Bending-torsional vibrations of aircraft wing

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Abstract. The problems of bending-torsional flutter of an aircraft wing are considered. Using equations of motion by the Bubnov-Galerkin method, based on the polynomial approximation of deflections and angle of twist, the problem is reduced to studying a system of ordinary integro-differential equations, where the independent variable is time. The solution of integro-differential equations with singular kernels is found by a numerical method based on quadrature formulas. The influence of physical-mechanical and geometric parameters on the flutter of the aircraft wing was studied. It was established that an account for the viscoelastic properties of the material of thin-walled aircraft structures leads to a 40-60% decrease in the critical flutter velocity. It is shown that an increase in the elongation parameter of an aircraft wing leads to a decrease in the flutter velocity.

1 Introduction

The aerodynamic stability of an aircraft wing in airflow is one of the most important factors determining the effectiveness of its application. Aeroelastic oscillations should be studied at the design stage since the occurrence of undamped elastic oscillations of parts of a real structure or the entire aircraft as a whole can lead to loss of controllability and even destruction [1, 2].

Studies of the bending-torsional flutter of aircraft wings have not lost their relevance, and at present, one of the key tasks is to find patterns that could be used in the future to develop a systematic approach to the analysis of various forms of flutter. The practical significance of this problem increases with the accumulation of experimental and calculated results of the study of flutter, and therefore it is necessary to search for effective ways to solve it [3, 4].

The study in [5] presents the bending-torsional flutter characteristics of an aircraft wing. The derivation of the proposed form of equations of motion from the Hamilton variational principle is given. Calculation results are compared with data published in the literature.

The formula for the aeroelastic analysis of an aircraft wing is given in [6]. The effect of aerodynamic nonlinearity on the critical flutter velocity is studied.

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The widespread use of composite materials in aviation technology has led to the need to study the problems of optimal design of thin-walled structures with viscoelastic properties. Following this, it is of considerable interest to analyze the features of similar problems for elastic thin-walled structures. Such an analysis is also interesting and important because the presence of damping properties of viscoelastic materials may be sufficient to prefer them for creating appropriate thin-walled structures from any material with the required properties.

The problem of considering the hereditary effects of deformable materials is of great theoretical and applied interest. Its solution effectively applies the theory of viscoelasticity to real processes. Therefore, the methods and problems of the theory of hereditary elasticity attract much attention from researchers. A significant number of publications are devoted to solving problems of calculating the characteristics of viscoelastic thin-walled structures [7–20]. Despite the numerous studies devoted to viscoelastic thin-walled structures, the bending-torsional flutter of the viscoelastic wing of aircraft has not been studied so far. This situation shows the relevance of this study.

The study aims to develop a mathematical model of bending-torsional oscillations of the wing in the airflow and to determine the flutter of the design.

2 Problem statement

The wing can perform vibrations of two main types: bending and torsional ones. However, due to the non-coincidence of the centers of gravity lines with the line of the centers of stiffness of the sections, purely bending or purely torsional oscillations of the wing are practically impossible. Regardless of the type of initial impulse (bending or torsional), the vibrations are always coupled - bending-torsional ones.

Consider a cantilever unswept wing of aircraft of finite span with a straight visco elastic axis normal to the root section. We assume that the wing is rigidly fixed in the massive fuselage, which is assumed to be fixed too. Thus, the effect of other parts of the aircraft, particularly the fuselage on the wing flutter, is excluded from the analysis.

Let us assume that the wing is streamlined by an airflow of velocity *V* directed along the *x*-axis, which coincides with the root chord of the wing. Under the airflow, the wing is deformed and is in a certain equilibrium state [3]. Then the equations of equilibrium of an element of a viscoelastic line, with coupled bending-torsional vibrations, can be written in the following form

$$\begin{cases} E\left(1-R^*\right)\dfrac{\partial^2}{\partial z^2}\left(J\dfrac{\partial^2 W}{\partial z^2}\right)+m(z)\dfrac{\partial^2 W}{\partial t^2}-m(z)\sigma\dfrac{\partial^2\theta}{\partial t^2}- \\ -C_y^\alpha \rho b V^2\left[\theta+\dfrac{1}{V}\dfrac{\partial\theta}{\partial t}\left(\dfrac{3}{4}b-x_i\right)-\dfrac{1}{V}\dfrac{\partial W}{\partial t}\right]=0 \\ G\left(1-R^*\right)\dfrac{\partial}{\partial z}\left(J_P\dfrac{\partial\theta}{\partial z}\right)+m(z)\sigma\dfrac{\partial^2 W}{\partial t^2}-\rho_m J_m\dfrac{\partial^2\theta}{\partial t^2}+ \\ +C_{mE}^\alpha \rho b^2 V^2\left[\theta+\dfrac{1}{V}\dfrac{\partial\theta}{\partial t}\left(\dfrac{3}{4}b-x_i-\dfrac{\pi b}{16C_{mE}^\alpha}\right)-\dfrac{1}{V}\dfrac{\partial W}{\partial t}\right]=0 \end{cases} \tag{1}$$

Where *m(z)*is the linear mass; J_m is linear mass moment of inertia relative to the center of stiffness; x_i is the distance from the leading edge of the wing to its stiffness axis; ρ_m, *E*, *G* are the density and moduli of elasticity of the material; *J(z)* is the main moment of inertia of the sections; $J_\rho(z)$ is the geometric torsional stiffness; θ is the angle of twist; W is the deflection; x_0 is the coordinate of the center of gravity of the section; ρ is the air density; ρ_m is the density of the material; $F(z)$ is the wing area per unit length (along the span); $C_y^\alpha, C_{mE}^\alpha$ are the aerodynamic characteristics of the wing; σ is the distance of the center of gravity of the wing section from the center of stiffness.

The alignment of the stiffness axis with the axis of the centers of gravity corresponds to the value of $\sigma = 0$, and the alignment of the stiffness axis with the axis of the wing foci corresponds to the value of $C_{mE}^\alpha = 0$.

In this case, the equations of motion take the following form:

$$\begin{cases} E\left(1-R^*\right)\dfrac{\partial^2}{\partial z^2}\left(J(z)\dfrac{\partial^2 W}{\partial z^2}\right)+m(z)\dfrac{\partial^2 W}{\partial t^2}-C_y^\alpha \rho b V^2\left[\theta+\dfrac{1}{V}\dfrac{\partial\theta}{\partial t}\left(\dfrac{3}{4}b-x_i\right)-\dfrac{1}{V}\dfrac{\partial W}{\partial t}\right]=0 \\ G\left(1-R^*\right)\dfrac{\partial}{\partial z}\left(J_p(z)\dfrac{\partial\theta}{\partial z}\right)-\rho_m J_m\dfrac{\partial^2\theta}{\partial t^2}-\dfrac{\pi}{16}\rho b^3 V\dfrac{\partial\theta}{\partial t}=0 \end{cases} \quad (2)$$

Thus, the task of studying the bending-torsional flutter of the wing is reduced to solving a system of sixth-order partial integro-differential equations. Following this, it is necessary to formulate three boundary conditions at each end of the wing.

Equations (1) must be integrated under the following boundary conditions:

a) At the left end of the wing, in the termination, the deflection, its first derivative concerning the z-coordinate, and the angle of twist must be zero:

$$W=0;\ \frac{\partial W}{\partial z}=0;\ \theta=0; \text{ for } z=0 \quad (3)$$

b) At the right load-free end of the wing, the bending, torque moments, and transverse force are zero:

$$EJ\frac{\partial^2 W}{\partial z^2}=0;\ \frac{\partial}{\partial z}\left(EJ\frac{\partial^2 W}{\partial z^2}\right)=0;\ GJ_\rho\frac{\partial\theta}{\partial z}=0 \text{ for } z=l. \quad (4)$$

3 Discrete model

The solution of system (2), which satisfies the boundary conditions of the problem, is sought in the following form:

$$W(z,t)=\sum_{n=1}^{N} W_n(t)\varphi_n(z) \quad (5)$$
$$\theta(z,t)=\sum_{n=1}^{N}\theta_n(t)\psi_n(z)$$

where

$$\left.\begin{aligned} \varphi_n(z)&=\sin\lambda_n\frac{z}{l}-sh\lambda_n\frac{z}{l}-a_n\left(\cos\lambda_n\frac{z}{l}-ch\lambda_n\frac{z}{l}\right) \\ \psi_n(z)&=\sin\frac{\pi n z}{2l} \end{aligned}\right\}$$

$$\lambda_n = \frac{2n-1}{2}\pi,\ a_n = \frac{\sin\lambda_n + sh\lambda_n}{\cos\lambda_n + ch\lambda_n}.$$

Substituting the expansion into system (1) and using the Bubnov-Galerkin method, we obtain a system of integro-differential equations (IDE). Introducing the following dimensionless quantities into the IDE

$$\frac{z}{l},\ \frac{x_i}{b},\ \frac{W}{l},\ \frac{h}{b}$$

and maintaining the previous notation, we have

$$\begin{cases} \sum\limits_{n=1}^{N}\Big\{D_{nk}\ddot{W}_{nk} - \overline{E}_{nk}\ddot{\theta}_{nk} - \gamma_1\Big(\ddot{\theta}_{nk}L_{nk} - \dot{W}_{nk}D1_{nk}\gamma_2\Big) + \\ +\gamma_3\left(1-R^*\right)A_{nk}W_{nk} - \gamma_1\gamma_2K_{nk}\theta_{nk}\Big\} = 0 \\ \sum\limits_{n=1}^{N}\Big\{U_{nk}\ddot{\theta}_{nk} - \gamma_4\overline{N}_{nk}\ddot{W}_{nk} - \gamma_5\Big(U_{nk}\theta_{nk}(t) - \gamma_6\dot{\theta}_{nk}U_{nk} - \\ -\gamma_7Z_{nk}\dot{W}_{nk}\Big) - \gamma_8\left(1-R^*\right)S_{nk}\theta_{nk}\Big\} = 0 \end{cases} \quad (6)$$

$$\gamma_1 = \frac{\partial G_y}{\partial\alpha}\cdot\bar{\rho}M^*\gamma\lambda,\ \gamma_2 = \frac{M^*}{\lambda},\ \gamma_3 = M_E g,\ \gamma_4 = \frac{Fb^2}{J_i}\cdot\frac{1}{\lambda},\ \gamma_5 = \bar{\rho}_b\cdot\frac{\partial G_{mi}}{\partial\alpha}\cdot \mathrm{M}^{*2}\gamma_4\cdot\gamma,$$

$$\gamma_6 = \left(\frac{V_0}{V}\right)\cdot\lambda\cdot\left(\frac{3}{4}\right) - \bar{x}_i - \frac{\pi}{16\dfrac{\partial C_{mi}}{\partial\alpha}},\quad \gamma_7 = \frac{V_0}{V},\ \gamma_8 = M_G\cdot\lambda\cdot\frac{Fb^2}{J_i}\cdot\gamma\cdot\left(\frac{h_0}{b}\right)^3\cdot\frac{\pi^2}{128}$$

When the stiffness axis is aligned with the axis of the center of gravity, and the stiffness axis is aligned with the wing foci axis, equations (6) take the following form:

$$\begin{cases} \sum\limits_{n=1}^{N}\Big\{D_{nk}\ddot{W}_{nk} - \gamma_1\Big(\dot{\theta}_{nk}L_{nk} - \dot{W}_{nk}D1_{nk}\gamma_2\Big) + \\ +\gamma_3\left(1-R^*\right)A_{nk}W_{nk} - \gamma_1\gamma_2K_{nk}\theta_{nk}\Big\} = 0 \\ \sum\limits_{n=1}^{N}\Big\{U_{nk}\ddot{\theta}_{nk} - \gamma_5\gamma_6\dot{\theta}_{nk}U_{nk} - \gamma_8\left(1-R^*\right)S_{nk}\theta_{nk}\Big\} = 0 \end{cases} \quad (7)$$

where $M^* = \dfrac{V}{V_0}$ is the Mach number, $\rho_b = \dfrac{\rho}{\rho_m}$, $\gamma = \dfrac{bl}{F_0}$, $g = \dfrac{J_0}{l^2F_0}$,

$\lambda = \dfrac{l}{b}$, $g_m = \dfrac{b^2F_0}{J_m}$, $M_G = \dfrac{G}{\rho_mV_0^2}$, $M_E = \dfrac{E}{\rho_mV_0^2}$,

$$D_{nk}=\int_0^1\left(1-\alpha^* z\right)\varphi_n(z)\varphi_k(z)dz,\ L_{nk}=\int_0^1\left(\frac{3}{4}-x_i\right)\psi_n(z)\varphi_k(z)dz,$$

$$D1_{nk}=\int_0^1\varphi_n(z)\varphi_k(z)dz,$$

$$A_{nk}=\int_0^1\left[6\alpha^*\varphi_n''(z)-6\alpha^*\left(1-\alpha^* z\right)^2\varphi_n'''(z)+\left(1-\alpha^* z\right)^3\varphi_n^{IV}(z)\right]\varphi_k(z)dz$$

$$U_{nk}=\int_0^1\psi_n(z)\psi_k(z)dz,\ S_{nk}=\frac{128}{\pi^2}\int_0^1\frac{\partial}{\partial z}\left(J_\rho\frac{\partial\theta}{\partial t}\right)\psi_k(z)dz$$

System (6) was integrated using the numerical method proposed in [9-11]. The calculation results are presented in Table 1.

4 Numerical results and discussion

Table 1 shows the critical values of the wing flutter velocity V_{cr} depending on the physical, mechanical, and geometrical characteristics of the wing.

As seen from the analysis of the results given in Table 1, the value of the coefficient V_{cr} in elastic $(A=0)$ and viscoelastic $(A=0.1)$ cases turns out to be 180 and 70, respectively. Thus, the viscoelastic properties of the material lead to a decrease in the flutter velocity by 61%.

The influence of the wing elongation parameter λ on the critical flutter velocity was studied. With an increase in the coefficient λ from 2 to 5, the critical velocity coefficient decreases from 67 *m/s* to 13 *m/s*.

The influence of the parameter of relative wing area γ on V_{cr} was studied. A noticeable increase in the parameter γ has a significant effect on the critical flutter velocity. The critical velocity decreases for wings of the type under consideration.

Table 1. The influence of rheological parameters on the critical flow rate

A	α	β	λ	γ	C_y^α	V_{cr} (m/s)
0 0.01 0.1	0.25	0.05	3	550	1.5432	180 140 70
0.1	0.1 0.5 0.75	0.05	3	550	1.5432	63 96 137
0.1	0.25	0.01 0.1	3	550	1.5432	65 61
0.1	0.25	0.05	2 4 5	550	1.5432	67 15 13
0.1	0.25	0.05	3	450 650	1.5432	90 60

5 Conclusion

A mathematical model of the problem of the bending-torsional flutter of a viscoelastic wing of aircraft was developed. The Boltzmann-Volterra integral model with weakly singular heredity kernels was used to describe the processes of structural deformation processes.

The influence of physical-mechanical and geometric parameters on the flutter of aircraft wingswas studied. It was established that an account for the viscoelastic properties of the material of thin-walled aircraft structures leads to a 40-60% decrease in the critical flutter velocity. It is shown that an increase in the elongation parameter of an aircraft wing leads to a decrease in the flutter velocity.

Based on the results obtained, it can be concluded that considering the viscoelastic properties of the material of structures leads to a decrease in the critical flutter velocity V_{cr}.

We also note that at flow velocity less than V_{cr}, the influence of the viscoelastic property of the material reduces the amplitude and frequency of oscillations. If the flow velocity exceeds V_{cr}, then the viscoelastic property of the material exerts a destabilizing effect.

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