Impact of rigid body and viscous-plastic rod of finite length

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Abstract. The impact of a rigid body and a rod of finite length made of a viscoplastic incompressible material moving towards each other at constant velocity is studied in the article. One-dimensional motion is considered, i.e., velocity, stress, and other parameters are considered to be averaged over the rod section. According to the model of a viscous-plastic material, the rod is divided into two parts: the region of plastic strain and the region that moves as a solid body after impact. The boundary between these regions is unknown and needs to be determined. As a result, we arrive at a problem with a free boundary, a well-known example of which is the Stefan problem. However, the formulation of the problem under consideration differs significantly from the Stefan problem. In addition, the function to be determined that describes the unknown boundary, in contrast to the Stefan problem, is not explicitly present in the boundary conditions. There are various numerical methods for solving problems with an unknown moving boundary. However, as is known, implementing these methods is associated with significant difficulties. In this article, the method of integral relations is used - a modification of the Karman-Pohlhausen method known in the boundary layer theory. The problem is reduced to the Cauchy problem for a system of nonlinear ordinary differential equations proposed to be solved by successive approximations and the Runge-Kutta method. Numerical calculations were performed. The influence of changes in the mass and magnitude of the velocities of the rod and rigid body on the size of the plastic strain region and the change in stresses on the contact surface is revealed.

1 Introduction

Numerous studies were devoted to the theory and problems of the impact of various rigid bodies (elastic, semi-elastic, and plastic ones) [3-6, 7-8, 9-11, 12-14, 17-20, 23-26]. In [14], extensive information is given, mainly on the theory of the impact of elastic bodies. At that, great attention is paid to Hertz's theory of considering local effects that arise in the vicinity of the points of contact between the surfaces of colliding bodies. Some semi-empirical solutions on the longitudinal impact of semi-elastic and elastic-plastic rods are also given. In [7], exact analytical solutions to the problems of impact and collision were obtained, taking into account friction on the lateral surface of the rod: longitudinal impact on a semi-infinite rod; the Saint-Venant problem; impact on a semi-infinite elastic-plastic rod by

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instantaneously imparting constant deformation and constant velocity to the end of the rod. Nonstationary motions of viscous-plastic bodies and viscous-plastic fluid flows were considered in [6, 7-8, 10, 12, 17-21, 28]. In [28], an analysis of exact and approximate solutions to nonstationary problems of viscous-plastic flows was given.

1.1 Statement of the problem of the collision of a rigid body and a viscoplastic rod

In this article, the authors study the collision problem between an absolutely rigid body and a rod of finite length l made of a viscous-plastic incompressible material moving towards each other. Time t is counted from the moment of impact, and the Ox-axis is directed along the axis of the rod. Let the rod move translationally in the direction of its axis with velocity $-v_{OC}$ before the impact, and the body moves with constant velocity v_T in the positive direction of the Ox-axis. The part of the surface of the colliding body and the section of the rod $(x=0)$ that come into contact are assumed plane and perfectly smooth. We consider one-dimensional motion; that is, stresses, velocity and other parameters are considered to be averaged over the section of the rod. As is known, the equation of motion has the following form [15].

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x}, \tag{1}$$

where $v(x,t)$ is the velocity of motion of the rod sections, $\sigma = \sigma(x,t)$ is the stress, and ρ is the density of the rod material.

It is assumed that the constitutive ratio of the material of the viscous-plastic rod has the form [7, 10]:

$$\frac{\partial v}{\partial x} = \begin{cases} \dfrac{\sigma + \sigma_0}{\mu}, (|\sigma| \geq \sigma_0) \\ 0, \quad (|\sigma| \leq \sigma_0) \end{cases}, \tag{2}$$

where $\sigma > 0$ is the ultimate stress, μ is the viscosity coefficient of the material.

In the case of an instantaneous impact, due to (1), the rod at $t>0$ is divided into two parts: the part of the rod adjoining the impacted end $(0 \leq x \leq x_0(t), x_0(0) = 0)$, where the viscous-plastic flow occurs, and the remaining part $(x_0(t) \leq x \leq l)$, which moves as a non-deformable rigid body. The equations of motion in these regions, according to (1) and (2), have the following form [7, 10]

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}, \quad a^2 = \frac{\mu}{\rho}, \quad 0 < x < x_0(t), t > 0, \tag{3}$$

$$\frac{\partial v}{\partial x} = 0, \qquad x_0(t) < x < l, t > 0. \tag{4}$$

from (4), it follows that in the rigid region.

$$v=-v_0(t),\ \left(x_0(t)\le x\le l,\ x(0)=0\right),\ v_0(0)=v_{0c} \tag{5}$$

where $-v_0(t)$ is the velocity of motion of the rigid region of the rod to be determined.
The equation of motion of a rigid region has the form

$$M_0\frac{dv_0(t)}{dt}=\sigma(x_0(t)+0,t)\cdot \mathrm{F}_c,\quad M_0=\rho\left(l-x_0(t)\right), \tag{6}$$

where M_0 is the mass of the rigid region of the rod, F_c is the cross-sectional area of the rod.

It is required to find a solution to equations (3) and (5) and function $x_0(t)$- the interface between the above two parts of the rod that satisfy the following conditions:

$$M_T\frac{dv}{dt}=F_c\cdot\sigma \quad \text{for } x=0,\ t>0;\ v(0,0)=v_T, \tag{7}$$

$$v(x_0(\mathrm{t})-0,t)=v(x_0(\mathrm{t})+0,t)=-v_0(\mathrm{t}),\ t>0;\ v_0(0)=v_{0c}, \tag{8}$$

$$\sigma(x_0(\mathrm{t})-0,t)=\sigma(x_0(\mathrm{t})+0,t)=-\sigma_0, \tag{9}$$

$$\frac{\partial v(x_0(\mathrm{t})-0,t)}{\partial x}=0, \tag{10}$$

where M_T is the mass of the rigid body.

2 Methods

We introduce dimensionless variables:

$$u(\xi,\tau)=\frac{v(x,\mathrm{t})}{v_T},\ \xi=\frac{x}{l},\ \tau=\frac{a^2t}{l^2},\ \xi_0(\tau)=\frac{x_0(t)}{l},\ u_0(\tau)=\frac{v_0(t)}{v_T}.$$

In this case, equations (3), (6), and conditions (7)-(10) are reduced to the forms (11), (12), and (13) - (15)

$$\frac{\partial u}{\partial\tau}=\frac{\partial^2 u}{\partial\xi^2},\ 0<\xi<\xi_0(\tau), \tag{11}$$

$$\frac{du_0}{d\tau}=-\frac{s}{1-\xi_0(\tau)},\ \tau>0, \tag{12}$$

$$\left.\frac{\partial u}{\partial\tau}\right|_{\xi=0}=-2m\left.\frac{\partial u}{\partial\xi}\right|_{\xi=0}-ms,\ u(0,0)=1, \tag{13}$$

$$u_0(0) = \frac{v_{0c}}{v_T} = V_0\,, \qquad (14)$$

$$u(\xi_0(\tau),\tau) = -u_0(\tau), \quad \frac{\partial u(\xi_0(\tau),\tau)}{\partial \xi} = 0, \qquad (15)$$

where $s = \dfrac{\sigma_0 l}{\mu v_T}$ is the Saint-Venant parameter, $m = \dfrac{\rho F_c l}{M_T} = \dfrac{M_c}{M_T}$; condition (9) is taken into account in (12), and the defining relation (2) is taken into account in (13).
Thus, the impact problem under consideration was reduced to solving a nonlinear problem with an unknown moving boundary - equations (11) and (12) with conditions (13) - (15). The formulated problem (11) - (15) is solved, as noted above, by the method of integral relations [8]. In this case, the solution to equation (11) is sought in the form of a polynomial, which for the *n*-th approximation has the following form

$$u_n(\xi\,,\tau) = \sum_{i=0}^{n+1} a_i(\tau)\left(\frac{\xi}{\xi_0(\tau)}\right)^i, n = 1,2,3,... \qquad (16)$$

where $a_i(\tau), (i = 0,1,2,..n)$ and $\xi_0(\tau)$ are the functions to be determined.
In the case of the first approximation, the sought-for solution, according to (16), has the following form

$$u_1(\xi\,,\tau) = a_0(\tau) + a_1(\tau)\frac{\xi}{\xi_0(\tau)} + a_2(\tau)\left(\frac{\xi}{\xi_0(\tau)}\right)^2, \qquad (17)$$

where functions $\xi_0(\tau)$ and $a_0(\tau), a_1(\tau),\ a_2(\tau)$ are to be defined. According to the method of integral relations, the desired solution satisfies equation (11) on average, i.e., following the integral relation

$$\int_0^{\xi_0(\tau)} \left[\frac{\partial u_1(\xi,\tau)}{\partial \tau} - \frac{\partial^2 u_1(\xi,\tau)}{\partial \xi^2}\right] d\xi = 0 \qquad (18)$$

Satisfying conditions (15), we find

$$a_1 = -2(a_0 + u_0),\ a_2 = a_0 + u_0\,. \qquad (19)$$

Further, satisfying condition (13), we obtain the following equation

$$\frac{da_0}{d\tau} = -2m\frac{a_0 + u_0}{\xi_0} - ms. \qquad (20)$$

Substituting (17) and (19) into (18) and after performing the integration, taking into account the second condition (15), we obtain the equation

$$(a_0 + u_0)\frac{d\xi_0^2}{d\tau} = 12(a_0 + u_0) + 2\xi_0^{\ 2}\frac{d}{d\tau}(2u_0 - a_0). \qquad (21)$$

Thus, the problem of finding an approximate solution to problem (11)-(15) was reduced to solving the Cauchy problem for a system of three nonlinear equations of the first order (12), (20), and (21) under the following initial conditions

$$\xi_0^{\ 2}(0) = 0,\ a_0(0) = u(0,0) = 1,\ u_0(0) = V_0\ (V_0 = \frac{v_{0c}}{v_T} > 0). \qquad (22)$$

This problem can be solved by the method of successive approximations by reducing the problem (12), (20) - (21) to an equivalent system of integral equations.
At the same time, using the contraction mapping principle [22], it is easy to prove that the following assertion holds: for small values τ, there is a unique solution to this system of integral equations.

3 Results and Discussion

Numerical calculations can be carried out by the method of successive approximations. However, the resulting error may not be small enough. Then, to obtain a solution with an allowable error, a large number of iterations may be required, the implementation of which is associated with cumbersome calculations. In this regard, the Cauchy problem for nonlinear differential equations (12), (20) - (21) is solved by the Runge-Kutta method of fourth-order accuracy using the Maple system. The calculation results are shown in Figs.1 and 2 for the case $u(0,0) = a_0(0) = 1$, $u_0(0) = V_0 = 1$.

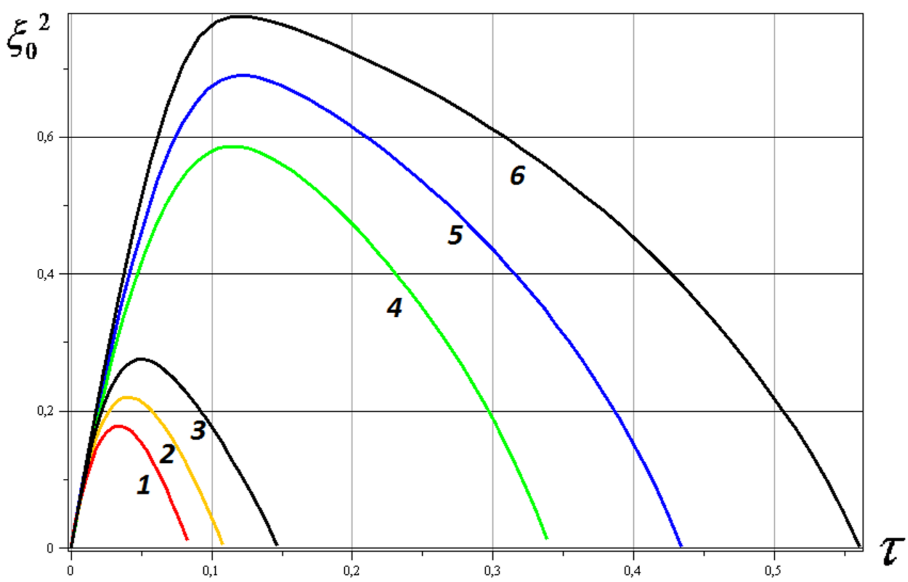

Fig. 1. Calculation results for three values of parameter $m = \frac{M_c}{M_T}$:

1 – m=1.5, 2 – m=1, 3 – m=0.5 for fixed Saint-Venant parameter *s=5;*
4 – m=1.5, 5 – m=1, 6 – m=0.5 for fixed Saint-Venant parameter *s=0.5.*

As seen from the figure, ξ_0 increases $\left(\frac{d\xi_0}{d\tau} > 0\right)$ (the viscous-plastic region $(0 < \xi < \xi_0)$ expands) up to certain time $\tau = \tau_M$. Then ξ_0 decreases and at some time $\tau = \tau_0 > \tau_M$ becomes zero (the viscous-plastic region disappears). In this case, the influence of the change in parameter $m = M_c / M_T$ - the ratio of the mass of the rod to the mass of the rigid body on the value of $\tau = \tau_M$ - the time the viscous-plastic strain region reaches the maximum size, is insignificant. The time of reaching the maximum size of the viscous-plastic strain region is practically the same for all values of *m*. However, time τ_0 of vanishing $(\xi_0 = 0)$ of the viscous-plastic strain region depends strongly on the parameter $m = M_c / M_T$, namely, the duration of the existence of the viscous-plastic strain region decreases quite significantly with an increase in *m:*

$\tau_0 = 0.5702$ for *m=0.5*; $\tau_0 = 0.2859$ for *m=1.5*.

As seen in Fig. 1, the Saint-Venant parameter also has a significant effect on the duration of the existence of a viscous-plastic strain region. The larger the parameter *s*, the shorter the duration of the existence of the region of the rod where viscous-plastic strain occurs:

τ_0 *=0.5274* for *s=0.5*; τ_0 *=0.1386* for *s=5*.

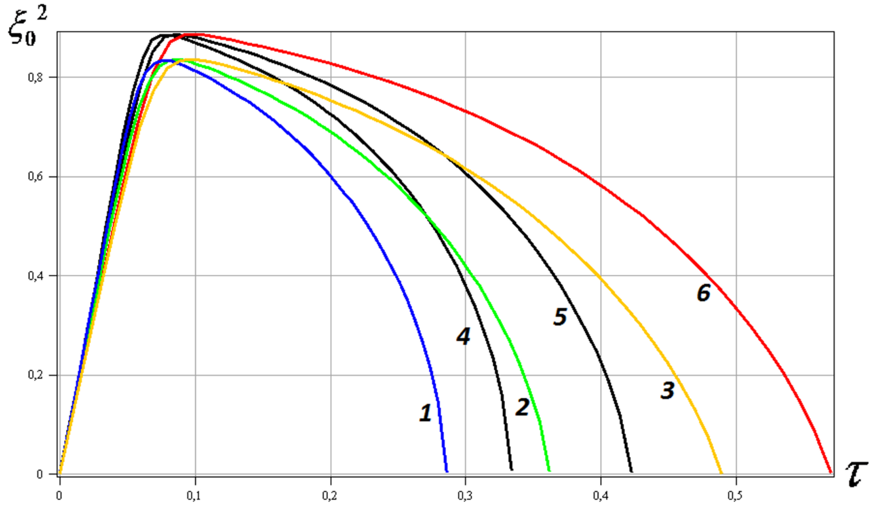

Fig. 2 Calculation results for two parameter values s=0.5 and s=5:
1 – *m=1.5 ; 2 – m=1, 3 – m=0.5* when V_0*=2; 4 – m=1.5 ; 5 – m=1, 6 – m=0.5* when V_0*=1.*

As seen from Fig.2, the larger the parameter *m*, and vice versa, the smaller *Vo*, the shorter the duration of viscoplastic deformation. At the same time, the influence of these parameters on the time to reach the maximum length of the viscoplastic deformation section is insignificant.

The pattern of the influence of the Saint-Venant parameter is similar for different initial velocities of the rod and rigid body. For example, for $V_0 = 0$, i.e., the impact of a rigid body on a fixed rod, the pattern of the influence of the Saint-Venant parameter is similar to the case for $V_0>0$ (Figs.1 and 2), but $\xi_0^2(\tau_M)$ and the duration τ_0 of the existence of a viscous-plastic region is less than in the case when the rigid body and the rod move towards each other at constant velocity $(V_0 > 0)$ (Table 1).

Table 1. Change in $\xi_0^{\ 2}(\tau_M)$ and τ_o depending on τ_M and s for *m=1*.

s	0.5	0.5	0.5	5	5	5
Vo	0	1	1.5	0	1	1.5
$\xi_0^{\ 2}(\tau_M)$	0.7124	0.8343	0.8653	0.2231	0.3488	0.3962
τ_0	0.2659	0.3674	0.3674	0.0613	0.1025	0.1194
τ_M	0.08	0.0847	0.086	0.03	0.045	0.051

Here, for *s=0.5* and *s=5,* the greater $V_0 = v_{0c}/v_T$, the greater $\xi_0(\tau_M)$ and τ_0. However, the time to reach the maximum size of the viscous-plastic strain region τ_M increases with an increase in *Vo* only for *s=5* and practically does not change for *s=0.5.*

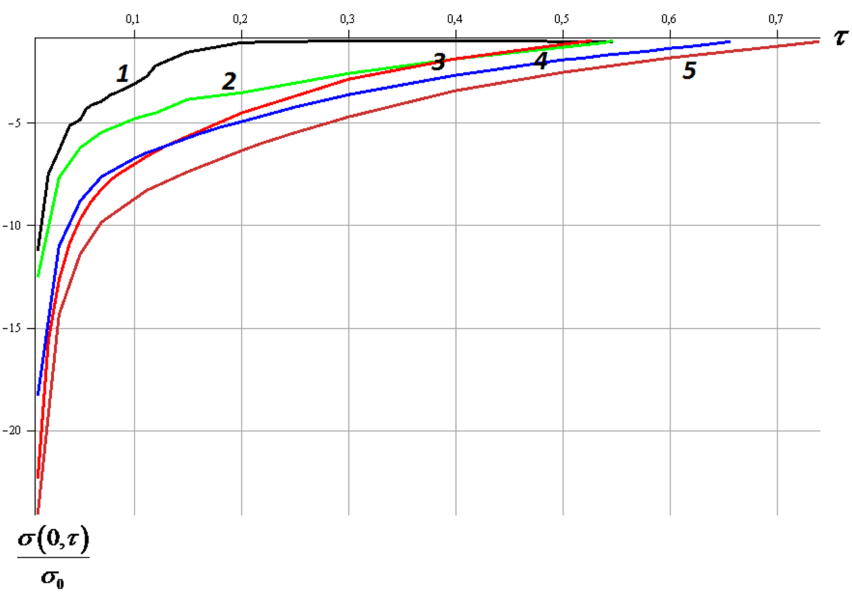

Fig. 3. Change in stresses $\sigma\left(\xi_0\left(\tau\right),\tau\right)$ at different values of parameter *m* and velocity V_0 for *a(0)=1, s=0.5:* curve 1 – (*m=1,* V_0*=0)* , curve 2 - (*m=0.5* , V_0*=0*), curve 3 - (*m=0.5* , V_0*=1)*, curve 4 - (*m=1* , V_0*=0.5*), curve 5 - (*m=1* , V_0*=1*).

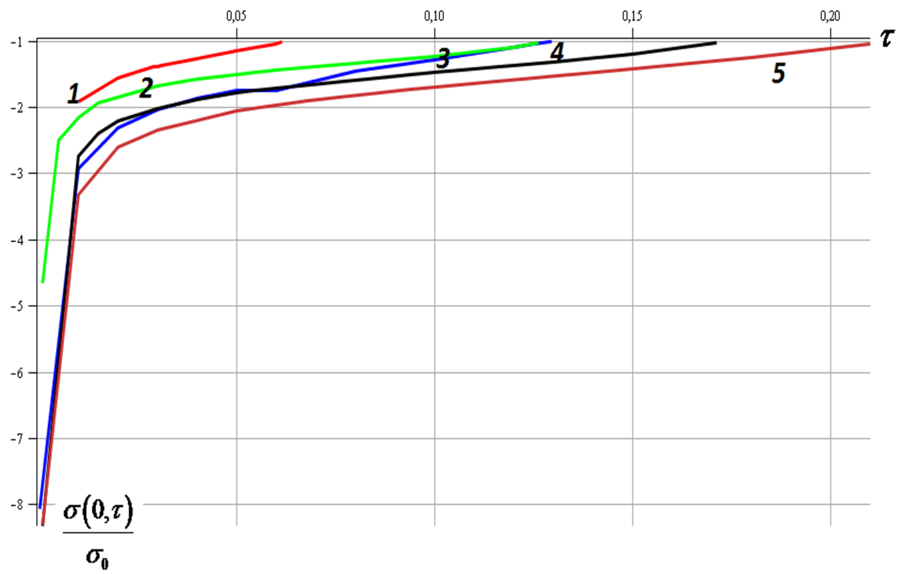

Fig. 4. Change in stress $\sigma\left(\xi_0\left(\tau\right),\tau\right)$ at different values of parameter *m* and velocity V_0 for *a(0)=1, s=5*: curve 1 - (*m=1,* $V_0=0$); curve 2 - (*m=0.5,* $V_0=0$); curve 3 - (*m=0.5 ,* $V_0=1$); curve 4 - (*m=1,* $V_0=0.5$); curve 5 - (*m=1 ,* $V_0=1$).

Figures 3 and 4 show graphs of the change in stresses relative to its threshold value $\left(\frac{\sigma(0,\tau)}{\sigma_0}\right)$ at the end of the rod. In all cases shown in Figs.3 and 4, the stress is $\sigma(0,\tau)\to-\infty\ as\ \tau\to 0$ and $\frac{\sigma(0,\tau)}{\sigma_0}\to-1\ as\ \tau\to\tau_0$. For the cases *s=0.5* and *s=5,* curves 1 and 2 intersect. This is explained for *s=0.5* by a decrease in the duration τ_0 of the existence of the viscous-plastic strain region with an increase in parameter *m* (Fig.1 and 2), and for *s=5* - by an increase in duration τ_0 with an increase in $V_0 = \frac{v_{0c}}{v_T}$ (Table 1). The intersection of the curves that show the relative stresses can occur at a small increase in these parameters.

Graphs of the change in velocity ($a_0\left(\tau\right)$) of the left end of the rod and velocity ($-u_0\left(\tau\right)$) of section $\xi=\xi_0\left(\tau\right)$ for different values of *Vo* $\left(a_0\left(0\right)=1\right)$ are shown in Figs.5 (*s=0.5, m=1*) and 6 *(s=5, m=1).*

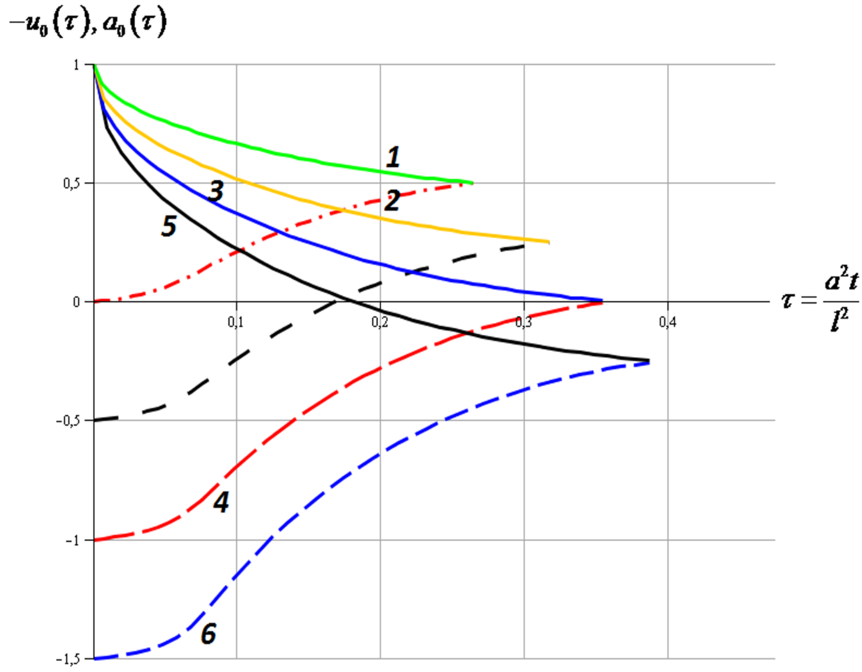

Fig. 5. Graphs of functions $a_0(\tau)$ (curves - 1,3,5) and $-u_0(\tau)$ (curves - 2,4,6) for s=0.5 ,m=1, $a_0(0)=1$: 1 and 2 (Vo=0); 3 and 4 (Vo=1); 5 and 6 (Vo=1.5) for s=0.5.

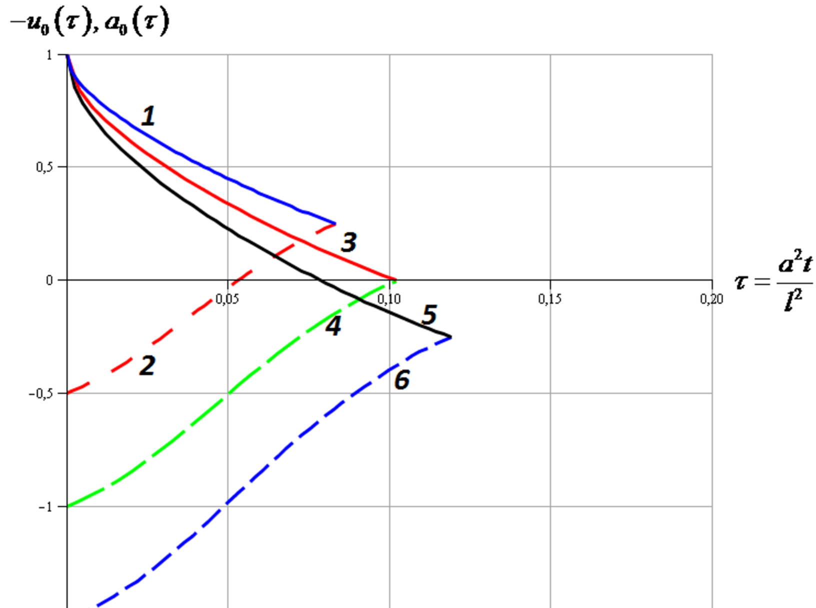

Fig. 6. Graphs of functions $a_0(\tau)$ (curves -1,3,5) and $-u_0(\tau)$ (curves -2,4,6) for *s=5*, *m=1*, *$a_o(0)=1$*:1 and 2(*Vo=0.5)*; 3 and 4 (*Vo=1)*; 5 and 6 (*Vo=1.5)* for *s=5*.

The closure of the curves represents, respectively, the functions $a_0(\tau)$ and $-u_0(\tau)$ occurs at the point $(\tau_0, a_0(\tau_0) = -u_0(\tau_0))$, i.e., at the moment of disappearance of the viscoplastic deformation area.

3.1 Verification of the proposed method

According to the studies in [7-8, 10], in which various problems were solved by the method of integral relations, satisfactory results were already obtained in the first approximation. This is confirmed by the following test example, which has an exact solution. The following function

$$v(\xi,\tau) = erfc(\frac{\xi}{2\sqrt{\tau}}), \quad \text{erfcz} = \frac{2}{\sqrt{\pi}}\int_z^\infty \exp(-x^2)dx \tag{23}$$

is an exact solution to equation (11) that satisfies the following initial and boundary conditions:

$$v(\xi, 0) = 0, 0 < \xi < \infty; \tag{24}$$

$$v(\xi, 0) = 1, v(\xi, \tau); \rightarrow 0 \; for \; \xi \rightarrow \infty, \tau > 0. \tag{25}$$

The first approximation, according to (17), has the following form

$$v_1(\xi \; ,\tau) = a_0(\tau) + a_1(\tau)\frac{\xi}{\xi_0(\tau)} + a_2(\tau)(\frac{\xi}{\xi_0(\tau)})^2, \tag{26}$$

Satisfying conditions (23) and (24), we obtain $a_0(\tau) = 1, \; a_1(\tau) = -2, \; a_2(\tau) = 1$.
In this case, equation (20) and its solution - the first approximation (26) - take the following form, respectively

$$\frac{d\xi_0^{\;2}}{d\tau} = 12, m.e. \; \xi_0^{\;2} = 12\tau, \tag{27}$$

$$v(\xi,\tau) \approx v_1(\xi \; ,\tau) = 1 - 2\frac{\xi}{\sqrt{12\tau}} + \frac{\xi^2}{12\tau}. \tag{28}$$

Figure 7 compares the graphs for the exact solution (23) with the first approximation (28). As follows from Fig.7, the first approximation agrees with the exact solution.

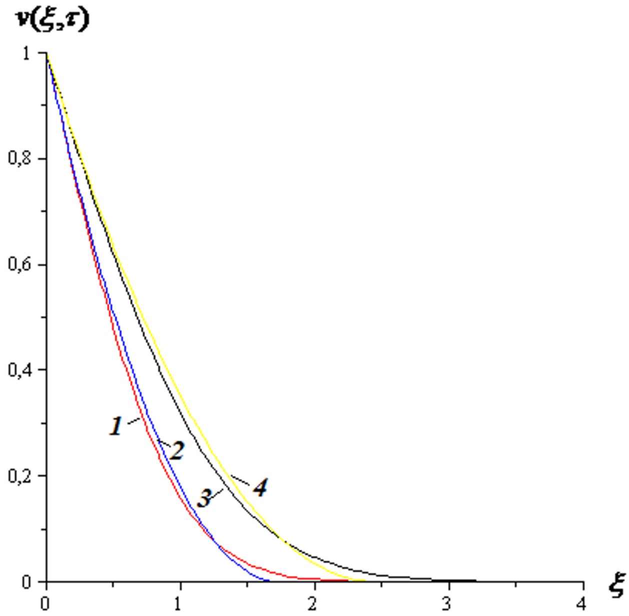

Fig.7. Graphs of $v(\xi,\tau)$ depending on ξ : 1 - exact solution (23), 2 - first approximation (28) for $\tau = 0.25$; 3 - exact solution (23), 4- first approximation (28) for $\tau = 0.5$.

4 Conclusions

1. The statement of the problem of the impact of a rigid body and a viscous-plastic rod was formulated.
2. Using the method of integral relations - a modification of the Karman-Pohlhausen method, the problem posed was reduced to the Cauchy problem for three nonlinear differential equations of the first order.
3. The results of calculations are shown in the form of graphs. A qualitative and quantitative analysis of the numerical results obtained was conducted.

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