Rayleigh and love surface waves with regard to seismic stress state of earth bed

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Abstract. This article presents the results of studies on the propagation of surface seismic waves, which directly affect the stability and strength of a high embankment of the subgrade. The subgrade is the foundation of the railway track. In an elastic half-space with a layer in which surface waves propagate, the conditions for the existence and propagation of a harmonic Rayleigh wave are considered. The equations of motion of a medium in a layer and half-space and their solutions in the form of plane waves propagating in phase along the contact boundary, some general laws of Rayleigh wave propagation are given. The accepted theories and calculation methods' main provisions are proposed for designing a durable and reliable subgrade and considering its seismic stress state.

1 Introduction

The Transport Strategy of the Central Asian countries is of great importance since the stable functioning of the transport complex is a necessary condition for the sustainable economic growth of these states. Transport services should focus on their recipients, namely the population, business, and the state, in addressing strategic issues of ensuring unity and security [1, 2].

The economic and geographical features of the countries of Central Asia make their economy one of the most cargo-intensive in the world, causing a high dependence on the transport system. Worldwide experience gained from earthquake disasters has shown that it is economically feasible to build various structures to successfully withstand surface seismic waves (Rayleigh and Love) with a fairly low probability of destruction. To this end, when designing, it is necessary to pay special attention to research, calculation, design, and during construction - to the quality of work and the impeccable execution of all design recommendations.

Regulatory documents [3, 4] regulate the procedure and conditions for maintaining the railway network's subgrade, drainage, fortification, and protective structures to ensure the uninterrupted and safe movement of trains at set speeds. The good condition of the subgrade and its structures is ensured by the conformity of its structures, the existing loads, and the implementation of scheduled preventive repairs. It is based on continuous current maintenance and periodic overhaul.

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The main requirements in the design of railway subgrades include the route choice, considering seismic zoning data. In highly seismic regions, it should bypass areas that are especially unfavorable in engineering and geological terms, in particular, zones of possible collapses, landslides, and avalanches.

Observing the requirements of the standards [3, 4], when designing, it is necessary to consider that the severity of the consequences of failure of the subgrade is far from the same. This circumstance is considered by the magnitude of the calculated seismicity, which, when designing the subgrade, can be much less than the strength of the maximum possible earthquake at a given construction site.

At the same time, the territory of the Central Asian countries is distinguished by complex geomorphology, hydrogeology, and neotectonics and is characterized by a seismic background of 9-10 points.

Among the seismic waves arising under the action of various sources, the most intense and long-lasting are most often surface waves (they are also called normal, interference, and channel waves, natural oscillations of layers, etc.). These waves are distinguished by specific dispersion and resonance and are used in several tasks: in determining the structure of the medium (especially surface layers and waveguides), determining the coordinates and properties of the source (especially its energy and mechanism), identifying underground explosions, microseismic zoning, tracing storms, etc.

Surface waves are known to be subdivided into Rayleigh and Love waves, which differ in speed and polarization: particle displacement in Rayleigh waves is parallel, and in Love waves - perpendicular to the vertical plane containing the propagation directions (some deviations from such polarization are possible near the source or in the presence of horizontal inhomogeneities of the environment).

2 Objects and methods of research

Various kinds of inhomogeneities of the medium (free surface, internal interfaces between media, low-velocity layers, etc.) lead to the appearance of qualitatively new, so-called surface waves. Their main property is the rapid decrease in the amplitude of oscillations of the medium with distance from the surface near which they propagate.

Theoretically, the existence of elastic waves propagating along a free surface, the amplitude of which rapidly decreases with depth, was shown in 1885 by Rayleigh. In 1926, Love showed the existence of surface waves in a layered homogeneous half-space when the speed of a transverse wave in a homogeneous layer is less than the speed of transverse waves in a half-space. These waves, which have only one displacement component, were called Love waves.

Initially, surface waves were studied only within the limits of low frequencies to seismology and seismic exploration. The main type of waves observed during earthquakes are Rayleigh waves. They carry most of the seismic energy and are localized near the free surface. Rayleigh wave energy decays with distance much more slowly than bulk wave energy [5-17].

Love waves. Love waves are the simplest type of surface waves, which are plane transverse waves with displacements parallel to the free surface and perpendicular to the wave propagation.

Consider an elastic half-space with a layer where horizontal polarization propagates (Fig. 1).

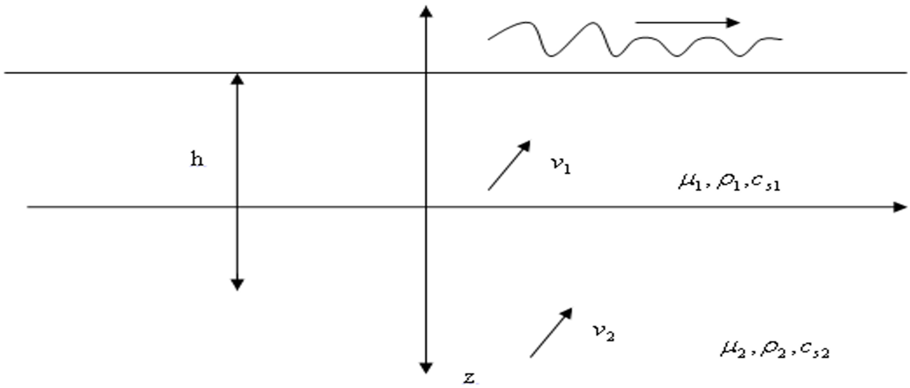

Fig. 1. Scheme of propagation of surface waves.

The components of the displacement vector $\vec{v}$ have the form:

$$u = w = 0, \quad v = v(x, z) \tag{1}$$

The equations of motion of the medium in the layer and half-space, respectively, have the form:

$$\rho_1 \frac{\partial^2 v_1}{\partial t^2} = \mu_1 \left(\frac{\partial^2 v_1}{\partial z^2} + \frac{\partial^2 v_1}{\partial x^2} \right), \qquad \rho_2 \frac{\partial^2 v_2}{\partial t^2} = \mu_2 \left(\frac{\partial^2 v_2}{\partial z^2} + \frac{\partial^2 v_2}{\partial x^2} \right), \tag{2}$$

where ρ_1, μ_1 are layer parameters; $\vec{v}_1$ is layer offset; ρ_2, μ_2 are half-space parameters; $\vec{v}_2$ is displacement in half space.

The surface $z = h$ is unloaded. The origin of coordinates is located at the interface between the layer and the half-space, which are in rigid contact.

We will seek the solution of equation (2) in the form of plane waves propagating in phase along the contact boundary

$$v_1 = A_1(z) e^{i(kx - \omega t)}, \quad v_2 = A_2(z) e^{i(kx - \omega t)} \tag{3}$$

where A_1, A_2 are amplitudes of waves in the layer and half-space, and the amplitude of the wave in half-space $(z \succ 0)$ decreases in depth with increasing *z*.

Substituting solutions (3) into (2), we can obtain the following ordinary differential equations:

$$\frac{d^2 A_1}{dz^2} + \left(\frac{\omega^2 \rho_1}{\mu_1} - k^2 \right) A_1 = 0, \qquad -h \le z \ge 0, \tag{4}$$

$$\frac{d^2 A_2}{dz^2}+\left(\frac{\omega^2 \rho_2}{\mu_2}-k^2\right)A_2=0, \qquad 0\le z\ge\infty. \quad (5)$$

General solutions of equations (4) and (5) can be represented as

$$A_1=A_0\sin(k\rho_1 z)+B_0\cos(k\rho_1 z), \qquad -h\le z\ge 0, \quad (6)$$

$$A_2=B_1 e^{-k\rho_1 z}+B_2 e^{-k\rho_2 z}, \qquad 0\le z\ge\infty, \quad (7)$$

Where

$$P_1=\sqrt{\frac{c^2}{c_{s1}^2}-1}, \;\; c_{s1}^2=\frac{\mu_1}{\rho_1};\; P_2=\sqrt{1-\frac{c^2}{c_{s2}^2}}, \; c_{s2}^2=\frac{\mu_2}{\rho_2}, \; c=\frac{\omega}{k}.$$

The amplitude of the wave in the half-space decays with increasing z. This condition is satisfied for $B_2=0$ in relation (7).

$$\tau_{yz}=0 \quad at \quad z=-h, \quad (8)$$

$$\tau_{yz}^1=\tau_{yz}^2, \quad v_1(x,0)=v_2(x,0) \quad at \quad z=0. \quad (9)$$

The condition (8) means that there is no shear stress at the free boundary of the layer.

Condition (9) means the continuity of shear stress and displacement at the interface between the layer and the half-space. Satisfying these conditions, we have

$$A_0\cos(s_1 h)+B_0\sin(s_1 h)=0, \qquad B_0-B_1=0, \qquad S_1\mu_1 A_0+S_2\mu_2 B_1=0 \quad (10)$$

where $S_1=P_1K, \quad S_2=P_2K.$

The condition for the existence of a non-trivial solution of system (10) is the equality to zero of the determinant composed of the coefficients A_0, B_0, and B_1, which leads to the following characteristic equation:

$$tg(s_1 h)=S_2\mu_2/S_1\mu_1. \quad (11)$$

It can (11) be seen from the relation that the condition for the existence of real roots of the equation (11) is the inequality

$$c_{s2}\succ c_{s1} \quad (12)$$

or the presence of a layer with a reduced velocity. In this case (12), the phase velocity determined from the equation satisfies the condition

$$c_{s2}\succ c\succ c_{s1}. \quad (13)$$

On the other hand, bearing in mind $k=-\frac{\omega}{c}$, equation (12) can be represented as

$$f(\omega, c) = 0, \tag{14}$$

which gives the dependence of the phase velocity c on the frequency. In other words, surface Love waves have a dispersion. The dispersion equation (14) for Love waves has many real roots. Different real roots correspond to Love waves of different orders (harmonics). This shows that the Love wave exists as a set of normal waves, each of which satisfies the equations of motion and boundary conditions and has its own law of distribution of displacements and stresses in the layer and half-space.

For some value of the root of equations (12) or (14), the system of algebraic equations (11) can be solved concerning one amplitude. Then the expressions for the displacement amplitudes become dependent on one arbitrary constant, and the displacement field takes the form:

$$\begin{aligned} v_1 &= A\cos(kP_1)(h+z)e^{i(kx-\omega t)}, \qquad -h \le z \le 0 \\ v_2 &= A\cos(kP_1)h\, e^{i(kx-\omega t)-kP_2 z}, \qquad z \ge 0. \end{aligned} \tag{15}$$

In particular, for k_2 h«1 $(\mathrm{k}_2 = \omega / c_{s2})$ the Love wave is described by the expressions

$$v_1 = Ae^{i(kx-\omega t)}, \qquad v_2 = Ae^{i(kx-\omega t)-kP_2 z}, \tag{16}$$

$$k = k_2\left[1 + 0.5(k_2 h)^2\left(\frac{\rho_1}{\rho_2}\right)^2\left(1 - \frac{c_{s1}^2}{c_{s2}^2}\right)^2\right], \tag{17}$$

$$kP_2 = k_2\left(1 - \frac{c_{s1}^2}{c_{s2}^2}\right)k_2 h\,\frac{\rho_1}{\rho_2}. \tag{18}$$

From relations (16-18), it can be seen that the displacement field in the layer under the condition k_2h«1 is constant, and in the half-space, it slowly decreases with distance from the boundary *z=0*. The slowness of the decrease in k_2h is due to the factor, which contains the exponent. The phase velocity $c = \omega / k$ is less than the phase velocity of the bulk wave in the half-space; the difference in velocities is of the order of $(k_2 h)^2$. At $k_2\mathrm{h} \to 0$, the phase velocity *c* tends to c_{s2}. The reverse picture arises as k_2h increases, where *c*, decreasing monotonically, tends in the limit to the phase velocity of transverse bulk waves in the layer. In this case, the displacements along the layer depth change according to relation (15) according to the cosine law. A qualitative picture of the change in phase and group velocities for harmonic Love waves is shown in (Fig. 2). It can be seen from the graph that the phase velocity *c* decreases monotonically with increasing k_1h and tends to c_{s1}, and the behavior of the group velocity c_{gr}, although it tends to c_{s1}, at $k_2\mathrm{h} \to \infty$ differs significantly from the behavior of the phase velocity, by the presence of a group velocity minimum after the region of strong dispersion.

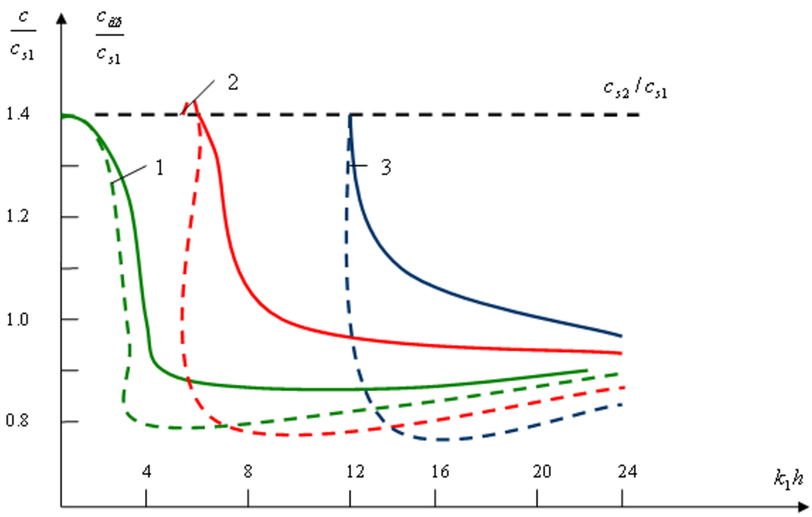

Fig. 2. Change of phase and group velocities.

For the asymptotic behavior of the field variation in the vicinity of the extrema c_{gr}, the Airy function is usually used. Without dwelling on this in detail, we only point out that for values of group velocities that are not extreme at large *t*, the amplitude changes in proportion to $t^{-1/2}$; in the vicinity of extreme values of the group velocity, the amplitude change is proportional to $t^{-1/3}$. The description of such wave processes using Airy functions or wave motion propagating at the velocities of extreme values of the group velocity are called "Airy waves" or "Airy phases".

Rayleigh waves. For the existence of the Love waves considered above, the most important is the inhomogeneity in the structure of the half-space; moreover, the velocity in the half-space of transverse body waves must be strictly greater than the velocity of transverse body waves in the layer. This means that when a wave interacts with inhomogeneities and sharp boundaries, it does not generate body waves of another type but can excite surface waves in the presence of a certain type of inhomogeneities (the velocity of a plane harmonic *SH* wave near the surface must be less than the velocity of transverse waves in a half-space or, in the general case, must there is a low-velocity zone).

Unlike Love waves, Rayleigh waves result from mutual interference of *P* and *SV* waves, generating each other when one of these waves falls on the free boundary of the half-space.

Let us consider the conditions for the existence and propagation of a harmonic Rayleigh wave with frequency ω along the flat boundary of a homogeneous, isotropic, ideally elastic half-space. Let the half-space occupy the region $z>0$ (Fig. 3).

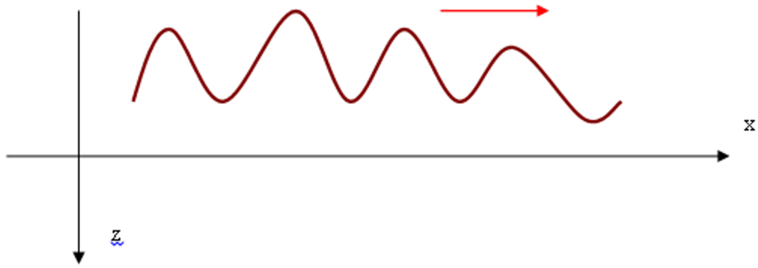

Fig. 3. Rayleigh Wave Propagation Diagram.

As noted above, by dividing the displacement field into gradient and vortex parts, the general equations of motion can be reduced to a system of wave equations. The obtained wave equations for harmonic plane waves are reduced to the Helmholtz equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} + k_e^2 \varphi = 0, \tag{19}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_t^2 \psi = 0 , \tag{20}$$

where φ and ψ longitudinal and transverse potentials describe displacement and stress fields; $k_e^2 = \omega^2 / c_P^2$; $k_t^2 = \omega^2 / c_s^2$; $c_P^2 = (\lambda + 2\mu) / \rho$; $c_s^2 = \mu / \rho$; k_e, k_t - wave numbers of longitudinal and transverse body waves.

Solutions of wave equations (19) and (20) corresponding to a harmonic surface wave can be represented as

$$\varphi = \Phi(z) e^{i(kx - \omega t)}, \qquad \psi = \psi(z) e^{i(kx - \omega t)}, \tag{21}$$

where $k = \omega / c$, c - as yet unknown phase velocity of the surface wave. Substituting (21) into (19) and (20), we obtain the following second-order linear ordinary differential equations

$$\frac{d^2 \Phi(z)}{dz^2} + \left(k_e^2 - k^2\right) \Phi(z) = 0, \tag{22}$$

$$\frac{d^2 \psi(z)}{dz^2} + \left(k_t^2 - k^2\right) \psi(z) = 0 . \tag{23}$$

Since the main characteristic feature of surface waves is the localization of the maximum values of displacement and stress amplitudes near the free surface, in equations (22) and (23), it is necessary to assume the following: $k^2 \succ k_t^2$, $k^2 \succ k_e^2$. Otherwise, the solutions of equations (22) and (23) would be periodic functions, and we would get ordinary plane waves that disappear in the entire body volume. Under the condition $k^2 \succ k_t^2 \succ k_e^2$, the solutions (22) and (23) will be.

Φ_o, ψ_o - permanent.

The solutions contained in (23) the plus sign in the exponent, with increasing *z* for *z>0*, lead to an unlimited increase in displacements and stresses. Therefore, in solutions (23), terms with a negative sign in the exponent have a physical meaning for surface waves. In this case, the wave amplitude will decrease with depth according to an exponential law. Thus, the φ, ψ expressions take the form

$$\varphi = A e^{-qz+i(kx-\omega t)}, \qquad q^2 = k^2 - k_e^2, \tag{24}$$

$$\psi = B e^{-sz+i(kx-\omega t)}, \qquad s^2 = k^2 - k_t^2. \tag{25}$$

On the free boundary of the half-space, the conditions

$$\lambda\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right) + 2\mu\left(\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right) = 0,$$

$$2\frac{\partial^2\varphi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2} = 0,$$

where φ, ψ – are related to displacements by the relations

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, \qquad w = \frac{\partial\varphi}{\partial z} - \frac{\partial\psi}{\partial x}$$

Substituting the solutions (24) and (25) into the boundary conditions, we have

$$\left(k^2 + s^2\right)A - 2ksB = 0, \qquad 2kqA + \left(k^2 + s^2\right)B = 0. \tag{26}$$

The conditions for the existence of non-trivial solutions of the system of equations with respect to A and B are given by the equality to zero of the determinant, composed of the coefficients at A and B. It gives an

$$4k^2qs - \left(k^2 + s^2\right)^2 = 0, \tag{27}$$

equation called the Rayleigh equation. Introducing the notation $\eta = k_t / k = c / c_s$, $\xi = k_e / k_t = c_s / c_p$, the equation can be reduced to the following polynomial form:

$$\eta\left[\eta^6 - 8\eta^4 + 8\left(3 - 2\xi^2\right)\eta^2 - 16\left(1 - \xi^2\right)\right] = 0. \tag{28}$$

3 Results and Discussion

The root $\eta_1 = 0$ does not meet the conditions of the problem since the equality $\eta = 0$ means that either $c = 0$ or $c_s = \infty$. The equality of the phase velocity to zero leads to the equality of the potentials to zero, the absence of perturbation. The equality $c_s = \infty$ means that the half-space is absolutely rigid. From the rest of the roots, you need to choose the one that meets the condition $q \succ 0$. Note that the equation (28) does not include the frequency ω. This means that the Rayleigh surface waves propagate at a constant speed c_R

independent on frequency, it has dispersion, and this is another difference between it and the Love wave.

The equation η_R, in addition to zero, has six roots. The values of these roots depend only on Poisson's ratio of the elastic medium. The Rayleigh wave corresponds to the roots of η_R lying between zero and one ($\eta \prec 1$ because $k \succ k_t$). It can be shown that for real media, when the Poisson's ratio changes from zero to 0.5, for any value, there is one and only one root of the equation, which is also the root of the equation. It can be shown that for real media, when the Poisson's ratio changes from zero to 0.5, for any value, there is one and only one root of the equation, which is also the root of the equation. To determine the root η_R for various values of *v* (Poisson's ratio), the following approximate formula can be obtained:

$$\eta_R = \frac{0.87 + 1.12\nu}{1+\nu}. \tag{29}$$

The c_s / c_p ratio practically changes for various substances within $1/\sqrt{2}$ to 0, corresponding to a change in v from 0 to 0.5. In this case, η changes from 0.87 to 0.96, the Rayleigh wave velocity c_R varies in the interval. (Fig. 4) shows a qualitative graph of the dependence η on *v*.

Knowing the phase velocity of the Rayleigh wave, it is possible to determine the relations between arbitrary constants *A, B* from the equations and express the potentials φ, ψ with an accuracy of one arbitrary constant as follows:

$$\varphi = Ae^{-q_R z + i(k_R x - \omega t)}, \qquad \psi = \frac{2ik_R q_R}{k_R^2 + s_R^2} Ae^{-s_R z + i(k_R x - \omega t)}. \tag{30}$$

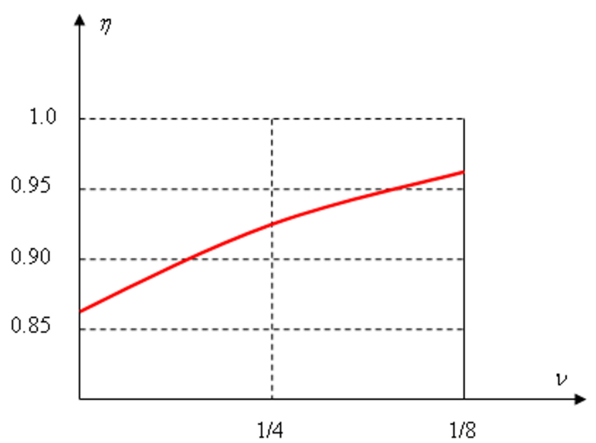

Fig. 4. Addiction η from ν

It can be seen from the formula (30) that the Rayleigh wave consists of inhomogeneous longitudinal and transverse waves propagating with the same phase velocity c_R along the boundary of the half-space and decays exponentially with distance from the boundary.

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Based on the formula for the potentials (30), one can obtain expressions for the displacements u, w in the Rayleigh wave.

Here are some general laws of Rayleigh wave propagation:

1. u, w offset components are phase-shifted by π/2;
2. The trajectories of particles in the medium are ellipses;
3. At the free surface up to depth $z = 0.2\lambda_R$, the particles rotate counterclockwise;
4. Starting from the depth $0.2\lambda_R$ (where $\lambda_R = 2\pi/\omega - c_R$ is the wavelength), the offset u changes sign and the direction of rotation changes; it will proceed clockwise;
5. The semi-major axis of the ellipses is perpendicular to the boundary of the half-space, and the eccentricity of the ellipses depends on the distance to the free surface and Poisson's ratio;
6. At an arbitrary point in the half-space, the time-averaged kinetic energy density in a Rayleigh wave is not equal to the time-average potential energy density;
7. The time-averaged kinetic energy in the wave (the integral of the kinetic energy density in the $0 \le z \prec \infty$ interval) is equal to the time-averaged total potential energy (the integral of the potential energy density in the $0 \le z \prec \infty$ interval);
8. A significant dependence of the time-averaged energy density on the depth z is observed.

Fig. 5 shows a qualitative picture of the change in the components of the displacement vector u, w depending on depth. (Fig. 6) shows changes in the energy density with depth.

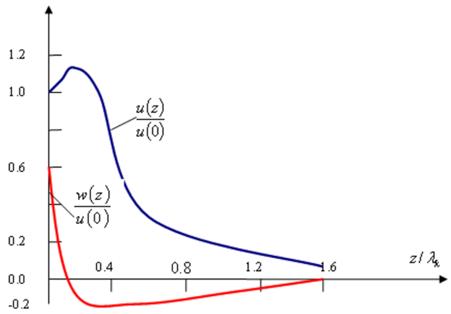

Fig. 5. Change in offsets in depth

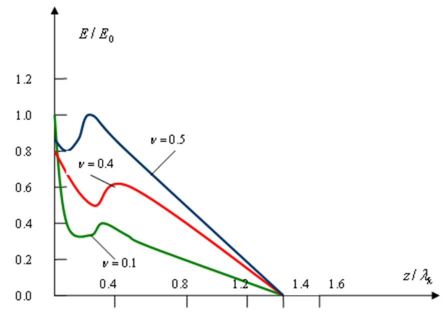

Fig. 6. Energy change with depth depending on the Poisson's ratio ν

The dynamic loads acting on the subgrade are different, but the impact of earthquakes is of greatest interest. The importance of the problem of seismic stability of the subgrade is determined by the catastrophic consequences of strong earthquakes acting on it. Designing an economical, durable, and reliable subgrade that can withstand inertial forces during intense earthquakes requires engineering and the use of scientific achievements in the field of seismic resistance theory of transport structures for various purposes [18-19].

Therefore, it is advisable to demonstrate the main provisions of the accepted theories and calculation methods, considering the seismic resistance of the "subgrade-base" system, considering the heterogeneity and heterogeneity of its constituent soil layers under the influence of strong earthquakes.

It should be expected that seismic vibrations of the ground of the embankment will cause significant damage to the subgrade in particular and to the whole railway track.

The actual data of destructive earthquakes again confirm that earthquakes of medium intensity cause significantly greater stresses and displacements than loads accepted by the norms. In reality, the subgrade, calculated according to the norms, will be overstressed, and the parameters of the actual reaction can only be determined using the above methodology.

4 Conclusion

1. It has been established that the coefficients of reflection of dispersive Rayleigh waves from thin vertical layers of increased velocity, calculated from the maximum amplitudes in the reflected wave, have the same values as in the absence of dispersion. However, the number of oscillations in a reflected wave is always less than in a direct one, and its spectrum is sharply shifted towards high frequencies.
2. It has been established that the coefficients of reflection of Rayleigh waves from low-velocity layers, thin ($d/\lambda_2 = 0.205$) and thick ($d/\lambda_2 = 0.82$), are approximately 0.2 ± 0.04 and practically do not change with increasing angle of incidence.
3. On the basis of theoretical studies, it was revealed that reflected surface waves carry information about the reflecting medium's elastic properties and the dimensions of the reflecting horizontal inhomogeneity.

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