

# Shear oscillations of bridge on sliding foundation during earthquake

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**Abstract.** The article investigates the vibration of the railway bridge on the sliding foundation with dry friction under the action of real earthquake records at the intensity of 7, 8, 9, and over 9 on the MSK-64 scale. Dynamic dry friction problems are non-linear since each slip's start and end times are determined during the problem-solving process; they depend on many design parameters and external influences. The simplified model of vertical and shear deformation of the bridge is proposed as oscillating intermediate support with the mass of the span connected to the girth rail by a rubber bearing part. In this case, all elements in the structural model are connected to each other, taking into account eccentricities. The finite element method discretizes the coordinate variables and the Newmark time method with matrices built at each step. Structural vibrations are studied based on four real earthquake records. It is shown that using a sliding foundation can significantly reduce the shear forces in the bridge supports depending on the structure mass, the dry friction coefficient, and the nature of the seismic effect.

## 1 Introduction

94.7% of bridges on the railways of Uzbekistan are railway reinforced concrete bridges [1]. Such bridges' advantages are relatively low construction cost, operating cost, and durability.

Earthquakes are naturally occurring broadband oscillatory ground motions caused by several causes, including tectonic ground motions, volcanism, rockburst, and man-made explosions. The most important of these are caused by tectonic plate collapsing and sliding along faults [2].

According to the preliminary report [3] on the consequences of the earthquake in Turkey that occurred in February 2023, the main destructions of structures were at the epicenter, where the vertical component of the ground acceleration was very large. The seismic wave's vertical component is not considered in many scientific works and building codes.

From the analysis of the consequences of earthquakes, it was found that supports are the vulnerable elements of bridges. Insulators are used as bearing parts of the bridge to prevent the failure of the abutments due to earthquake effects. Numerical and experimental studies of full-scale isolators have significantly improved seismic isolation methods for bridge structures. In [4, 5], a simplified procedure for optimizing isolation systems for bridge

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structures is proposed. The seismic response of the example-designed structure is investigated using non-linear time history analysis and the effective contribution of the isolators' non-linear hysteretic behavior and the supports' elastic response.

The results of shake table tests carried out on a model bridge structure were described, and the seismic effect of the isolation system was tested in [6].

As another way of reducing the impact of earthquakes on objects, it is proposed to use horizontal barriers in the form of metasurfaces made of granular metamaterials with wideband phononic crystal properties [7, 8]. Granulated metamaterials are used as seismic isolation devices for ground, above-ground and underground structures [9, 10]. It should also be noted that seismic cushions of granulated metamaterials are also used to protect the supporting structures of large-span bridges [11]. For this purpose, a system of territorial seismic protection based on granulated metamaterials was developed during the construction of the Rion – Antirion bridge across the Corinth Gulf (Greece). The metamaterial is placed between the foundation and the support, which causes the support to slide on the foundation during the earthquake. It thus reduces the stress level in the support body [12, 13].

In [14], the vibrations of buildings on sliding foundations with dry friction under real earthquakes of intensity 8 and 9 on MSK-64 scale were studied. A unique algorithm for calculating displacements, velocities, accelerations, and shear forces due to the simultaneous action of horizontal and vertical components of the seismogram record was developed. A four-story and a nine-story building under a set of three earthquake records were investigated. It is shown that the shear vibration of the building is significantly affected by the vertical component of the seismic effect.

Accounting for friction between the foundation and the structure during earthquakes using a rigid plastic model with the development of a unique algorithm for solving the non-linear problem [14, 15] showed the effectiveness of seismic isolation based on sliding friction.

For certain types of buildings, sliding foundations using fluoroplastic are an effective way of seismic insulation [16, 17].

The base isolation method is one method of passive energy loss that increases a structure's resistance to earthquakes. It is important to control the energy that passes from the foundation or ground to the top of the structure. To do this, a flexible insulation layer is installed between the superstructure and the substructure. This reduces the deformation in the structural elements and changes the basic natural vibration period of the structure. This avoids a resonance state between the ground's acceleration and the structure's vibrations. Different types of insulation systems and their use are discussed in [18–23].

In [24], the problem of preventing dangerous displacements caused by seismic effects by using dampers with dry friction elements is considered. A description of seismically isolated foundation structures is given.

In [25], the results of experimental verification of the effectiveness of several low-cost geotechnical seismic friction isolation methods are presented. A total of 11 types of layers of different materials were considered and analyzed. The test results showed that the reduction in earthquake impact depending on the type of earthquake and the property of the layer material used could be significant. In [26], a detailed analysis of the seismic safety performance of building structures using rubber cushions under seismic effects of different intensities was carried out.

The Friction Pendulum Support System (FPS) is widely used in constructing buildings, bridges, and other structures to improve their seismic performance; in Japan, the FPS was first used to construct the Shimizu Bridge in 2020 on the Tokai – Hokuriku Expressway. To confirm the results of the design of a real bridge with FPS, a series of static and dynamic tests were carried out, and the FPS parameters were evaluated. The friction coefficient was

estimated from the force and displacement data at the sliding points, and there was good agreement between the static tests in situ and the laboratory tests. The results of the free vibration tests, including measured displacements and accelerations, showed agreement with the simulation results based on a simplified bridge model with FPS [27].

Calculating bridges and overpasses on the impact of earthquakes with their existing records allows to analyze their stress-strain state [28]. Based on the above tasks, mathematical models and algorithms are created to study complex seismic and dynamic processes taking into account friction between foundations and supports of railway bridges under the effect of seismic waves. Using the effect of dry friction between the foundation and the support makes it possible to increase the seismic resistance of the bridge by selecting the coefficient of dry friction, which leads to a decrease in stresses in its elements. At the same time, cost savings will be achieved by making the necessary modifications to the design of the railway bridge.

## 2 Objects and methods of research

Let the horizontal and vertical movements of the earth's surface be given as a seismogram of a real earthquake. We will assume that the bridge's foundation acquires the same displacements and that the superstructure is separated from the foundation by a double-layer fluoroplastic or other sliding material with an appropriate dry friction coefficient. In the horizontal direction, we will use the Coulomb – Amonton dry friction model as a model for the interaction between the foundation and the superstructure. In the vertical direction, we assume they are absolutely rigidly connected.

Surface seismic waves have a complex structure. They consist of a vertical displacement and two horizontal displacements. Bridges also have three dimensions. Structural elements of the bridge experience simultaneous compression – tension, bending, and shear. Concentrated masses can be located at certain points. We replace each type of structural element with the corresponding model depending on the deformations they experience. We discretize the elements of the bridge by the finite element method. As a result, we obtain a system of ordinary linear or non-linear differential equations [14, 29].

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{Q(t)\} \quad (1)$$

$$\{U\} = \{U_{st}\}, \quad \{\dot{U}\} = 0, \quad \text{at } t=0,$$

where  $[M]$  is the mass matrix,  $[K]$  is the stiffness matrix,  $[C] = \alpha \cdot [M] + \beta \cdot [K]$  is the viscosity matrix,  $\{U\}$  is the displacement vector,  $\{U_{st}\}$  is the displacement vector at the initial moment of time. The condition for the interaction of the mass  $M_0$  with the foundation is

$$u_0 = u_g - u_r, \quad \text{if } |F_0| < |F_{fr}| \quad \text{i.e., when moving together,} \quad (2)$$

$$F_0 = F_{fr}, \quad \text{when sliding} \quad (3)$$

$$v_0 = v_g \quad (4)$$

where  $u_0$ ,  $v_0$  are the horizontal and vertical displacements of the grillage,  $u_g$ ,  $v_g$  are the horizontal and vertical displacements of the foundation, i.e., approximated functions of the digitised earthquake seismogram,  $u_r$  is the displacement value at the time at the start of the

current joint movement of the foundation and the grillage, i.e., the difference between the displacement values of the foundation and the grillage (at the initial moment of time  $u_r = 0$ ),  $F_0$  is the unknown value of the bonding force between the grillage and foundations,  $F_{fr} = \text{sign}(\dot{u}_g - \dot{u}_0) \cdot f \cdot P$  is the value of dry friction force,  $f$  is the coefficient of dry friction,  $P$  is the pressure force on the lower foundation in the dynamic process, if vertical vibrations are not taken into account, then this is part of the bridge weight corresponding to the vertical pressure force on the lower foundation.

It should be noted that the vertical oscillations are independent of the horizontal oscillations of the bridge and the horizontal oscillations are dependent on the vertical oscillations of the bridge through the condition (3), as the pressure force on the sliding foundation changes during the vertical oscillations [14].

Let us consider a simplified bridge model in the form of oscillations of the intermediate support with the mass of the span connected to the girth rail with a tangential support part considering eccentricities. The grillage is connected to the support with eccentricity. In the case under consideration, the intermediate support consists of six supports. In the model, these six supports with square cross-sections will be replaced by one equivalent support working in compression and shear due to the small height of each support. The equivalent shear stiffness of the model is determined as follows

$GF = 6kl$ , where  $k = 12 \cdot \frac{EJ}{l^3}$ ;  $l$  is the height of supports;  $E$  is the modulus of elasticity;  $J$  is the moment of inertia of the cross-section of one support.

If the masses are taken concentrated at the nodal points in the calculation model, then, in the joint motion, the displacement  $u_0$  is determined by equality (2), and the equation of motion of mass  $M_1$  has the form [14]

$$M_1 \ddot{u}_1 + k_1 u_1 + c_1 \dot{u}_1 - k_2 (u_2 - u_1) - c_2 (\dot{u}_2 - \dot{u}_1) = k_1 u_0 + c_1 \dot{u}_0. \quad (5)$$

In this case  $Q_1 = k_1 u_0 + c_1 \dot{u}_0$ , the remaining elements of the vector  $\{Q\}$ , corresponding to the horizontal displacements of the concentrated masses, are equal to zero. The equation of the vertical motion of mass  $M_1$ , which is similar to (5),  $M_1 g$  is added to the right side. The elements of the vector  $\{Q\}$  corresponding to the vertical displacements of concentrated masses are equal to the values of weights of the corresponding concentrated masses.

Sliding with dry friction occurs only when condition (3) is fulfilled. The considered problem (1), (2), (3) is non-linear; there are no conditions for calculation of the unknown function  $F_0$ , and during the dynamic process the dimensions of the matrices  $[M]$  and  $[K]$ . When sliding the equation for the mass  $M_0$  takes place [14]

$$M_0 \cdot \ddot{u}_0 - k_1 (u_1 - u_0) - c_1 (\dot{u}_1 - \dot{u}_0) = F_{fr}, \text{ here with } Q_0 = F_{fr}.$$

To solve the problem as a whole, we will use the unique algorithm presented in [14]. The Newmark method solves the problem [30], which uses an implicit two-layer finite difference approximation scheme. The direct use of a digitized seismogram of an earthquake gives gross errors in calculating velocities and accelerations at the nodal points. Therefore, we take accelerogram records as the basis for calculations and use them to calculate earthquake velocities and displacements using Newmark's formulae. The

displacements are approximated by the Hermite spline function, which makes it possible to carry out the calculations in the required time increments.

### 3 Results and discussion

As an example, a railway bridge with a length of 29.6 meters is considered to be located on the Navoi – Bukhara section of a high-speed electrified railway. This bridge is calculated for seismic resistance using a specially designed program, considering the impact in the form of real earthquake records [1].

The bridge structure includes many elements, including supports, bearing parts, girth rails, and spans. Total number of supports is 6 pcs, dimensions: height is 2 m, the cross-section is 0.35 m×0.35 m. The total cross-sectional area of the supports  $F=0.735 \text{ m}^2$ .

The structure is constructed mainly of reinforced concrete material – concrete of strength class B25, modulus of elasticity  $E=30000 \text{ MPa}$ , Poisson's ratio  $\nu=0.2$ . The support part has the following characteristics:  $EF/h=235092 \text{ MN/m}$ ,  $GF/h=1200 \text{ MN/m}$ .

The weights of the foundation, support, girth rail, and span were calculated. Total weight: grillage is 7800 kg, all supports (6 pcs.) are 8250 kg, girth rail is 11500 kg, and two spans are 57800 kg.

Calculation results of the railway bridge are obtained based on real records of the following earthquakes [31]:

1. Tabas (Iran) – 000187 (16.09.1978, more than 9 on the MSK-64, maximum horizontal and vertical accelerations are  $10.17 \text{ m/s}^2$  and  $7.78 \text{ m/s}^2$ , velocities are 0.88 m/s and 0.33 m/s, displacements are 0.3446 m and 0.0927 m, digitizing step is 0.005 s, duration is 78.398 s);

2. Gazli (Uzbekistan) – 000074 (17.05.1976, 9 on the MSK-64, maximum horizontal and vertical accelerations are  $7.22 \text{ m/s}^2$  and  $13.163 \text{ m/s}^2$ , velocities are 0.62 m/s and 0.57 m/s, displacements are 0.18 m and 0.216664 m, digitizing step is 0.005 s, duration is 28 s);

3. Duzce (Turkey) – 006501 (12.11.1999, 8 on the MSK-64, maximum horizontal and vertical accelerations are  $1.5822 \text{ m/s}^2$  and  $0.63 \text{ m/s}^2$ , velocities are 0.10 m/s and 0.0485 m/s, displacements are 0.0147 m and 0.0089 m, digitizing step is 0.005 s, duration is 50.055 s);

4. Boshroyeh (Iran) – 000181 (16.09.1978, 7 on the MSK-64, maximum horizontal and vertical accelerations are  $0.734 \text{ m/s}^2$  and  $0.7637 \text{ m/s}^2$ , velocities are 0.0948 m/s and 0.0682 m/s, displacements are 0.0217 m and 0.0109 m, digitizing step is 0.005 s, duration is 41.645 s).

Three finite elements, taking into account the operation of each type of finite element, and 6 nodal points are used to discretize the shear model of the railway bridge. The characteristics of the finite elements of the two different types are given through their respective ordinal numbers. The number of connections with eccentricity is 3.

The bending stiffness of one support is determined by the formula

$$k = 12 \cdot \frac{EJ}{l^3} = 12 \cdot \frac{0.037 \cdot 10^9}{2^3} = 0.0555 \cdot 10^9 \text{ N/m.} \quad (6)$$

The number of supports is 6, so the total shear stiffness will be

$$6k = 6 \cdot 0.0555 \cdot 10^9 = 0.333 \cdot 10^9 \text{ N/m.} \quad (7)$$

Then the equivalent shear stiffness of the supports will be equal to

$$\frac{GF}{l} = 6k. \quad (8)$$

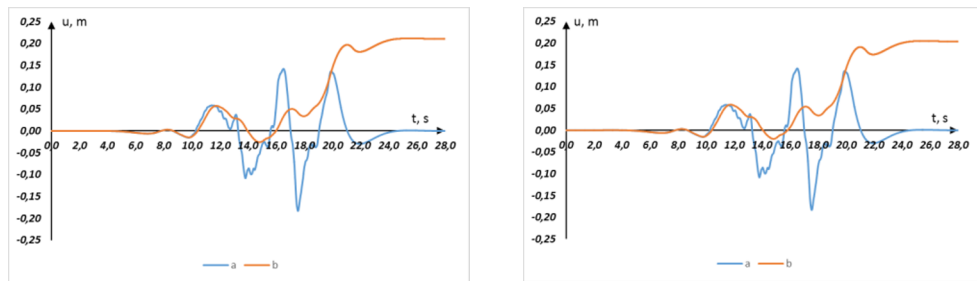
The following values of concentrated masses are accepted: concentrated mass at the center of mass of the grillage is  $M_0 = 9862.5$  kg (the mass of the grillage and 1/4 part of the support mass), the mass located in the middle of the support  $M_1 = 4125$  kg (1/2 part of the support mass), the mass of the girth rail  $M_2 = 13562.5$  kg (the mass of the girth rail and 1/4 part of the support mass), the mass of the span structure  $M_3 = 57800$  kg.

For the numerical solution of problems with dry friction, it is necessary to select a time step to ensure the required solution accuracy. In our calculations, the time step was equal to 0.001 s.

The results in the form of displacement figures and tables of the maximum values of shear forces in the elements are given below.

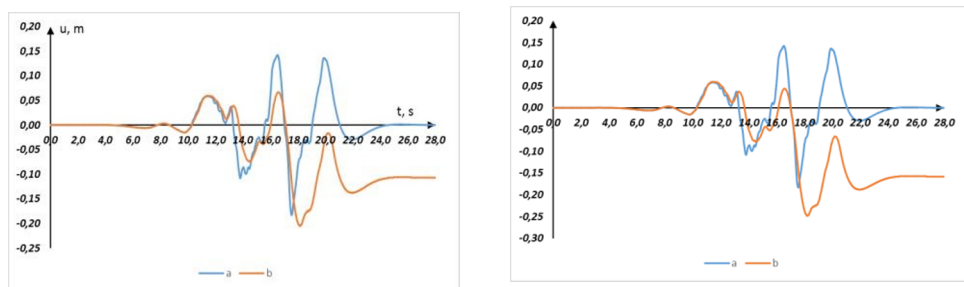
Figures 1-3 show the results of calculations of the changing displacements of the grillage and of the foundation of the railway bridge during the time under the Gazli earthquake, considering only the horizontal impact and simultaneously the horizontal as well as vertical impacts of real earthquake records.

In Figure 1, the dry friction coefficient value is equal to  $f=0.01$ . The occurrence of the first slip is associated with an increase in foundation acceleration. The transition from sliding with dry friction to co-motion is then reversed and vice versa. By the end of the process, the shear of the grillage concerning the foundation (residual shear) is 0.212 m for horizontal action and 0.200 m for vertical movement.



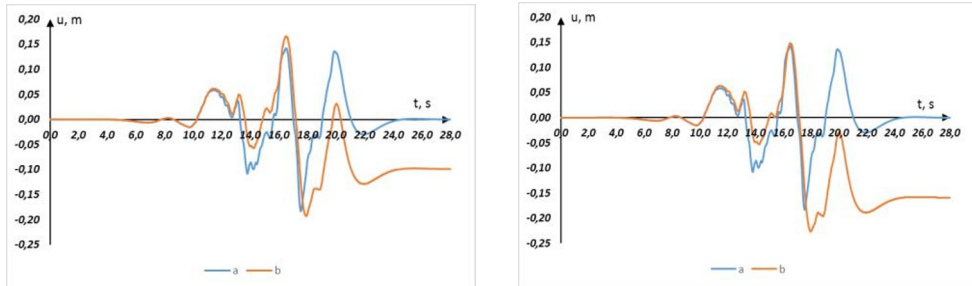
**Fig. 1.** Horizontal displacements of foundation (a) and grillage (b) over time of the railway bridge considering horizontal (left) and simultaneous horizontal and vertical impacts (right) ( $f=0.01$ ).

Figure 2 shows the same graphs when the dry friction coefficient equals  $f=0.05$ . Increasing the value of the dry friction coefficient results in a lower residual shear value. In this case, the residual shear is 0.11 m for horizontal action and 0.16 m for vertical movement. At the same time, the residual shear increases when the vertical movement is taken into account.



**Fig. 2.** Horizontal displacements of foundation (a) and grillage (b) over time of the railway bridge considering horizontal (left) and simultaneous horizontal and vertical impacts (right) ( $f=0.05$ ).

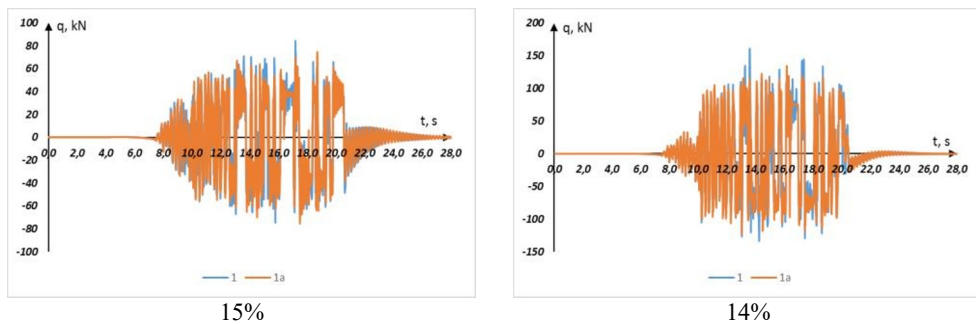
Figure 3 shows that the dry friction coefficient value equals  $f=0.1$ . By the end of the process, the residual shear is 0.09 m for horizontal action and 0.16 m for vertical movement. At the same time, the residual shear increases when the vertical movement is considered, as in the case of  $f=0.05$ .



**Fig. 3.** Horizontal displacements of foundation (a) and grillage (b) over time of the railway bridge considering horizontal (left) and simultaneous horizontal and vertical impacts (right) ( $f=0.1$ ).

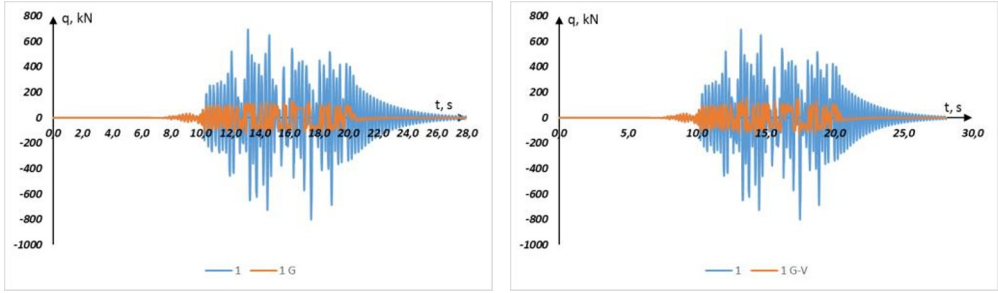
From Figures 2 and 3, you can see the vertical action's effect on the horizontal vibrations of the railway bridge.

Figure 4 shows the influence of the vertical component of the seismic wave on the shear force in the support element in contact with the grillage. Accounting for the vertical component leads to a change in time of the transverse force, while its maximum value increases by about 15%.

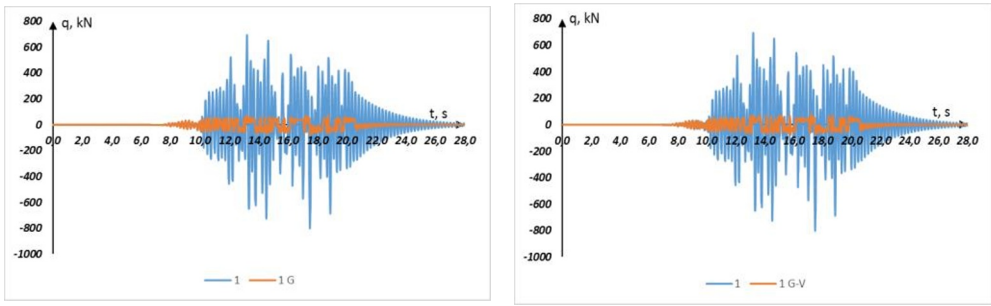


**Fig. 4.** Changing the transverse force in the supporting element takes into account the vertical earthquake component (1) and without it (1a) with dry friction coefficient  $f=0.05$  (left) and with dry friction coefficient  $f=0.1$  (right).

Figures 5-6 show the results of calculations of the change in shear forces in the support element depending on the time under the Gazli earthquake for cases without and with slip. These graphs show that using a sliding foundation with dry friction coefficients  $f=0.1$  and  $f=0.05$  allows for reducing the maximum value of the shear force by 6.1 and 11 times, respectively, concerning the case of the absence of the sliding foundation. For the railway bridge, considering the vertical movement makes it possible to reduce its value by 5.2 and 9.1 times, respectively.



**Fig. 5.** Changing the transverse force in the supporting element without slip (blue line) and with slip (brown line), taking into account horizontal (left) and simultaneous horizontal and vertical forces (right).



**Fig. 6.** Changing the transverse force in the supporting element without slip (blue line) and with slip (brown line), taking into account horizontal (left) and simultaneous horizontal and vertical forces (right).

Tables 1-4 show the results of calculations of changes in the transverse force in structural elements during the time under the records of Tabas, Gazli, Duzce, and Boshroyeh earthquakes for cases without and with sliding elements, taking into account horizontal and simultaneous horizontal and vertical forces.

**Table 1.** Maximum values of transverse forces in structural elements under the records of Tabas earthquake

q <sub>1max</sub> is only the horizontal impact, q <sub>2max</sub> is horizontal and vertical impacts (kN)									
№	No sliding	Dry friction coefficients							
		f = 0.01		f = 0.05		f = 0.1		f = 0.2	
		q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>
1	2718	17.5	19.4	71.3	83.1	154	178	253	305
2	2700	21.5	23.6	85.1	115	185	190	294	317
3	2570	29.6	31.8	115	133	251	218	396	348



**Table 2.** Maximum values of transverse forces in structural elements under the records of Gazli earthquake

q <sub>1max</sub> is only the horizontal impact, q <sub>2max</sub> is horizontal and vertical impacts (kN)									
№	No sliding	Dry friction coefficients							
		f = 0.01		f = 0.05		f = 0.1		f = 0.2	
	q <sub>max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>
1	801	18.5	18.6	72.8	84.4	131	154	232	281
2	799	22.6	22.7	87	93.8	142	169	261	297
3	774	31.6	32	115	116	181	194	331	366

**Table 3.** Maximum values of transverse forces in structural elements under the records of Duzce earthquake

q <sub>1max</sub> is only the horizontal impact, q <sub>2max</sub> is horizontal and vertical impacts (kN)									
№	No sliding	Dry friction coefficients							
		f = 0.01		f = 0.05		f = 0.1		f = 0.2	
	q <sub>max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>
1	590	15.7	16.6	59.8	60.8	107	105	193	193
2	580	18.7	19.6	68.5	66.6	117	115	199	199
3	544	25.7	26	87.4	85.5	134	136	214	214

**Table 4.** Maximum values of transverse forces in structural elements under the records of Boshroyeh earthquake

q <sub>1max</sub> is only the horizontal impact, q <sub>2max</sub> is horizontal and vertical impacts (kN)									
№	No sliding	Dry friction coefficients							
		f = 0.01		f = 0.05		f = 0.1		f = 0.2	
	q <sub>max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>	q <sub>1max</sub>	q <sub>2max</sub>
1	164	12.8	13.1	52.7	51.9	90.7	90.9	164	164
2	162	14.6	14.8	56.5	55.9	92.6	93.2	162	162
3	155	19.6	19.4	62.8	62.9	96.8	97.6	155	155

From the analysis of the tables, the following conclusions can be drawn: sliding foundations allow for a significant reduction of transverse forces in the bridge supports; by adjusting the coefficient of dry friction, i.e., by selecting appropriate sliding materials, the mechanical characteristics of the supports can be adjusted to the intensity of the earthquake at the construction site; for small earthquake intensities, the principle of reducing forces in structural elements may not be used, as their values are not very high. In addition, the vertical component of the seismic action, in this case, has almost no effect on the maximum shear force values. If it is necessary to reduce the maximum value of transverse force, then the use of the sliding foundation should be with a dry friction coefficient equal to less than 0.1.

## 4 Conclusion

The seismic insulation of the foundations of bridges and overpasses makes it possible to reduce the value of maximum transverse force in the supports by several times, depending on the mass of the structure, the dry friction coefficient, and the nature of the seismic effect, i.e., its intensity and frequency composition. At low earthquake intensities, the vertical component of the seismic effect can be disregarded. For earthquakes of intensity 8 or higher on the MSK-64 scale, the influence of the vertical component of the seismic effect is significant, especially in the epicentral zone of the earthquake.

The available records of past earthquakes make it possible to select a set of records with suitable characteristics depending on the construction site of bridges and overpasses for their calculations for seismic effects, considering the seismic isolation on their foundations. Using the effect of dry friction between the foundation and the support makes it possible to increase the seismic resistance of the bridge by reducing the stresses in its elements by adjusting the dry friction coefficient. At the same time, savings will be achieved by making the necessary modifications to their design.

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