Non-stationary problem of high earth dam under seismic impact

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Abstract. The design, construction, and operation of high earth dams on the territory of seismogenic zones of Uzbekistan set the task for researchers to improve the calculation methods and algorithms for various kinds of loads to which the structure is subjected. At present, the application of the dynamic method (a wave theory) to the calculation of an earth structure, i.e., the dynamic stress-strain state of high earth dams under seismic impact within the framework of wave theory, is the most difficult problem in the mechanics of a deformable rigid body. In the example of the operating Charvak high earth dam, the numerical method of finite differences solved the problem of studying the stress-strain state with shear dynamic impact on the base (such as the records of a real seismogram of an earthquake). At the boundaries of the considered finite region of the base, the so-called radiation conditions are set, i.e., diffraction is not considered. The result of solving the non-stationary problem is presented as isolines of displacements and stresses along the dam's body, depending on time. At that, the most vulnerable zones of the considered earth dam were identified.

1 Introduction

At present, one of the most important priority tasks for developing the national economy is to ensure the safe and reliable operation of structures in seismic regions, such as the territory of the Republic of Uzbekistan. In particular, the category of especially important structures includes earth hydro-technical structures (dams of hydroelectric power stations, dam reservoirs) since their possible destruction during an earthquake can lead to catastrophic consequences and death of people.

At present, various methods of calculation are used to assess the strength under loads to which the body of the water-retaining structure itself is subjected. The current normative methods for calculating earth dams for seismic resistance are based on the linear-spectral theory. This method considers a structure as a cantilever rod (fixed to the base). A one-dimensional problem is solved to determine the frequencies and modes of vibrations. One of the disadvantages of this method is that it does not consider the non-one-dimensional nature of the oscillation and the real physical and mechanical characteristics of the soil of

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the structure and the subsoil adjacent to it. A sufficient number of publications are devoted to the dynamic calculation of high earth dams under static (basic) and dynamic (special combinations of loads) loads, taking into account the linear and non-linear laws of soil properties. The tasks are solved mainly by the finite element method [1-4].

In the publications of foreign scientists and researchers [5-8], special attention is paid to solving specific problems related to assessing the stress-strain state (SSS) and the dynamic behavior of earth dams, taking into account various impact factors.

A plane design model representing the cross-section of an earth dam is considered in the article; the base consists of siltstone. In contrast to similar studies, the article deals with the problem of determining the stress-strain state of an earth structure together with the foundation. The seismic impact on the structure is realized through the base. In the numerical calculation of the problem, an explicit scheme of the numerical finite difference method of second-order accuracy was used to solve a system of differential equations. A non-stationary problem was solved for the elastic "soil dam-foundation" scheme in the example of the high earth dam of the Charvak HPP (the height is 169 m).

The researchers in [9-12] solved the problems of statics for the high earth dam of the Charvak HPP. The problem of studying the stress state of the dam was solved by considering the forces of gravity and hydrostatics [9-12].

The tasks were solved using the numerical finite element method. The service life of the dam for more than 50 years makes it possible to solve the problem in an elastic formulation. Comparison of the results of the solved problems of statics with the data of long-term field studies showed the reliability of the solutions obtained [9-12].

2 Objects and methods of research

The equations of motion of a plane elastic system "earth dam-foundation" are based on the basic equations of continuum mechanics:

$$\begin{aligned} &\rho_i \frac{\partial^2 u^i}{\partial t^2} = \frac{\partial \sigma^i_{xx}}{\partial x} + \frac{\partial \tau^i_{xy}}{\partial y}, \\ &\rho_i \frac{\partial^2 \upsilon^i}{\partial t^2} = \frac{\partial \tau^i_{yx}}{\partial x} + \frac{\partial \sigma^i_{yy}}{\partial y}, \end{aligned} \quad (i=1,\ 2) \qquad (1)$$

where index *i=1* refers to the base, *i=2* - to the dam. ρ_i is the density, u^i, υ^i are the displacement vector projections on the *x, y*-coordinate axes; $\sigma^i_{xx}, \sigma^i_{yy}, \tau^i_{xy}$ are the normal and shear stresses.

The Cauchy relations are presented in the following form:

$$\begin{aligned} &\varepsilon^i_{xx} = \frac{\partial u^i}{\partial x},\ \ \varepsilon^i_{yy} = \frac{\partial \upsilon^i}{\partial y}, \\ &\varepsilon^i_{xy} = \frac{\partial u^i}{\partial y} + \frac{\partial \upsilon^i}{\partial x}. \end{aligned} \quad (i=1,2) \qquad (2)$$

Following Hooke's elastic law, the relationship between the stress and strain tensors is represented as:

$$\sigma_{xx}^{i}=\frac{E_i}{1-\nu_i^{2}}(\varepsilon_x^{i}+\nu\varepsilon_y^{i})$$
$$\sigma_{yy}^{i}=\frac{E_i}{1-\nu^{2}}(\nu_i\varepsilon_x^{i}+\varepsilon_y^{i}) \qquad (i=1,2) \tag{3}$$
$$\tau_{xy}^{i}=\frac{E_i}{2(1+\nu_i)}\varepsilon_{xy}^{i}$$

where ν_i is Poisson's ratio; E_i is linear deformation modulus.

Boundary conditions are:

on the surface of the upstream slope of the dam

$$p_x=\sigma_{xx}^{2}l_1+\tau_{xy}^{2}m_1,$$
$$p_y=\tau_{xy}^{2}l_1+\sigma_{yy}^{2}m_1. \tag{4}$$

where l_1, m_1 are the direction cosines; p_x, p_y are the stress components from hydrostatic pressure,

on the crest of the dam

$$\tau_{xy}^{2}=0; \quad \sigma_{yy}^{2}=0 \tag{5}$$

on the downstream face

$$\sigma_{xx}^{2}l_2+\tau_{xy}^{2}m_2=0,$$
$$\tau_{xy}^{2}l_2+\sigma_{yy}^{2}m_2=0. \tag{6}$$

where l_2, m_2, are the direction cosines.

$$u^{2}=0; \ \frac{\partial u^{2}}{\partial t}=0$$
$$\upsilon^{2}=0; \ \frac{\partial \upsilon^{2}}{\partial t}=0 \tag{7}$$

on the free surface of the dam foundation

$$\sigma_{yy}^{1}=0$$
$$\tau_{yx}^{1}=0 \tag{8}$$

At the contact boundary between the base and the dam, the no-slip (continuity) condition is satisfied

$$u^1 = u^2, \quad \upsilon^1 = \upsilon^2$$
$$\sigma_{xx}^1 = \sigma_{xx}^2, \; \tau_{xy}^1 = \tau_{xy}^2 \tag{9}$$
$$u^1 = u^2, \quad \upsilon^1 = \upsilon^2$$
$$\sigma_{yy}^1 = \sigma_{yy}^2, \; \tau_{yx}^1 = \tau_{yx}^2 \tag{10}$$

Displacements and velocities in the entire area of the earth dam at the initial time (*t=0*) are zero.

The time when the wavefront approaches the lower part of the dam base was taken as the initial condition t=0, i.e.

$$u^1 = u^0$$
$$\upsilon^1 = \upsilon^0 \tag{11}$$

here the superscript *0* corresponds to displacements in the incident wave.

The problem is solved numerically by the method of finite differences using an explicit scheme, and therefore we select the calculated finite domain for the base of the dam, the so-called fictitious boundary, and set the boundary conditions along the contour of the calculated domain. The displacement for the dam foundation soil u^1, υ^1 is represented as the sum of the incident ($u^{1,0}, \upsilon^{1,0}$) and reflected ($u^{1,1}, \upsilon^{1,1}$) waves, i.e. $u^1 = u^{1,1} + u^{1,0}, \;\; \upsilon^1 = \upsilon^{1,1} + \upsilon^{1,0}$. Subject to the boundary conditions, the determined displacements, and stresses for the dam foundation soil are also presented as the sum of incident and reflected waves, and radiation conditions are set on fictitious boundaries.

On the fictitious boundary of the base from the side of the upstream slope, the following conditions are set:

$$\sigma_{xx}^{1,1} = \rho_1 a_1 \frac{\partial u^{1,1}}{\partial t}, \;\; \tau_{xy}^{1,1} = \rho_1 b_1 \frac{\partial \upsilon^{1,1}}{\partial t} \tag{12}$$

On the fictitious boundary of the base from the side of the downstream slope, the following conditions are set:

$$\sigma_{xx}^{1,1} = -\rho_1 a_1 \frac{\partial u^{1,1}}{\partial t}, \;\; \tau_{xy}^{1,1} = -\rho_1 b_1 \frac{\partial \upsilon^{1,1}}{\partial t} \tag{13}$$

On the bottom of the fictitious boundary of the base, the following conditions are set:

$$\sigma_{yy}^{1,1} = \rho_1 a_1 \frac{\partial \upsilon^{1,1}}{\partial t}, \;\; \tau_{xy}^{1,1} = \rho_1 b_1 \frac{\partial u^{1,1}}{\partial t} \tag{14}$$

The calculated area of the earth dam, taking into account the deformable subsoil, is conditionally divided into three areas with a step: *hx=hx1* horizontally (*x*-axis) (upper retaining prism); *hx=hx2* (dam crest), *hx=hx3* (lower retaining prism); *hy* vertically (Fig.1).

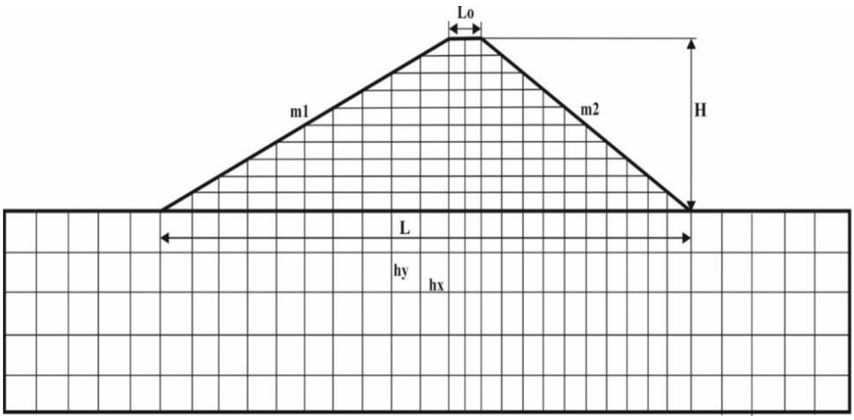

Fig. 1. Partition of the calculation domain of the dam

For internal nodes of the computational domain at (*i+1/2, j*) at time *kτ*, the components of the strain tensor in the difference form have the following form

$$\varepsilon^k_{xx,i+1/2,j} = \frac{u^k_{i+1,j} - u^k_{i,j}}{hx},\ \varepsilon^k_{yy,i+1/2,j} = \frac{\upsilon^k_{i+1,j+1} + \upsilon^k_{i,j+1} - \upsilon^k_{i+1,j-1} - \upsilon^k_{i,j-1}}{4hy} \tag{15}$$

$$\varepsilon^k_{xy,i+1/2,j} = \frac{u^k_{i+1,j+1} + u^k_{i,j+1} - u^k_{i+1,j-1} - u^k_{i,j-1}}{4hy} + \frac{\upsilon^k_{i+1,j} - \upsilon^k_{i,j}}{hx}$$

for the node (*i,j+1/2*) at *kτ*,

$$\varepsilon^k_{xx,i,j+1/2} = \frac{u^k_{i+1,j+1} + u^k_{i+1,j} - u^k_{i-1,j+1} - u^k_{i-1,j}}{4hx},\ \varepsilon^k_{yy,i,j+1/2} = \frac{\upsilon^k_{i,j+1} - \upsilon^k_{i,j}}{hy} \tag{16}$$

$$\varepsilon^k_{xy,i,j+1/2} = \frac{u^k_{i,j+1} - u^k_{i,j}}{hy} + \frac{\upsilon^k_{i+1,j+1} + \upsilon^k_{i+1,j} - \upsilon^k_{i-1,j+1} - \upsilon^k_{i-1,j}}{4hx}.$$

where on the upper retaining prism, it is *hx=hx1*, on the lower retaining prism - *hx=hx3*; on the crest of the dam - *hx= hx2*.

Since the problem under consideration does not take into account the water pressure on the upstream slope, i.e., $p_x = p_y = 0$, then conditions (4) have the form

$$\begin{aligned} \sigma_{yy} &= \sigma_{xx} d_1^2 \\ \tau_{xy} &= -\sigma_{xx} d_1 \end{aligned} \tag{17}$$

where $d_1 = l_1 / m_1$.

By substituting Hooke's law (3) and Cauchy relations (2) into equality (17), we define

$$\frac{\partial \upsilon}{\partial y} = D_{11}\frac{\partial u}{\partial x},$$
$$\frac{\partial u}{\partial y} = D_{21}\frac{\partial u}{\partial x} - \frac{\partial \upsilon}{\partial x} \tag{18}$$

where $D_{11} = \dfrac{d_1^2(\lambda+2G)-\lambda}{\lambda+2G-\lambda d_1^2}$, $D_{21} = -\dfrac{d_1(\lambda+2G+\lambda D_{11})}{G}$, $\lambda = \dfrac{2\nu G}{1-2\nu}$ -.

According to (18), we obtain an expression for determining the contour displacements

$$\upsilon_{i,j+1}^k = 2hy\left(D_{11}\frac{u_{i+1,j}^k - u_{i-1,j}^k}{2hx1}\right) + \upsilon_{i,j-1}^k,$$
$$u_{i,j+1}^k = 2hy\left(D_{21}\frac{u_{i+1,j}^k - u_{i-1,j}^k}{2hx1} - \frac{\upsilon_{i+1,j}^k - \upsilon_{i-1,j}^k}{2hx1}\right) + u_{i,j-1}^k \tag{19}$$

i=j, j=1,…,J1-1, k=1,…,K.

For the downstream slope (6), similar to (19), we obtain

$$\upsilon_{i,j+1}^k = 2hy\left(D_{12}\frac{u_{i+1,j}^k - u_{i-1,j}^k}{2hx3}\right) + \upsilon_{i,j-1}^k,$$
$$u_{i,j+1}^k = 2hy\left(D_{22}\frac{u_{i+1,j}^k - u_{i-1,j}^k}{2hx3} - \frac{\upsilon_{i+1,j}^k - \upsilon_{i-1,j}^k}{2hx3}\right) + u_{i,j-1}^k \tag{20}$$

i=I2+j, j=1,…,J1-1, k=1,…,K

where $d_2 = l_2/m_2$, $D_{12} = \dfrac{d_2^2(\lambda+2G)-\lambda}{\lambda+2G-\lambda d_2^2}$, $D_{22} = -\dfrac{d_2(\lambda+2G+\lambda D_{12})}{G}$.

Taking into account the boundary condition at the crest of the dam (5) and Hooke's law (3), the formula for calculating the normal stress σ_{xx} is

$$\sigma_{xx} = \frac{4G(\lambda+G)}{\lambda+2G}\frac{\partial u}{\partial x}$$

at nodes with half-integer grid numbers, shear stresses τ_{xy} and normal stresses σ_{yy} are zero (at the crest of the dam) and σ_{xx} is determined from (21)

$$\sigma_{xx,i+1/2,j}^k = \frac{4G(\lambda+G)}{\lambda+2G}\frac{u_{i+1,j}^k - u_{i,j}^k}{hx2} \tag{21}$$

i=I1, …,I2-1, j=J1, k=1,…,K.

The finite-difference equation of motion of the dam and its foundation for each domain has the following form

$$\rho\frac{(u_{i,j}^{k+1}-2u_{i,j}^{k}+u_{i,j}^{k-1})}{\tau^2}=\frac{\sigma_{xx,i+1/2,j}^{k}-\sigma_{xx,i-1/2,j}^{k}}{hx}+\frac{\sigma_{xy,i,j+1/2}^{k}-\sigma_{xy,i,j-1/2}^{k}}{hy} \tag{22}$$
$$\rho\frac{(\upsilon_{i,j}^{k+1}-2\upsilon_{i,j}^{k}+\upsilon_{i,j}^{k-1})}{\tau^2}=\frac{\tau_{yx,i+1/2,j}^{k}-\tau_{yx,i-1/2,j}^{k}}{hx}+\frac{\sigma_{xy,i,j+1/2}^{k}-\sigma_{xy,i,j-1/2}^{k}}{hy}$$

with boundary conditions (9) - (11), and initial conditions (17). Next, it is necessary to determine the displacements in the nodes of the dam for each time point.

Difference equation (22) approximates the equation of motion (1), with order O (τ^2 , hx^2 , hy^2). Condition ($\tau<h/c_1$), where $h=min(hx,hy)$ is the condition for the stability of the scheme.

3 Results and discussion

To perform dynamic calculations, it is necessary to set the inertial and elastically damping properties of a hydro-technical structure and the seismic impact in the form of an accelerogram of foundation oscillations. Since the impact parameters are random variables, therefore, a single calculation of the structure for the action specified by the earthquake accelerogram should be considered the implementation of a random process. One of the most common and rather conservative ways [13-15] is using not one but several calculated accelerograms (a package of accelerograms), taking their maximum values as the calculated forces.

When calculating a structure for a package of accelerograms, the latter must satisfy several requirements. In the most general case, these requirements include:

a) the absence of serious distortions in the calculated accelerograms;

b) an account for the correlation between the calculated intensity, amplitude, and the dominant frequency of the impact.

These requirements are strict, and the calculations are time-consuming. Therefore, dynamic calculations are now widely used, in which a short-time process is taken as the impact, and a seismic impact model is used in the calculations. The proof of the reliability of the developed methods and algorithm for solving the problem under consideration was the solution to the test problem (Lamb problem) [16-17].

The model proposed by I.L. Korchinsky [18, 20] is taken as a seismic impact

$$\frac{\partial^2 u_0}{\partial t^2}=Ae^{-st}\sin\omega t \tag{23}$$

where A is the maximum acceleration value, s is the attenuation coefficient, ω is the dominant frequency of the impact.

In the problem under consideration, the seismic impact on the dam foundation in the horizontal direction (seismogram) is taken in the following form

$$u_0=\frac{A}{(\omega^2+s^2)^2}\left(e^{-st}\left(s^2\sin(\omega t)-\omega^2\sin(\omega t)+2\omega s\cos(\omega t)\right)-2\omega s\right) \tag{24}$$

which represents the model (23) for $A=0.2g$ and coefficient $s=0.3$, which is equivalent to an 8-point earthquake.

For example, the Charvak earth dam was considered with the following parameters: height $H=168m$; crest width $Lo=12m$; coefficients of laying slopes: upstream $m_1=2.2$, downstream – $m_2=2.2$.

The design domain of the earth dam is divided vertically with a step $hy=2m$, and horizontally: $hx1=4.4m$, core $hx2=2m$, downstream slope $hx3=4.4m$. The numerical grid of the model has 7476 nodes. Longitudinal wave propagation velocity in the dam's body is $V_p=1000m/s$; Poisson's ratio $\nu=0.3$, soil density $\rho=1950\ kg/m^3$. The time step for the calculation is $t=0.001s$, which satisfies the stability condition for calculating the difference scheme.

As the impact on the dam foundation, the law of change of displacements is taken in the horizontal direction in the form of (24) and in the vertical direction $\upsilon_0=0$, i.e., there is no vertical component.

The calculations were performed at frequencies $f=1Hz$ and $f=5Hz$, which enter the frequency range of strong earthquakes and are the key parameters of many building codes when assessing seismic risk.

Isolines of velocities of (horizontal) tangential stresses *(t=0.3 sec)* at frequency $f=1Hz$ are shown in Figures 2-3, and the dam's foundation is assumed to be non-deformable. It can be seen that at the initial time point, a shear wave propagates along the dam at a large amplitude. Vertical oscillations of the dam occur due to wave reflection from the upstream and downstream slopes.

After the time point $t=0.3s$, the shear wave front travels a distance of approximately 160m (80x2m) at a dam height of 168m. In the lower part of the retaining prisms, the particle velocities in the horizontal direction have a negative value (*-0.1m/s),* which shows that the particles in this part of the dam have changed their direction. Figure 3 at the time point $t=0.3sec$ shows the isolines of the maximum values of shear stresses $\tau_{max},$ and its maximum value of *0.41 MPa* occurs in the area of the dam crest.

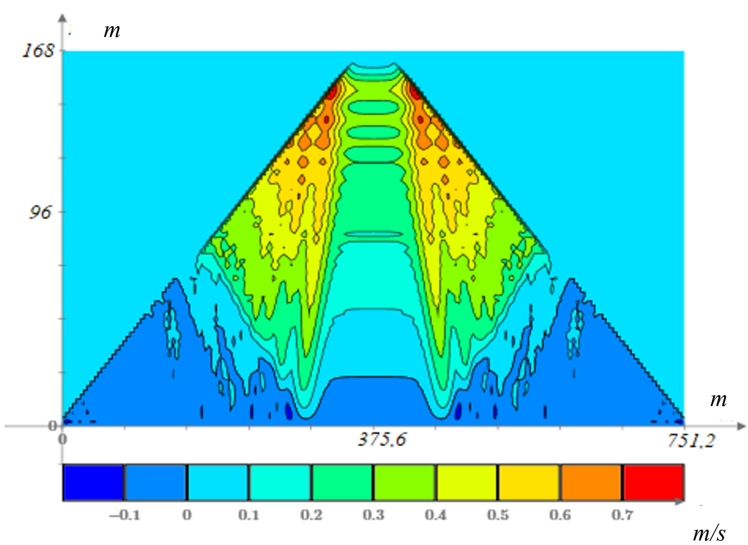

Fig. 2. Isolines of horizontal velocity $\dfrac{\partial u}{\partial t}$ *m/s* in the body of the dam at *t=0.3s* and frequency $f = 1\ Hz$.

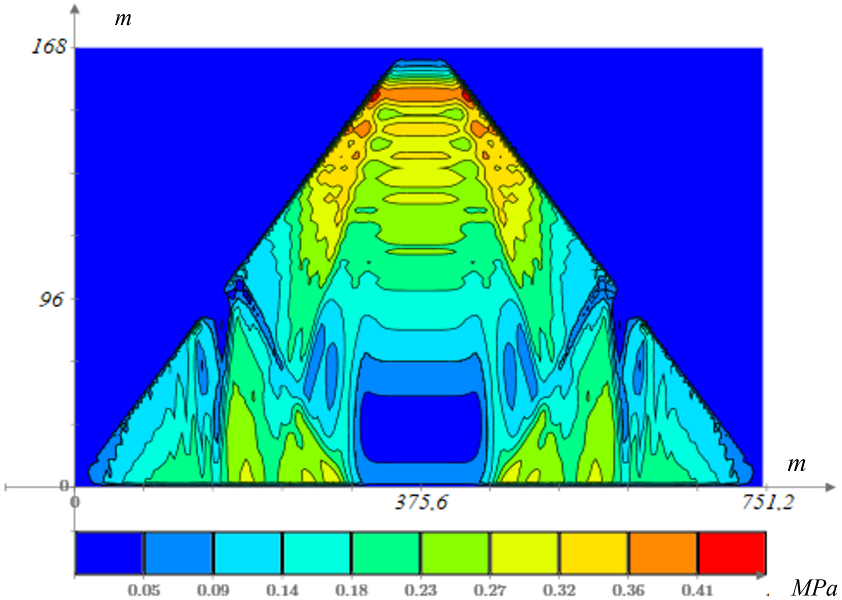

Fig. 3. Isolines of the maximum value of shear stress $\tau_{\max}\,(MPa)$ in the body of the dam at $t=0.3s$ and frequency $f=1$ Hz.

Figure 4 shows the distribution of shear stress in the body of the dam and foundation at time $t=10$ *sec* and frequency $f=1$ Hz.

Acceleration amplitude according to formula (23) is $A=0.2g$, coefficient $s=0.3$. Longitudinal wave propagation velocity at the dam base is $a_1=1000$ *m/s*, density $\rho_1=1950 kg/m^3$, Poisson's ratio $v_1=0.3$. For the body of the dam, $a_2=600$ *m/s*, density $\rho_2=1750 kg/m^3$, Poisson's ratio $v_2=0.3$ are taken.

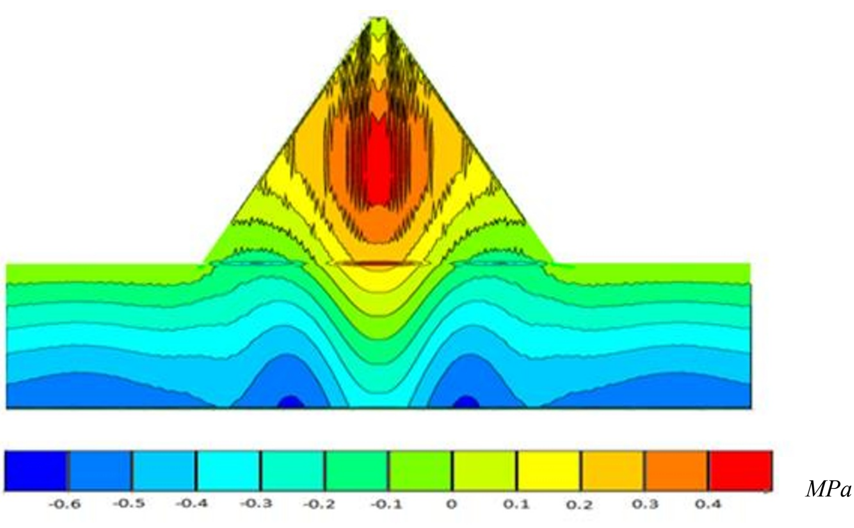

Fig. 4. Distribution of shear stress $\tau_{\max}\,(MPa)$ in the body of the dam and at the bases at $t=10s$.

4 Conclusion

In contrast to the studies conducted in [1-8], in this article, following the current regulatory methods for calculating structures for seismic effects, the stress-strain state was calculated using the example of the high earth dam of the Charvak HPP, which has been in operation for about 50 years. A real record of earthquakes was taken as a seismic impact. The problem was solved numerically using the finite difference method.

Thus:

- a mathematical formulation was developed, and a solution to a non-stationary problem was derived by the numerical method of finite differences. In the example of a specific operated high earth dam, the calculation of the stress state together with the base and the physical and mechanical parameters of the soil of the structure and the base was conducted under shear seismic impact;
- the results of solving the problem are of horizontal velocities, normal, tangential, and maximum tangential stresses in the body of the dam at the frequency of seismic action $f=1Hz$ and $f=5Hz$, which enter the frequency range of strong earthquakes.
- for the dam under consideration, it was shown that low-frequency seismic impacts are more unfavorable than high-frequency ones. At that, the crest and slopes are the most vulnerable areas of high earth dam in terms of the possibility of soil sliding.

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