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Cross-diffusion systems with convective transport

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Abstract. The properties of cross-diffusion systems, which have double nonlinearity and include convective transfer, are investigated. This means that two factors are taken into account in the system: diffusion (random movement) and convection (transfer with the participation of the medium flow). The study of the properties of such systems makes it possible to understand how the interaction of these factors can influence the behavior of a population. The simulation of the processes of multicomponent cross-diffusion systems of a biological population with convective transfer on a computer is described. This means that with the help of numerical methods and computer models, models have been created that make it possible to simulate and study these systems. Such modeling helps to get an idea about the behavior of a cross-diffusion system under various conditions and system parameters. Estimates are obtained for solving the Cauchy problem of multicomponent cross-diffusion systems with convective transfer, which are analytical estimates of solutions. The study of the qualitative properties of the system made it possible to perform a numerical experiment depending on the values included in the system of numerical parameters.

1 Introduction

The utilization of mathematical models and their numerical solutions is an essential aspect of the rapid development of science and technology [1]. Applied mathematics investigates mathematical models for various physical and biological phenomena [2]. These models usually use linear differential equations with general solutions [3]. However, in practical situations, nonlinearities often arise, and we must use linear models to correctly depict the underlying physical processes [4-5]. Many practical problems are expressed as nonlinear differential equations or systems of equations with a specified product under boundary conditions [6]. These multidimensional problems can be solved analytically in some cases, but most of the time, we must rely on approximate solutions [7,8]. Approximate calculation methods can help us obtain solutions and analyze them, allowing us to better understand the processes under investigation [2,9]. In the numerical solution of mathematical physics problems, finite difference or grid methods are commonly used. The theory of numerical methods faces two primary issues [3]:

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Constructing a discrete (differential) approximation of the equations of mathematical physics, assessing the approximation error, investigating the stability of the differential scheme's solution, and the accuracy of the resulting differential scheme [10].
Solving the differential equation system by explicit (correct) or iterative methods while ensuring the efficiency of the computational algorithm [11].

2 Methods

When convective transport $u_i(t,x)(i=\overline{1,n})$ is present, for a system with n components, the local mass balance equations can be expressed as a system [1,12].

$$\frac{\partial u_i}{\partial t}=\nabla(u_{3-i}^{s_{3-i}}\nabla u_i)-div(l_i(t)u_i)+\varepsilon u_i^{p_i}u_{3-i}^{q_i}, \quad t>0, \quad x\in R^N, \; i=(1,2). \tag{1}$$

here $u_{3-i}^{s_{3-i}}$ is diffusion coefficients, $l_i(t)$ is speed of convective transfer, ε, p_i, q_i are positive numerical parameters, and solutions of the cross-diffusion system of the biological population.

Numerous studies have investigated different properties of system solutions, such as solution localization and asymptotic behavior of self-similar solutions. The numerical aspects of solving system (1) have been discussed in [3,13], while the properties of the solution have been studied using the method of nonlinear separation in [2,14]. In [1], the asymptotics of self-similar systems have been examined. In this study, we focus on the case where N = 2 and construct self-similar solutions of the system of equations for (1) as follows:

$$\frac{1}{2}j_i x\frac{df_i}{dx}+x^{1\text{-}N}\frac{d}{dx}\left(x^{N-1}f_{3-i}^{s_{3-i}}\frac{df_i}{dx}\right)+y_i(f_i^{p_i}f_{3-i}^{q_i}\text{-}f_i)=0, \tag{2}$$

$$u_i(t,x)=\overline{u_i}(t)\omega_i(\tau,x), \tag{3}$$

$$\omega_i(\tau,x)=f_i(\xi), \quad |\xi|=(\textstyle\sum_{i=1}^{N}(x_i-\int_0^t l_i(t)dt))^{\frac{1}{2}}/\tau^{\frac{1}{2}}(t). \tag{4}$$

Here functions

$$\bar{u}_i(t)=d_i(T\pm t)^{-\alpha_i}, \tag{5}$$

$$\frac{d\bar{u}_i}{dt}=\varepsilon\bar{u}_i^{p_i}\bar{u}_i^{q_i} \tag{6}$$

is chosen as the solution of the system of equations and

$$\tau=d_1^{s_1}\frac{(T\pm t)^{1-\alpha_1 s_1}}{\alpha_1 s_1-1}, \quad d_i=\left[\alpha_{3-i}^{q_i}\alpha_i^{1-p_{3-i}}\right]^{\frac{1}{2}}, \alpha_1 s_2-1>0, \sqrt{a^2+b^2},$$
$$\alpha_1 s_2=\alpha_2 s_1, \quad \varphi_i=d_1^{s_1}d_{3-i}^{-s_{3-i}}, \quad \psi_i=\frac{\alpha_i\varphi_i}{\alpha_i s_i-1}, \tag{7}$$
$$\alpha_i=(1-p_{3-i}+q_i)p^{-1}, \quad p=q_1q_2-(1-p_1)(1-p_2), \quad i=1,2$$

after modification

$$f_i(\xi)=\overline{f}_i(\xi)y_i(\eta)\ ,\ \eta=-\ln(a-\xi^2)\ ,\ \overline{f}_i(\xi)=A_i(a-\xi^2)^{\frac{1}{s_i}}\ . \tag{8}$$

We obtain the following expression from (1):

$$\left[y_i'+a_{i1}y_i\right]'+\left[a_{i2}+a_{i6}y_{3-i}^{-s_{3-i}}+a_{i3}a_{3-i,1}+a_{i3}y_{3-i}^{-1}y_{3-i}'\right]\cdot\left[y_i'+a_{i1}y_i\right]+ \tag{9}$$
$$+a_{i4}y_iy_{3-i}^{-s_{3-i}}+a_{i5}y_i^{p_i}y_{3-i}^{q_i-s_{3-i}}=0.$$

Here

$$a_{i1}=-\frac{1}{s_i},\ a_{i2}=1+a_{i1}+\frac{N}{2ae^{\eta}-2},\ a_{i3}=\frac{1}{2}s_{3-i},$$

$$a_{i4}=\frac{1}{4}\psi_iA_{3-i}^{-s_{3-i}}(ae^{\eta}-1)^{-1},\ a_{i5}=\frac{1}{4}\psi_iA_i^{p_i-1}A_{3-i}^{q_i-s_{3-i}}\frac{e^{-l_i\eta}}{a-e^{-\eta}},$$

$$a_{i6}=\frac{1}{4}\varphi_iA_{3-i}^{-s_{3-i}},\ l_i=1+\frac{q_i}{s_{3-i}}+\frac{p_i-1}{s_i},\quad i=1,2; \tag{10}$$

$$b_{i1}=\frac{2-s_i}{2s_i^2},\ b_{i2}=-\frac{1}{4s_i}\varphi_iA_{3-i}^{-s_{3-i}},$$
$$b_{i3}=\frac{1}{4a}\psi_iA_i^{p_i-1}A_{3-i}^{q_i-s}. \tag{11}$$

Equations for the existence of a solution of the system (9)

$$y_i(\eta)=y_i^0+o(1),\quad \eta\to+\infty,\ i=1.2,\ 0<y_i^0<+\infty \tag{12}$$

can be obtained by solving nonlinear algebraic systems in the form of y_i^0 (i=1,2), and the solutions:

$$\begin{array}{ll} 1. & l_i=0,\ b_{i1}+b_{i2}z_{3-i}^{-s_{3-i}}+b_{i3}z_i^{p_i-1}z_{3-i}^{q_i-s_{3-i}}=0,\\ 2. & l_1=0,\ l_2>0,\quad b_{11}+b_{12}z_2^{-s_2}+b_{13}z_1^{p_i-1}z_1^{q_1-s_2}=0,\ b_{21}+b_{22}z_1^{-s_1}=0,\\ 3. & l_1>0,\ l_2=0,\quad b_{11}+b_{12}z_2^{-s_2}=0,\ b_{21}+b_{22}z_1^{-s_1}+b_{23}z_1^{q_2-s_1}z_2^{p_2-1}=0,\\ 4. & l_i>0,\ b_{i1}+b_{i2}z_{3-i}^{-s_{3-i}}=0. \end{array} \tag{13}$$

Investigating the system's qualitative properties suggests a numerical experiment using the numerical parameter system should be conducted. Asymptotic solutions were constructed as an initial approximation for this purpose. To linearize the system, Newton's, Picard's, and special linearization methods were utilized for the numerical solution of the problem [1-3,15]. The numerical experiment's results indicate that the proposed approach is effective.

3 Results and discussion

The method of undetermined coefficients involves constructing a differential scheme as a linear combination of the unknown grid function values at the nodal points of the stencil. The coefficients of this combination are determined by requiring that the differential equation represented by the scheme be approximated to the highest possible order on the grid layers [16].

For example, for the equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}\,. \tag{14}$$

The now differential equation we can find the approximate differential equation in the template. In this case, a differential scheme will be in the form [17]

$$Sh\left(\mathrm{x_i},t_j\right)=\left\{\left(\mathrm{x_i},t_j\right)\quad\left(\mathrm{x_{i-1}},t_{j+1}\right)\quad\left(\mathrm{x_i},t_{j+1}\right)\quad\left(\mathrm{x_{i+1}},t_{j+1}\right)\right\}, \tag{15}$$

we find the approximate differential equation

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=0\,. \tag{16}$$

u_{i-1}^{j+1}, u_i^{j+1}, u_{i+1}^{j+1} grid functions spread the Taylor line around the point (x_i,t_j):

$$u_i^{j+1}=u_i^j+\tau\frac{\partial u\left(x_i,t_j\right)}{\partial t}+\frac{\tau^2}{2}\frac{\partial^2 u\left(x_i,t_j\right)}{\partial t^2}+\frac{\tau^3}{6}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial t^3}+\frac{\tau^4}{24}\frac{\partial^4 u\left(x_i,t_j\right)}{\partial t^4}+\ldots$$

$$u_{i-1}^{j+1}=u_i^j-h\frac{\partial u\left(x_i,t_j\right)}{\partial x}+\tau\frac{\partial u\left(x_i,t_j\right)}{\partial t}+\frac{h^2}{2}\frac{\partial^2 u\left(x_i,t_j\right)}{\partial x^2}+\frac{\tau^2}{2}\frac{\partial^2 u\left(x_i,t_j\right)}{\partial t^2}-$$

$$-h\tau\frac{\partial^2 u\left(x_i,t_j\right)}{\partial x\partial t}-\frac{h^3}{6}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x^3}+\frac{\tau^3}{6}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial t^3}+\frac{h^2\tau}{2}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x^2\partial t}-\frac{h\tau^2}{2}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x\partial t^2}+\ldots$$

$$u_{i+1}^{j+1}=u_i^j+h\frac{\partial u(\mathrm{x}_i,\mathrm{t}_j)}{\partial x}+\tau\frac{\partial u(x_i,t_j)}{\partial t}+\frac{h^2}{2}\frac{\partial^2 u(\mathrm{x}_i,\mathrm{t}_j)}{\partial x^2}+\frac{\tau^2}{2}\frac{\partial^2 u(\mathrm{x}_{i,}\mathrm{t}_j)}{\partial t^2}+$$

$$+h\tau\frac{\partial^2 u\left(x_i,t_j\right)}{\partial x\partial t}+\frac{h^3}{6}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x^3}+\frac{\tau^3}{6}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial t^3}+\frac{h^2\tau}{2}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x^2\partial t}+\frac{h\tau^2}{2}\frac{\partial^3 u\left(x_i,t_j\right)}{\partial x\partial t^2}+\ldots \tag{17}$$

In this case

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=\alpha u\left(x_i,t_j\right)+\beta\Big[u\left(x_i,t_j\right)-hu'_x\left(x_i,t_j\right)+u'_t\left(x_i,t_j\right)+$$

$$+0,5h^2u''_{xx}\left(x_i,t_j\right)+0,5\tau^2u''_{tt}\left(x_i,t_j\right)-h\tau u''_{xt}\left(x_i,t_j\right)-\frac{h^3}{6}u'''_{xxx}\left(x_i,t_j\right)+\frac{\tau^3}{6}u'''_{ttt}\left(x_i,t_j\right)+$$

$$+\frac{h^2\tau}{2}u'''_{xxt}\left(x_i,t_j\right)-\frac{h\tau^2}{2}u'''_{xtt}\left(x_i,t_j\right)+\frac{h^4}{24}u''''_{xxxx}\left(x_i,t_j\right)-\frac{h^3\tau}{6}u''''_{xxxt}\left(x_i,t_j\right)+$$

$$+\frac{h^2\tau^2}{4}u''''_{xxtt}\left(x_i,t_j\right)-\frac{h\tau^3}{6}u''''_{xttt}\left(x_i,t_j\right)+\frac{\tau^4}{24}u''''_{tttt}\left(x_i,t_j\right)+\ldots\Big]+$$

$$+\gamma\left[u\left(x_i,t_j\right)+\tau u_t'\left(x_i,t_j\right)+\ 0,5\tau^2 u_{tt}''\left(x_i,t_j\right)+\frac{\tau^3}{6}u_{ttt}'''\left(x_i,t_j\right)+\frac{\tau^4}{24}u_{ttt}'''\left(x_i,t_j\right)+...\right]+$$

$$+\mu\left[u\left(x_i,t_j\right)+hu_x'\left(x_i,t_j\right)+u_t'\left(x_i,t_j\right)+\right.$$

$$+0,5h^2u_{xx}''\left(x_i,t_j\right)+0,5\tau^2u_{tt}''\left(x_i,t_j\right)+h\tau u_{xt}''\left(x_i,t_j\right)+\frac{h^3}{6}u_{xxx}'''\left(x_i,t_j\right)+\frac{\tau^3}{6}u_{ttt}'''\left(x_i,t_j\right)+$$

$$+\frac{h^2\tau}{2}u_{xxt}'''\left(x_i,t_j\right)+\frac{h\tau^2}{2}u_{xtt}'''\left(x_i,t_j\right)+\frac{h^4}{24}u_{xxxx}''''\left(x_i,t_j\right)+\frac{h^3\tau}{6}u_{xxxt}''''\left(x_i,t_j\right)+$$

$$\left.+\frac{h^2\tau^2}{4}u_{xxtt}''''\left(x_i,t_j\right)+\frac{h\tau^3}{6}u_{xttt}''''\left(x_i,t_j\right)+\frac{\tau^4}{24}u_{tttt}''''\left(x_i,t_j\right)+...\right]. \tag{18}$$

After expanding the Taylor series and simplifying similar terms, we obtain the following expression [18]:

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=\left[\alpha+\beta+\gamma+\mu\right]u\left(x_i,t_j\right)+h\left(\mu-\beta\right)u_x'\left(x_i,t_j\right)+$$

$$+\tau\left(\beta+\gamma+\mu\right)u_t'\left(x_i,t_j\right)+0,5h^2\left(\beta+\mu\right)u_{xx}''\left(x_i,t_j\right)+0,5\tau^2\left(\beta+\gamma+\mu\right)u_{tt}''\left(x_i,t_j\right)+$$

$$+h\tau\left(\mu-\beta\right)u_{xt}''\left(x_i,t_j\right)+\frac{h^3}{6}\left(\mu-\beta\right)u_{xxx}'''\left(x_i,t_j\right)+\frac{\tau^3}{6}\left(\beta+\gamma+\mu\right)u_{ttt}'''\left(x_i,t_j\right)+$$

$$+0,5h^2\tau\left(\beta+\mu\right)u_{xxt}'''\left(x_i,t_j\right)+0,5h^2\tau\left(\mu-\beta\right)u_{xtt}'''\left(x_i,t_j\right)+\frac{h^4}{24}\left(\beta+\mu\right)u_{xxxx}''''+$$

$$+\frac{h^3\tau}{6}\left(\mu-\beta\right)u_{xxxt}''''\left(x_i,t_j\right)+\frac{h^2\tau^2}{4}\left(\beta+\mu\right)u_{xxxt}''''\left(x_i,t_j\right)+\frac{h\tau^3}{6}\left(\mu-\beta\right)u_{xttt}''''\left(x_i,t_j\right)+$$

$$+\frac{h\tau^3}{6}\left(\mu-\beta\right)u_{xttt}''''\left(x_i,t_j\right)+\frac{\tau^4}{24}\left(\beta+\mu\right)u_{tttt}''''\left(x_i,t_j\right)+... \tag{19}$$

In the last equation, we require that the following be done:

$$\begin{cases}\alpha+\beta+\gamma+\mu=0\\ \beta+\gamma+\mu=\dfrac{1}{\tau}\\ \beta+\mu=-\dfrac{2}{h^2}\\ \mu-\beta=0\end{cases}. \tag{20}$$

Solving this system of equations enables us to determine the values of unknown parameters [19]

$$\alpha,\beta,\gamma,\mu:$$

$$\alpha=-\frac{1}{\tau},\quad \beta=\mu=-\frac{1}{h^2},\ \ \gamma=\frac{1}{\tau}+\frac{1}{h^2}. \tag{21}$$

At these values of the parameters

$$\alpha u_i^j+\beta u_{i-1}^{j+1}+\gamma u_i^{j+1}+\mu u_{i+1}^{j+1}=\left(\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}\right)_{(x_i,t_j)}-0,5\tau u''_{tt}\left(x_i,t_j\right)+\frac{\tau^2}{6}u'''_{ttt}\left(x_i,t_j\right)-$$
$$-\frac{h^2}{12}u''''_{xxxx}\left(x_i,t_j\right)-\frac{\tau^2}{2}u''''_{xxxt}\left(x_i,t_j\right)+-\frac{h^2\tau^4}{12}u''''_{tttt}\left(x_i,t_j\right)+.... \tag{22}$$

α,β,γ,μ If we substitute the found values of the parameters into equation (16), we obtain the form of the difference scheme we have constructed [20]:

$$\frac{u_i^{j+1}-u_i^j}{\tau}-\frac{u_{i-1}^{j+1}-2u_i^{j+1}+u_{i-1}^{j+1}}{h^2}=\frac{\partial u\left(x_i,t_j\right)}{\partial t}-\frac{\partial^2 u\left(x_i,t_j\right)}{\partial t^2}+O\left(\tau,h^2\right). \tag{23}$$

$$Sh\left(\mathrm{x}_i,t_j\right)=\left\{\left(\mathrm{x}_i,t_j\right)\quad\left(\mathrm{x}_{i-1},t_{j+1}\right)\quad\left(\mathrm{x}_i,t_{j+1}\right)\quad\left(\mathrm{x}_{i+1},t_{j+1}\right)\right\}. \tag{24}$$

Let us construct a differential scheme

$$\bar{\omega}_h=\left\{x_i=ih,\quad h>0,\quad i=0,1,...,n,\quad hn=M\right\}, \tag{25}$$

and for time

$$\bar{\omega}_\tau=\left\{t_j=j\tau,\quad \tau>0,\quad j=0,1,...,m,\quad \tau\, m=T\right\},\ T>0 \tag{26}$$

We construct the following scheme [21]:

$$\begin{cases}\dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h}\left(a_{i+1}\dfrac{y_{i+1}^{j+1}-y_i^{j+1}}{h}-a_i\dfrac{y_i^{j+1}-y_{i-1}^{j+1}}{h}\right)+l_i^{j+1}\dfrac{y_{i+1}^{j+1}-y_{i-1}^{j+1}}{2h}+k_{1i}^{j+1}y_i^{j+1}\left(1-\left(w_i^j\right)^{\beta_1}\right),\\ \dfrac{w_i^{j+1}-w_i^j}{\tau}=\dfrac{1}{h}\left(b_{i+1}\dfrac{w_{i+1}^{j+1}-w_i^{j+1}}{h}-b_i\dfrac{w_i^{j+1}-w_{i-1}^{j+1}}{h}\right)+l_i^{j+1}\dfrac{w_{i+1}^{j+1}-w_{i-1}^{j+1}}{2h}+k_{2i}^{j+1}w_i^{j+1}\left(1-\left(y_i^{j+1}\right)^{\beta_2}\right).\end{cases} \tag{27}$$

To find a solution to the different equations, we use the iteration method constructed as follows [22]:

$$\begin{cases}\dfrac{\overset{s+1}{y}{}_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h}\left(\overset{s}{a}_{i+1}\dfrac{\overset{s+1}{y}{}_{i+1}^{j+1}-\overset{s+1}{y}{}_i^{j+1}}{h}-\overset{s}{a}_i\dfrac{\overset{s+1}{y}{}_i^{j+1}-\overset{s+1}{y}{}_{i-1}^{j+1}}{h}\right)+l_i^{j+1}\dfrac{\overset{s+1}{y}{}_{i+1}^{j+1}-\overset{s+1}{y}{}_{i-1}^{j+1}}{2h}+k_{1i}^{j+1}\overset{s+1}{y}{}_i^{j+1}\left(1-\left(w_i^j\right)^{\beta_1}\right),\\ \dfrac{\overset{s+1}{w}{}_i^{j+1}-w_i^j}{\tau}=\dfrac{1}{h}\left(\overset{s}{b}_{i+1}\dfrac{\overset{s+1}{w}{}_{i+1}^{j+1}-\overset{s+1}{w}{}_i^{j+1}}{h}-\overset{s}{b}_i\dfrac{\overset{s+1}{w}{}_i^{j+1}-\overset{s+1}{w}{}_{i-1}^{j+1}}{h}\right)+l_i^{j+1}\dfrac{\overset{s+1}{w}{}_{i+1}^{j+1}-\overset{s+1}{w}{}_{i-1}^{j+1}}{2h}+k_{2i}^{j+1}\overset{s+1}{w}{}_i^{j+1}\left(1-\left(y_i^{j+1}\right)^{\beta_2}\right).\end{cases} \tag{28}$$

The difference scheme (28) concerning the function $\overset{(s+1)}{y}{}^{j+1}$ and $\overset{(s+1)}{w}{}^{j+1}$ will be linear. The functions of the previous time step are taken as the initial iteration: $\overset{(0)}{y}{}^{j+1}=y^j$ and

$\overset{(0)}{w}{}^{j+1} = w^j$. If the conditions $\max\limits_i \left| \overset{(s+1)}{y_i} - \overset{(s)}{y_i} \right| \le \varepsilon$ and $\max\limits_i \left| \overset{(s+1)}{w_i} - \overset{(s)}{w_i} \right| \le \varepsilon$ are satisfied, then the convergence of the iteration is ensured.

A computational experiment was carried out for various parameter values (Table 1, Table 2). The results of the experiment with fast diffusion are shown in Table 1. As an initial approximation u_0, v_0 we took the functions:

$$u_0(x,t) = (T+t)^{-\alpha_1}(a+\xi^{\gamma})^{q_1},\ v_0(x,t) = (T+t)^{-\alpha_2}(a+\xi^{\gamma})^{q_2},\ \xi = (\int_0^t c(y)dy - x)/\tau^{\frac{1}{p}},$$

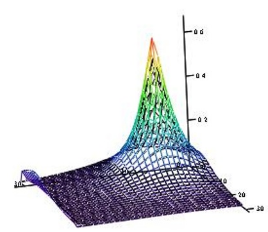
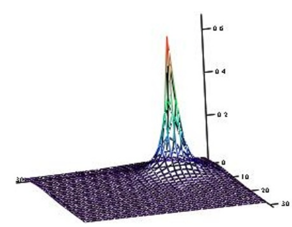
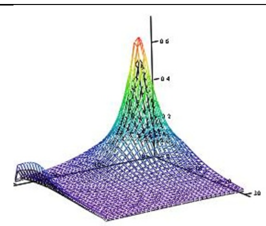
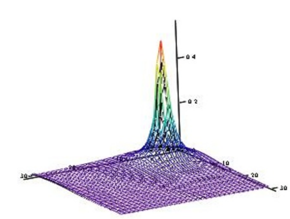
$$\gamma = \frac{p}{p-1},$$

$$c(t) = 1/(T+t)^n,\ n \ge 1,\ n < 1, \int c(y)dy = (T+t)^{1-n}/(1-n),\ \alpha_1 = \frac{1}{\beta_1 - 1},$$

$$\alpha_2 = \frac{1}{\beta_2 - 1},$$

$$q_i = \frac{(p-1)}{p+m_i-3},\ p+m_i-3<0,\ i=1,2. \tag{29}$$

Table 1. Results of numerical experiments

Parameter values	Results of the counting experiment at the beginning of the moment	results of the counting experiment in the final moment of time
$m_1 = 0.8,\ m_2 = 0.7,\ p = 2.1$ $eps = 10^{-3}$ $\beta_1 = 2\quad \beta_2 = 5$ $m_i + p - 3 < 0$ $n = 3$		
$m_1 = 0.4,\ m_2 = 0.5,\ p = 2.2$ $eps = 10^{-3}$ $\beta_1 = 2\quad \beta_2 = 2$ $m_i + p - 3 < 0$ $n = 5$		

The results of the computational experiment with slow diffusion are given in Table 2. As the initial approximation u_0, v_0 we took the functions:

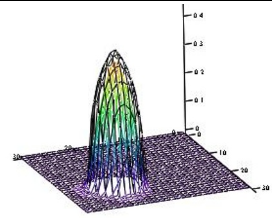
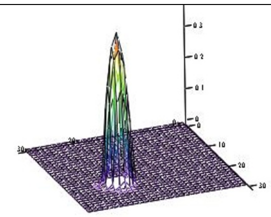
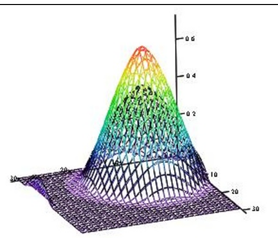
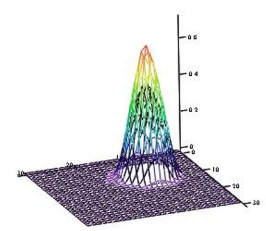
$$u_0(x,t) = (T+t)^{-\alpha_1}(a-\xi^{\gamma})_+^{q_1},\ v_0(x,t) = (T+t)^{-\alpha_2}(a-\xi^{\gamma})_+^{q_2},\ \xi = (\int_0^t c(y)dy - x)/\tau^{\frac{1}{p}};$$

$$c(t)=1/(T+t)^n,\ n\ge 1,\ n<1,\ \int c(y)dy=(T+t)^{1-n}/(1-n)\ ;\alpha_1=\frac{1}{\beta_1-1},\ \alpha_2=\frac{1}{\beta_2-1},\quad (30)$$

$$q_i=\frac{(p-1)}{p+m_i-3},\ p+m_i-3>0,\ i=1,2,\quad u(x,t)=v(x,t)\equiv 0,$$

$$|x|\ge\int_0^t c(y)dy-a^{(p-1)/p}\tau^{\frac{1}{p}},\ \tau(t)=(T+t)^{1-\alpha_i(m_i+p-3)}/[1-\alpha_i(m_i+p-3)],\ \alpha_i=\frac{\beta_i+1}{\beta_1\beta_2-1},\ i=1,2,\ \beta_1\beta_2>1.$$

Table2. Results of numerical experiments

Parameter values	Results of the counting experiment at the beginning of the moment	results of the counting experiment in the final moment of time
$m_1=1.9,\ m_2=5,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $m_i+p-3>0$ $n=3$		
$m_1=1.5,\ m_2=2,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $m_i+p-3>0$ $n=5$		

Due to the correct choice of the initial approximation, the number of iterations in the computational experiments performed does not exceed six. The experiment was carried out for various values of the parameters of the system of equations. The number of iterations is given in Table 3.

Table 3. Number of iterations for different parameters

eps	m_1	m_2	p	β_1	β_2	k	Average number of iterations
10^{-3}	4.1	4.0	4.4	1.0	1.0	0.5	3
10^{-5}	5.7	5.4	3.0	2.0	2.0	3.0	4
10^{-3}	3.7	3.3	4.0	2.0	0.5	0.1	3
10^{-5}	2.5	2.4	3.1	2.0	0.5	0.5	4
10^{-3}	5.1	5.3	3.5	3.0	0.3	1.5	3
10^{-5}	3.0	3.2	3.0	3.0	3.0	1.0	6
10^{-3}	5.0	5.2	3.0	10.0	5.0	2.0	2
10^{-5}	2.7	2.5	5.4	3.0	2.0	2.0	6
10^{-3}	3.7	3.5	7.4	2.0	3.0	3.0	3
10^{-3}	3.0	3.5	7.0	14.0	7.0	2.0	5

4 Conclusion

Studies have been carried out on the properties of cross-diffusion systems with double nonlinearity and convective transfer. Computer simulations of multicomponent cross-diffusion systems processes with convective transfer were carried out. This means that with the help of computer programs and numerical methods, models have been created that make it possible to simulate and study the behavior of such systems. Such modeling allows one to understand the dynamics and interaction of components in cross-diffusion systems, taking into account convective transport. The properties of cross-diffusion systems with double nonlinearity and convective transfer have been studied. These studies can help better understand the dynamics and interactions of components in such systems and apply the knowledge gained in various areas where it is important to consider cross-diffusion and convective transport, such as biology and population ecology.

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