}{A_2^2 + B_2^2} A_2.$$

Here

$$A_1 = \bar{A}\bar{M}_1 + \bar{B}M_1, \quad B_1 = \bar{A}M_1 - \bar{B}\bar{M}_1,$$

$$A_2 = \left(A_1^2 + B_1^2\right) - A_1 C - B_1 D, \quad B_2 = \left(B_1 C - A_1 D\right),$$

$$C = \sin M_1 ch\bar{M}_1, \quad D = -\cos M_1 sh\bar{M}_1,$$

$$\bar{A} = \sin M_1 sh\bar{M}_1, \quad \bar{B} = \cos M_1 ch\bar{M}_1, \; M_1 = \frac{\alpha_0}{\sqrt{2}}, \bar{M}_1 = \frac{\alpha_0}{\sqrt{2}},$$

$$i^{\frac{3}{2}}\alpha_0 = \frac{\alpha_0}{\sqrt{2}}(1-i) = M_1 - \bar{M}_1 i.$$

Now, putting the value of z_0 and $\bar{k}$ into formula $\sqrt{\bar{k}\, z_0}L = \bar{\chi} + \bar{\beta}i$, $\bar{\chi}$, $\bar{\beta}$ is found:

$$\sqrt{\bar{k}\, z_0}L = L\sqrt{\frac{3\gamma^*}{h^2}}\sqrt{\frac{1}{3}}\sqrt[4]{\bar{R}^2+\bar{L}^2}\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right) = \sqrt{\frac{\gamma^*}{h^2}}L\sqrt[4]{\bar{R}^2+\bar{L}^2}\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right).$$

Here $\alpha_0 = \sqrt{\frac{\omega}{\nu}}h, \; \nu = \frac{\eta}{\rho}, \; \varphi = arctg\frac{\bar{L}}{\bar{R}}.$

From this formula, $\bar{\chi}, \bar{\beta}$ is found accordingly. That is

$$\bar{\chi} = \sqrt{\frac{\gamma^*}{h^2}}L(\sqrt[4]{\bar{R}^2+\bar{L}^2}\cos\frac{\varphi}{2}),$$

$$\bar{\beta} = \sqrt{\frac{\gamma^*}{h^2}}L(\sqrt[4]{\bar{R}^2+\bar{L}^2}\sin\frac{\varphi}{2}).$$

Here, $\bar{\chi}$ is a dimensionless coefficient that determines the attenuation of the wave along the length; $1/\bar{\beta}$ – ox is a dimensionless quantity characterizing the propagation speed of the pulse wave along the axis.

3 Results and Discussion

The propagation speed of the pulse wave is determined by the following formula $c = \omega L/\bar{\beta}$. From this formula, the dimensionless form of the propagation speed of the pulse wave is found as follows:

$$c = \frac{\omega L}{\bar{\beta}} = \frac{\omega L}{\sqrt{\frac{\gamma^*}{h^2}}L(\sqrt[4]{\bar{R}^2+\bar{L}^2}\sin\frac{\varphi}{2})} = \frac{\nu}{h}\sqrt{\frac{1}{\gamma^*}}\alpha_0^2\left(\sqrt[4]{\bar{R}^2+\bar{L}^2}\sin\frac{\varphi}{2}\right)^{-1},$$

$$\frac{c}{c_0} = \alpha_0^2\sqrt{\frac{1}{\gamma^*}}\left(\sqrt[4]{\bar{R}^2+\bar{L}^2}\sin\frac{\varphi}{2}\right)^{-1}. \qquad (7)$$

Here $c_0 = 5\cdot\nu/h$ is the base pulse wave propagation speed.

Based on the formula (7) determined as a result of solving the problem, the propagation speed of the pulse wave depending on the vibration frequency parameter, is depicted in Figure 1.

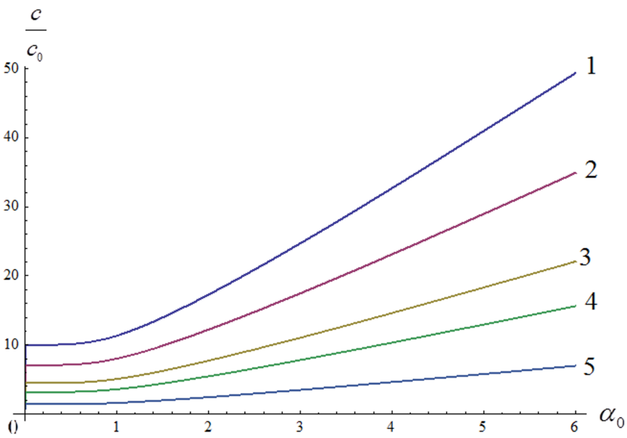

Fig. 1. Variation of pulse wave propagation speed at different values of wall permeability coefficient γ^* depending on vibration frequency parameter 1-0.01; 2-0.02; 3; 0.05; 4-0.1;5-0.5

The variation of the magnitude, which is the inverse of the magnitude of the wave attenuation obtained concerning the wavelength, depending on the parameter of the oscillation frequency, is determined by the following formula: $\frac{\partial \ln p}{\partial x} = -\frac{\bar{\chi}}{L}$.

Here 1 characterizes the decay of the wave.

Integrating $\frac{\partial \ln p}{\partial x} = -\frac{\bar{\chi}}{L}$, we get:

$\ln p = -\frac{\chi}{L}x + \ln c$ and $x = 0$ *et* $p = p_0$, for being $\ln p_0 = \ln c$, and the result is as follows:

$$\frac{p}{p_0} = e^{-\chi\lambda}.$$

Here $\lambda = x/L.$

This formula above represents the inverse of the magnitude of the wave attenuation taken concerning the wavelength.

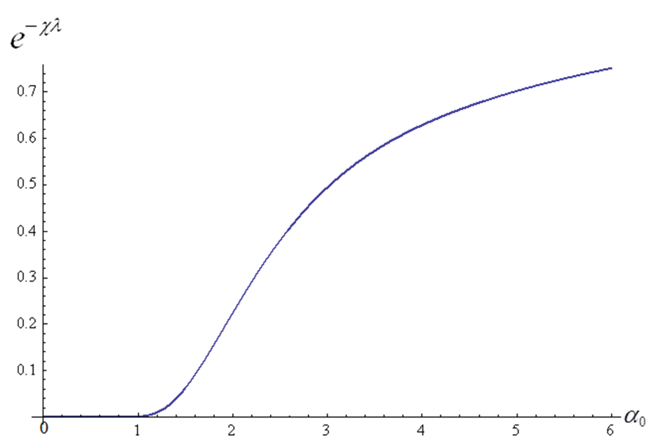

Fig. 2. Variation of magnitude, which is inverse of magnitude of wave attenuation obtained concerning wavelength, as function of oscillation frequency parameter.

4 Conclusions

Figure 1 shows the pulse wave propagation speed variation depending on the vibration frequency parameter. At sufficiently small values of the vibration frequency parameter, it was found that the propagation velocity of the pulse wave can be found by formula $c_0 = 5\nu / h$, and this formula was adopted as the formula representing the propagation velocity of the base pulse wave. It is shown in the figure that the propagation speed of the pulse wave does not significantly differ from the propagation speed of the base pulse wave at small values of the vibration frequency parameter. At large values of the vibration frequency parameter, the propagation speed of the pulse wave is significantly different from the base speed, and its increase ensures an increase in the propagation speed of the pulse wave.

It can be seen from Fig. 2. that at small values of the vibration frequency parameter, wave extinction almost does not occur, while at its large values, the wave extinction index increases significantly. Although the variation of the magnitude, which is inverse to the magnitude of the wave attenuation obtained concerning the wavelength, depending on the oscillation frequency parameter, is caused by the wall permeability coefficient, it is almost independent of its variation.

References

1. Ageeva A. I., Osiptsova A. N. Pulsating Flow of a Viscous Fluid over a Cavity Containing a Compressible Gas Bubble. Fluid Dynamics, Vol. 56. (2021).
2. Akbar,N. S. Non-Newtonian model study for blood flow through a tapered artery with a stenosis. Alexandria Engineering Journal, 55(1), 321-329. (2016).
3. Begjanov A. Sh., Inoyatov M.O. Simplification of differential equations of elastic viscous fluid motion in pipes and ducts. Central asian journal of theoretical and applied sciences. Vol. 02(03), pp. 49-53. (2021).
4. Ferras L. L., Nóbrega J. M., Pinho F. T. Analytical solutions for Newtonian and inelastic non-Newtonian flows with wall slip. Journal of Non-Newtonian Fluid Mechanics Vol. 175, pp.76–88 (2012).
5. R.P. Chhabra Non-Newtonion Fluids: An Introduction. India (2010).
6. R.P. Chhabra, J.F.Richardson Non-Newtonion Flow and applied rheology, (Engineering applications). 2008.
7. Bao K., Lavrov A., and Nilsen H. M. Numerical modelling of non-newtonian fluid flow in fractures and porous media. In ECMOR XV-15th European Conference on the Mathematics of Oil Recovery p. 494. EAGE Publications BV. (2017).
8. Fusi L., Housiadas K. D., and Georgiou G. C. Flow of a Bingham fluid in a pipe of variable radius. Journal of Non-Newtonian Fluid Mechanics, 285, 104393. (2020).
9. Mogilevskii E. I. Non-Newtonian Fluid Film Flowing Down an Inclined Plane with a Periodic Topography. Fluid Dynamics, Vol. 56(6), pp. 786–798. (2021).
10. Navruzov K., Kujatov N., and Begjanov A. Stationary flow of a viscous fluid in a flat channel with permeable walls (in the example of blood circulation). European journal of molecular and clinical medicine, (2020).
11. Navruzov K., Begjanov A.Sh., Shukurov Z.K. Method for determining hydraulic resistance during fluid flow in pipes. Electronic journal of actual problems of modern science, education and training. II part.p.1-10. (2019).
12. Navruzov K.N., Rajabov S.X., Shukurov Z.K, Begjanov A., Babajonova Y. On the

reduction of the resistance in the central arterial vessel. Asian Journal of Research. Vol.12(12), pp. 20-31. (2017).
13. Bali R., and Gupta N. Study of non-Newtonian fluid by K–L model through a non-symmetrical stenosed narrow artery. Applied Mathematics and Computation, Vol. 320, pp.358-370. (2018).
14. Rubenstein D.A., Wti Yin and Mary D.Frame. Biofluid mechanics. Stony brook, NY, USA, Second edition (2015).
15. Zakin J.L., Ni C.C., Hansen R.J., Reischman M.M. Laser Doppler velocimetry studies of early turbulence. Phys. Fluids, Vol. 20(10), p. 85-88. (1977).
16. Ziabicki A. Fundamentals of fibber formation. The science of fibber spinning. London: Wiley, (1976).
17. Ding Z., and Jian Y. Electrokinetic oscillatory flow and energy conversion of viscoelastic fluids in microchannels: a linear analysis. Journal of Fluid Mechanics, 919, A20. (2021).
18. Zahorski S. Unsteady shearing flows and plane shear waves in simple fluids. Archiwum Mechaniki Stosowanej, Vol. 32(2), pp.285-298. (1980).
19. Zakin J.L., Ni C.C., Hansen R.J., Reischman M.M. Laser Doppler velocimetry studies of early turbulence. Phys. Fluids, Vol. 20(10), pp. 85-88. (1977).
20. Zohreh Sheidaei, Pooria Akbarzadeh. Analytical solution ofthelow Reynolds third-grade non-Newtonian fluids ow insiderough circular pipes. Acta Mechanica Sinica. (2021).
21. Shukurov Z. K., Yuldoshev B. S., and Begjanov A. Investigation of hydraulic resistance of pulsating flows of viscous fluid in elastic pipe. In E3S Web of Conferences, Vol. 365, p. 03026. (2023).
22. Kh. Fayziev, T. Sultanov and E. Toshmatov. Methods for calculating the balance during the period of draw-off, drainage of sludge collector cells and construction of secondary embankment dams. In E3S Web of Conferences Vol. 264, 03048 (2021).
23. Kh. Z. Rasulov, E. S. Toshmatov, and D. Zh. Artykbaev Earthquake–Resistant Steepness of Slope Structures. AIP Conference Proceedings Vol. 2612, p.020006 (2023).
24. Abdukadirov S., Aisenberg-Stepanenko M., Yuldoshev B.Sh., Zhabborova Kh. Impact of a plane compression wave on a cylindrical shell filled with fluid. AIP Conference Proceedings Vol. 2612, (2023).