Pulsating flows of viscous fluid in flat channel for given harmonic fluctuation of flow rate

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Abstract. In research work, it is pointed out the issue of a pulsating flow of a viscous incompressible fluid in a flat channel for a given harmonic fluctuation of the fluid flow rate. The study of the generation of pulsating current is used in biological mechanics, in particular, in the use of microchip systems. In addition, to ensure a constant flow of liquid, pneumatic micropumps that periodically squeeze liquid from empty volumes are widely used. In such systems, the installation of pulsating flow is shown to be economically beneficial. The transfer function of the amplitude-phase rate response is determined; with the help of these functions, the ratio of the tangential shear pressure on the channel fence to the average acceleration over the channel section is determined. In Figure 1, it can be seen that the ratio of $K_и = 0$ in $\frac{\tau_{нс}}{\tau_{0кс}}$ is close to one and α_0^2 is less than one. If α_0^2 takes on values greater than unity, then the ratio of $K_и = 0$ to $\frac{\tau_{нс}}{\tau_{0кс}}$ is greater than unity, and it has been shown to increase with increasing frequency of dimensionless oscillation. It was shown that in an unsteady flow of liquid, even in cases where liquid acceleration is equal to zero, it was studied that the stress on the wall of the channel exceeds its quasi-stationary value. $\frac{\tau_{нс}}{\tau_{0кс}}$ with the increase of the ratio $K_и$, the increase of the parameter is explained by the fact that the change of the tensile stress on the wall advances in phases concerning the average speed along the section. Calculations determine that the non-stationary shear pressure on the channel fence increases non-monotonically with the acceleration of the liquid particle at low oscillation frequencies. The shear pressure reaches its maximum value, then decreases with increasing dimensionless oscillation rate, and asymptotically approaches the values without accelerated flow.

1 Introduction

The study of the pulsating flow of a viscous incompressible fluid in a flat channel for a given harmonic fluctuation of the flow rate can be applied in biological mechanics,

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particularly for the operation of microchip systems [1-3]. These systems are designed to diagnose the functioning of various human organs and target drug delivery to them. In addition, to ensure a constant flow of liquid, pneumatic micro pumps are often used in biomedical arrangeations with a periodic displacement of liquid from free volumes [4, 5]. In such systems, arranging with a pulsating flow rate may be economically advantageous. As the authors know, at present, there is no information on the effect of flow rate pulsations on fluctuations in the coefficients of hydraulic resistance and friction resistance.

However, these studies are essential for calculating the pressure gradient and other hydrodynamic characteristics that have special space in some biomedical and other technological studies [1-3]. Thus, studying the characteristics of the tangential shear pressure on the fence during an oscillatory flow of a viscous fluid and other flow parameters is of great importance.

Pulsating flows of viscous fluids in rigid and elastic pipes were studied in the works of B.C.Gromeka [6]. In them, he determined the propagation velocities of the pressure pulse wave and their attenuation. Later, the oscillatory flow of a viscous fluid in a pipe was studied in the work of I.B.Crendal [7]. He, solving the issues of the oscillatory flow of a viscous fluid in an endless round pipe, derived formulas for determining the acceleration profile, fluid flow, and impedance during the propagation of a sinusoidal pressure wave. A few years later, P.Lambosii published his findings in [8], where he obtained the same acceleration profile, and he also calculated the viscosity resistance.

J.R.Uomersley in [9] re-deduced P.Lambosia's solution. His distinctive qualitative results were that a phase shift between pressure fluctuations and fluctuations on average acceleration over the section was detected, and secondly, a non-monotonic distribution of acceleration profiles formed.

For the first time, the influence of superimposed fluctuations of the average acceleration over the cross-section on the flow of a viscous fluid with a laminar flow in a pipe was published in an experimental work [10]. The so-called "annular effect" of Richardson was obtained at relatively high oscillation frequencies, which appear as a maximum on the profile of the oscillating component of the longitudinal acceleration in a narrow near-fence layer, the thickness of which decreases with increasing oscillation rate. In the rest of the pipe, the liquid oscillates as a whole following the fluctuation of the average acceleration over the section. In [11], experiments were also carried out on pipes with an internal diameter of 40 mm, in which the piston creates harmonic changes in the fluid flow rate near zero. This graph pointed out obtained from oscillograms, on which local velocities were recorded using an electrothermal anemometer at various points in the pipe section. It can be seen from the graphs that the local accelerations have the maximum values near the fence. These experimental results agree with the results of [10]. Unsteady pulsating flows of a viscous fluid in a round cylindrical pipe of infinite length under a harmonic changing pressure gradient were studied in [12, 13]. The calculation formulas were obtained for determining the distribution of acceleration and fluid flow by solving the issue. Numerical calculations have shown that in a pulsating flow at lower values of the dimensionless oscillation rate, the acceleration, flow rate, and other hydrodynamic parameters from the zero initial state are established slowly, relatively at high oscillation frequencies, and close to the parameters of the establishment of a non-pulsating flow. In an oscillating flow at high oscillation frequencies, these parameters are set almost instantly. Pulsating flows of a viscous incompressible fluid were studied in [14, 15] in a rectangular channel. The issue is solved by the finite difference method. The optimal parameters of the difference scheme are determined, and data are obtained on the amplitude and phase of oscillations of the longitudinal acceleration, the hydraulic resistance coefficient, and other flow parameters. At low oscillation frequencies, it is shown that all hydrodynamic parameters fluctuate according to the laws of the average acceleration over the cross-section. And at high values of oscillation frequencies, the dependences of the hydrodynamic quantities on the dimensionless oscillation rate have the same change in character for various rectang1``ular channels. The influence of the aspect ratio of a rectangular channel on the hydrodynamics of a pulsating flow is also analyzed.

In the process listed above, the field of fluid acceleration is mainly studied for various modes of change in the pressure gradient. The change in tangential and normal pressure arising during movement has been studied relatively little. In most cases, fluids were replaced by a sequence of flows with a quasi-stationary distribution of hydrodynamic quantities in hydrodynamic models of unsteady flows. However, the structure of unsteady flows differs from the structure of stationary flows, and in such cases, such a replacement should be justified in each specific case. The legitimacy of studying quasi-stationary characteristics for determining the field of shear pressure in a non-stationary flow of a viscous fluid is far from being resolved.

In this paper, we study pulsating flows of a viscous fluid in a flat channel when harmonic oscillations of the fluid flow rate are superimposed on the flow.

2 Method

Consider the issue of solving a slow pulsating flow of a viscous incompressible fluid between two fixed parallel planes extending in both directions to limitless. Let us denote the distance between the fences as $2h$. Axis OX runs horizontally in the middle of the channel along the flow. Axis OX is directed perpendicular to the OY axis.

The fluid flow occurs symmetrically along the channel axis. Then the differential equation of motion of a viscous incompressible fluid has the following form [16-21].

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

where u is longitudinal speed; p is pressure; ρ is density; μ is dynamic viscosity; t is time.

The oscillatory flow of a viscous fluid occurs due to a given harmonic fluctuation of the fluid flow rate or the averaged longitudinal acceleration over the channel section.

$$Q = a_Q cos\omega t = \mathrm{Re}\, a_Q e^{i\omega t},\ < u >= a_u cos\omega t = \mathrm{Re}\, a_u e^{i\omega t} \quad (2)$$

where a_Q and a_u are the amplitudes of the liquid flow rate and the amplitudes of the averaged longitudinal acceleration over the channel section.

In this case, the flow occurs symmetrically along the channel axis, and in this regard, the no-slip condition is satisfied in the channel fence, then the longitudinal acceleration on the channel fence is equal to zero. Then the boundary conditions will be:

$$u = 0 \text{ at } y = h,\ \frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \quad (3)$$

Due to the linearity, the equation (1) of the longitudinal acceleration, pressure, and shear pressure on the fence can be written as follows

$$u(y,t) = \mathrm{Re}\, u_1(y) e^{i\omega t},\ p(x,t) = \mathrm{Re}\, p_1(x) e^{i\omega t},\ \tau(t) = \mathrm{Re}\, \tau_1 e^{i\omega t} \quad (4)$$

Substituting these expressions into equation (1), we obtain

$$\frac{\partial^2 u_1(y)}{\partial y^2} - \frac{\rho i\omega}{\mu} u_1(y) = \frac{1}{\mu}\frac{\partial p_1(x)}{\partial x} \tag{5}$$

The solutions of equation (5), taking into account the boundary conditions (3), has the form

$$u_1(y) = \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\cos\left(i^{3/2}\alpha_0 \frac{y}{h}\right)}{\cos\left(i^{3/2}\alpha_0\right)}\right) \tag{6}$$

where $\alpha_0 - \sqrt{\frac{\omega}{2}}h$ is the Uomersley vibrational number (dimensionless vibration rate). Using the equation

$$\tau_1 = -\mu \frac{\partial u_1(y)}{\partial y}\Big|_{y=h} \tag{7}$$

sort out the tangential shear pressure on the fence

$$\tau_1 = -h\left(-\frac{\partial P}{\partial x}\right)\frac{1}{i\alpha_0^2}\left(\frac{i^{3/2}\alpha_0 \sin\left(i^{3/2}\alpha_0\right)}{\cos\left(i^{3/2}\alpha_0\right)}\right) \tag{8}$$

Now I will integrate both parts of the formula (6) concerning the variable y in the range from $-h$ to h; we will find the formulas for the fluid flow

$$Q_1 = 2h\left[\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\sin\left(i^{3/2}\alpha_0\right)}{\left(i^{3/2}\alpha_0\right)\cos\left(i^{3/2}\alpha_0\right)}\right)\right] \tag{9}$$

Taking into account in formula (9) $Q_1 = 2h < u_1 >$, we find the average longitudinal acceleration over the channel section

$$< u_1 > = \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1 - \frac{\sin\left(i^{3/2}\alpha_0\right)}{\left(i^{3/2}\alpha_0\right)\cos\left(i^{3/2}\alpha_0\right)}\right) \tag{10}$$

Here $\rho i\omega$ can be written in the form

$$\rho i\omega = i\frac{\omega}{\nu}h^2 \cdot \frac{\mu}{h^2} = i\alpha_0^2 \frac{\mu}{h^2} \tag{11}$$

Taking into account (11), formula (10) takes the form:

$$< u_1 >= -\frac{h}{3\mu}\tau_1 \cdot \frac{3i^{\frac{3}{2}}\alpha_0 \cos\left(i^{\frac{3}{2}}\alpha_0\right) - \sin\left(i^{\frac{3}{2}}\alpha_0\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)^2 \sin\left(i^{\frac{3}{2}}\alpha_0\right)} \tag{12}$$

Using the formula (12), we contemplate the transfer function $W_{\tau,u}(i\omega)$ for the shear pressure on the fences, as

$$W_{\tau_1,u_1}(i\omega) = \frac{\tau_1(i\omega)}{u_1(i\omega)} \tag{13}$$

From equation (13), we get

$$W_{\tau_1,u_1}(i\omega) = \frac{h}{3\mu}\frac{\tau_1(i\omega)}{< u_1(i\omega) >} = -\frac{\left(i^{\frac{3}{2}}\alpha_0\right)^2 \sin\left(i^{\frac{3}{2}}\alpha_0\right)}{3(i^{\frac{3}{2}}\alpha_0 \cos\left(i^{\frac{3}{2}}\alpha_0\right) - \sin\left(i^{\frac{3}{2}}\alpha_0\right))} \tag{14}$$

The transfer function (14) is sometimes called amplitude-phase rate response (AFCH). These functions make it possible to determine the dependence of the shear pressure on the channel fence on time for a given law of change in the average longitudinal acceleration over the channel section.

3 Results and Discussion

To determine the dependence of the shear pressure on the channel fence in a non-stationary flow of averages over the longitudinal acceleration section, we use the transfer function (14). In this regard, we consider the law of the change in the average longitudinal acceleration over the channel section $< u_1 >= a_{u_1} \cos\omega t$. Where a_{u_1} is the amplitude of the averaged longitudinal acceleration over the channel section. Using the formulas for the averaged longitudinal acceleration over the channel section, it is possible to determine the dependence of the shear pressure on the fence between the averaged longitudinal acceleration over the channel section.

In this case, the change in shear pressure on the fence is determined as follows.

$$\tau_1 = a_{\tau_1} \cos(\omega t + \varphi_{\tau_1}) \tag{15}$$

Where a_{τ_1} is amplitude of shear pressure on the channel fence; φ_{τ_1} is phase shift between the value τ_1 and $< u_1 >$. Using the ratio

$$\cos(\omega t + \varphi_{\tau_1}) = \cos\omega t \cos\varphi_{\tau_1} - \sin\omega t \sin\varphi_{\tau_1} .$$

And considering that $\frac{\partial < u_1 >}{\partial t} = -a_{u_1}\omega \sin\omega t$ we bring equation (15) to the form.

$$\tau_1 = (\frac{a_{\tau_1}}{a_{u_1}}\cos\varphi_{\tau_1}) < u_1 > + (\frac{a_{\tau_1}}{a_{u_1}}\sin\varphi_{\tau_1})\frac{1}{\omega}\frac{\partial < u_1 >}{\partial t} \quad (16)$$

The values $(\frac{a_{\tau_1}}{a_{u_1}}\cos\varphi_{\tau_1})$ and $\frac{a_{\tau_1}}{a_{u_1}}\sin\varphi_{\tau_1}$ are, respectively, the real and imaginary parts of the transfer function (14); therefore, from (14), we obtain

$$W_{\tau_1,u_1} = -\frac{1}{3}(\frac{(i^{3/2}\alpha_0)^2\sin(i^{3/2}\alpha_0)}{i^{3/2}\alpha_0\cos(i^{3/2}\alpha_0) - \sin(i^{3/2}\alpha_0)}) = \chi + \beta i \quad (17)$$

Here $\chi = (\frac{a_{\tau_1}}{a_{u_1}}\cos\varphi_{\tau_1})$, $\beta = \frac{a_{\tau_1}}{a_{u_1}}\sin\varphi_{\tau_1}$.

Then (17) the formula takes the form

$$\frac{h}{3\mu}\frac{\tau_1}{< u_1 >} = W_{\tau_1,u_1} = \chi + \beta\frac{1}{\omega}K_н \quad (18)$$

Here, $K_н = \frac{\partial < u_1 >}{< u_1 > \partial t}$ χ and β are dimensionless quantities, t is dimensional quantities, so it must be converted to a dimensionless form using the $t = \frac{h^2\rho}{3\mu}t^*$ transformation.

Taking this into account in (18), we obtain the calculation formula.

$$\frac{\tau_{нс}}{\tau_{0кс}} = \chi + \frac{3\beta}{\alpha_0^2}K_н \quad (19)$$

Here $\tau_{0кс} = \frac{3\mu}{h} < u_1 >$ and $\tau_1 = \tau_{нс}$. Using the formula (19), we plot the change in the relative shear pressure on the fence in an unsteady flow depending on the dimensionless oscillation rate.

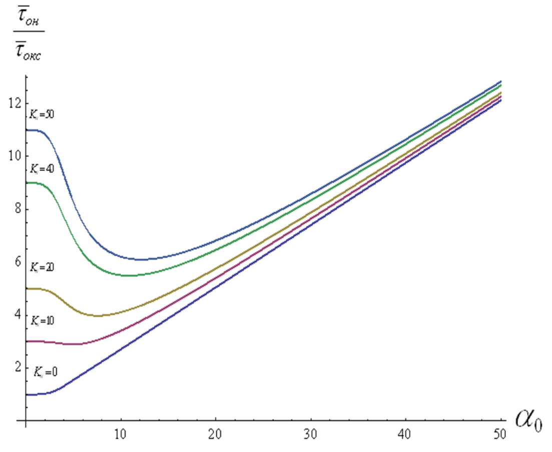

Fig. 1. Change in relative shear pressure in a non-stationary flow depending on the dimensionless oscillation rate at various values of fluid acceleration $K_н$.

The constructed graphs in Drawings-1 show that at $K_и = 0$ the ratio $\frac{\tau_{нс}}{\tau_{0кс}}$ is close to unity, while α_0^2 is less than one. If α_0^2 takes on greater values than unity, then even at $K_и = 0$, the ratio $\frac{\tau_{нс}}{\tau_{0кс}}$ becomes greater than unity and increases with increasing dimensionless oscillation rate.

This suggests that the shear pressure on the channel fence during unsteady fluid flow can exceed their quasi-stationary values even at those times when the fluid acceleration is zero. The ratio $\frac{\tau_{нс}}{\tau_{0кс}}$ increases with an increase in the $K_и$ parameter, which explains the change in shear pressure on the fence, which occurs with a phase advance compared to the average speed over the cross-section.

Therefore, the considered features in changes in the shear pressure on the fence for a given harmonic fluctuation of the flow rate are caused by violating the parabolic law of the distribution of local velocities over the channel section.

Calculations show that in the near-surface layer, the velocities change in phase with the change in shear pressure on the fence along the channel, while in the central part of the flow, they remain in half phase, with the phase of shear pressure on the fence.

4 Conclusions

Issues of a pulsating flow of a viscous incompressible fluid in a flat channel are solved for a given harmonic oscillation of the fluid flow rate. The transfer function of the amplitude-phase rate characteristics is determined with the help of these functions; the ratio of the tangential shear pressure on the channel fence to the average acceleration over the channel section is determined. Calculations show that the non-stationary shear pressure on the channel fence increases non-monotonically with the acceleration of the liquid particle at low oscillation frequencies. Reaching a maximum value, then decreasing with an increase in the dimensionless oscillation rate and asymptotically approaching the value without accelerated flow.

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