Mathematical modeling of hydrodynamic resistance in an oscillatory flow of a viscoelastic fluid

Kuralboy Navruzov[1], *Suxrob* Rajabov[2]*, and *Muzaffar* Ashurov[2]

[1]Urgench State University, Urgench, Uzbekistan
[2]Kattakurgan branch of Samarkand State University, Samarkand, Uzbekistan

Abstract. The problems of the oscillatory flow of a viscoelastic fluid in a flat channel for a given harmonic oscillation of the fluid flow rate are solved based on the generalized Maxwell model. The transfer function of the amplitude-phase frequency characteristics is determined. Using this function, the dependence of the hydrodynamic resistance on the dimensionless oscillation frequency is studied for various values of the elastic Deborah number and the concentration of the Newtonian fluid. It is shown that in an oscillatory flow of a viscoelastic fluid, the hydrodynamic resistance decreases depending on the Deborah number. With an increase in this number, the decrease becomes more pronounced than before. This effect allows us to evaluate the hydrodynamic resistance for a given law; the change in the longitudinal velocity averaged over the channel section and for the motion of a viscoelastic fluid in an unsteady flow allows us to determine the dissipation of the mechanical energy of the medium, which is important in the regulation of hydraulic and pneumatic systems.

1 Introduction

The study of pulsating flows of a viscoelastic fluid in a flat channel and a cylindrical pipe under the influence of harmonic oscillations of the pressure gradient or when harmonic oscillations of the flow rate are superimposed on the flow is of practical interest. In [1], the flow of viscoelastic fluids along a long pipe under the periodic pressure gradient was studied. The distinctive features of this flow are shown in comparison with the corresponding flow of the Newtonian fluid. The inertialess oscillatory flow of a viscoelastic fluid in an infinite circular pipe under an oscillatory pressure gradient was studied in [2]; it was shown that in an oscillating flow, the longitudinal velocity profiles were symmetric and there was a significant phase shift between the pressure gradient and velocity. The phase shift was absent in pulsating flows, and the axial velocity changed asymmetrically relative to its average oscillation period. Laminar oscillatory flows of Maxwell and Oldroyd - B viscoelastic fluids were studied in [3, 4]. Many interesting features missing in the Newtonian fluid flows were shown in the study.

*Corresponding author: rajabovsuxrob72@gmail.com

The results of the study given in [3] show that in the inertialess mode for $\mathrm{Re} \ll 1$, the properties of the flow depend on three characteristic lengths. Wavelength λ_0 and attenuation length of viscoelastic shear waves $x_0 = \left(\frac{2\nu}{\omega_0}\right)^{1/2}$, here ν is the kinematic viscosity; ω_0 is the oscillation frequency, and a is the characteristic transverse size of the system. In this regard, the lengths are divided into three scales and three independent dimensionless groups: $\frac{t_\vartheta}{\lambda}$ (viscosity before the relaxation time), De (relaxation time to oscillation period) and X (viscosity coefficient). At the same time, the oscillatory flow regions are divided into two systems corresponding to the "wide" $\left(\frac{a}{x_0} > 1\right)$ and "narrow" $\left(\frac{a}{x_0} < 1\right)$ systems. In wide systems, the oscillations are limited by near-wall flows and in the central core by frictionless flows. In narrow systems, transverse waves across the entire system and cross its center, leading to constructive resonances followed by a sharp increase in the amplitude of the velocity profile.

In [5], unsteady flows of a viscoelastic fluid were analyzed on the Oldroyd-B model in a round infinite cylindrical pipe under a time-dependent pressure gradient for the following cases: a) the pressure gradient changes with time according to the exponential law; b) the pressure gradient changes according to the harmonic law; c) the pressure gradient is constant. In all cases, formulas were obtained for the velocity distribution, fluid flow, and other hydrodynamic values in a pulsating flow. The problem of unsteady oscillatory flow of a viscoelastic fluid in a round cylindrical pipe was considered in [6] based on the Maxwell model. Formulas for determining dynamic and frequency characteristics were obtained. Numerical experiments are used to study the influence of the oscillation frequency and the relaxation properties of the liquid on the tangential shear stress on the wall. It was shown that the viscoelastic properties of the fluid and its acceleration are the limiting factors for using the quasi-stationary approach.

In recent decades, electro-kinetic phenomena, including electro-osmosis, flow potential, electrophoresis, and sedimentation potential, have received much attention and provided many applications in micro and nanochannels. In this regard, in [7, 8], the electro-kinetic flow of viscoelastic fluids through flat channels under the influence of an oscillatory pressure gradient was studied. It was assumed that the fluid flow is laminar and unidirectional; therefore, the fluid flow has a linear mode. The surface potentials are considered small, so the Poisson-Boltzmann equation is linearized. A resonant behavior appears in the flow when the elastic properties of the Maxwell fluid predominate. The resonant phenomenon amplifies the electro-kinetic effects and the efficiency of electro-kinetic energy conversion. Unsal B., Ray S., Marx U., Wallis H., Inman W., Domanskiy K., Tsangaris S., Vlachakis N.W. for determination of hydrodynamic resistance. The oscillatory flow was studied by such scientists as [9-14].

In the publications listed above, the velocity field of fluids is mainly investigated for various modes of change in the pressure gradient. The change in tangential and normal stresses arising in the flow was studied insufficiently. In most cases, in hydrodynamic models of unsteady flows, liquids were replaced by a sequence of flows with a quasi-stationary distribution of hydrodynamic values. However, the structure of unsteady flows differs from the structure of stationary flows, and such a replacement must be justified in each specific case. The question of the relevancy of studying quasi-stationary characteristics for determining the field of shear stresses in nonstationary flows of viscous and viscoelastic fluids is far from being resolved. Naturally, under such conditions, it becomes necessary to use hydrodynamic models of nonstationary processes that consider the change in the hydrodynamic characteristics of the flow depending on time. It should be noted that, in the general case, hydrodynamic characteristics in pipeline transportation could not be determined from characteristics that correspond to stationary flow conditions.

In this paper, the authors studied the oscillatory flow of a viscoelastic fluid on the generalized Maxwell model in a flat channel when harmonic oscillations of the fluid flow rate are superimposed on the flow. The transfer function of the amplitude-phase frequency characteristics (APFC) is determined. This function is used to analyze the change in hydrodynamic resistance during an oscillatory flow of an elastic-viscous fluid depending on the dimensionless oscillation frequency.

2 Materials and Methods

Let us consider the problems of a slow oscillatory flow of a viscous elastic incompressible fluid between two fixed parallel planes extending in both directions to infinity. Let us denote the distance between the walls by $2h$. The $0x$-axis runs horizontally in the middle of the channel along the flow. The $0y$-axis is directed perpendicular to the $0x$-axis. The flow of a viscoelastic fluid occurs symmetrically along the channel axis. The differential equation of motion of a viscous elastic incompressible fluid in stress has the following form [15-20]:

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}-\frac{\partial \tau}{\partial y} \tag{1}$$

where u is the longitudinal velocity; p is the pressure; ρ is the density; τ is the tangential stress; t is time.

The rheological equation of the state of the fluid is taken in the form of the generalized Maxwell equation [3, 7]

$$\tau=\tau_s+\tau_p,\ \tau_s=-\eta_s\frac{\partial u}{\partial y},\ \lambda\frac{\partial \tau_p}{\partial t}+\tau_p=-\eta_p\frac{\partial u}{\partial y} \tag{2}$$

Here λ is the relaxation time; τ_s is the tangential stress of the Newtonian fluid; τ_p is the tangential stress of the Maxwell fluid; τ is the tangential stress of the solution; η_s is the dynamic viscosity of the Newtonian fluid; η_p is the dynamic viscosity of the Maxwell fluid. Scientists such as Momoniat E., Ali F., Khan I. Thorough review of nonstationary and stationary and second-order MHD fluid flow in Refs used FDM (finite difference method) and L1 schemes to obtain numerical solutions to the process [21–24]. The following equality is fulfilled between dynamic viscosities [3, 7]

$$\eta_0=\eta_s+\eta_p$$

where η_0 is the dynamic viscosity of the solution.

Substituting (2) into the equation of motion (1) for the fluid velocity, we obtain

$$\rho\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=-\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x}+\eta_s\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2}+\eta_p\frac{\partial^2 u}{\partial y^2} \tag{3}$$

We consider that the oscillatory flow of a viscoelastic fluid occurs due to given harmonic fluctuations of the fluid flow rate or the longitudinal velocity averaged over the channel section.

$$Q = a_Q cos\omega t = \mathrm{Re}\, a_Q e^{i\omega t}, \qquad <u>= a_u cos\omega t = \mathrm{Re}\, a_u e^{i\omega t}$$

where a_Q and a_u are the amplitudes of the fluid flow rate and the amplitudes of the longitudinal velocity averaged over the channel section, respectively. In this case, the flow moves symmetrically along the channel axis, and the no-slip condition is satisfied for the channel wall, i.e., the longitudinal velocity on the channel wall is zero. Then the boundary conditions are:

$$u = 0 \text{ for } y = h, \qquad \frac{\partial u}{\partial y} = 0 \quad \text{for} \quad y = 0 \tag{4}$$

Due to the linearity of equation (3), the longitudinal velocity, pressure, and shear stress on the wall can be written as

$$u(y,t) = \mathrm{Re}\, u_1(y) e^{i\omega t},\ p(x,t) = \mathrm{Re}\, p_1(x) e^{i\omega t},\ \tau(t) = \mathrm{Re}\, \tau_1 e^{i\omega t} \tag{5}$$

Substituting (5) into equation (3), we obtain

$$\frac{\partial^2 u_1(y)}{\partial y^2} - \frac{\rho i\omega}{\eta_0}\left(X + \frac{Z}{(1+i\omega\lambda)}\right)^{-1} u_1(y) = \frac{1}{\eta_0}\left(X + \frac{Z}{(1+i\omega\lambda)}\right)^{-1} \frac{\partial p_1(x)}{\partial x} \tag{6}$$

here $X = \frac{\eta_s}{\eta_0}$, $Z = \frac{\eta_p}{\eta_0}$, $X + Z = \frac{\eta_s}{\eta_0} + \frac{\eta_p}{\eta_0} = 1$

The following functions are the fundamental solutions of equation (6) without the right-hand side

$$\cos\left(\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)y\right) \text{ and } \sin\left(\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)y\right)$$

here

$$\eta(i\omega) = \left(\frac{1+i\omega\lambda}{1+i\omega\lambda X}\right)^{1/2},\ \alpha_0 = \sqrt{\frac{\rho\omega}{\eta_0}}h.$$

And the solution of the non-homogeneous part has the following form

$$\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)$$

Thus, the general solution to equation (6) is

$$u_1(y) = C_1 \cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) + C_2 \sin\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) + \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right) \tag{7}$$

To determine the constant coefficients C_1 and C_2 , we use boundary conditions (4)

$$\frac{\partial u_1(y)}{\partial y}=-C_1\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)\sin\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right)+C_2\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)\cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right) \tag{8}$$

for $y=0$, (8) has the following form

$$0=C_2\frac{i^{3/2}\alpha_0}{h}\eta(i\omega)$$

from here, it is easy to find

$$C_2=0$$

C_1 is determined from (7) under the condition that $u(y,t)=0$ for $y=h$

$$C_1=-\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\frac{1}{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}$$

As a result of this, we determine the velocity:

$$u_1(y)=\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1-\frac{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\frac{y}{h}\right)}{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}\right) \tag{9}$$

where $\alpha_0-\sqrt{\frac{\omega}{\nu_0}}h$ is the Womersley vibrational number (dimensionless vibration frequency); ν_0 is the kinematic viscosity of the solution.

Using the following equation

$$\tau_1(i\omega)=-\frac{\eta_0}{\eta^2(i\omega)}\frac{\partial u_1(y)}{\partial y}\Big|_{y=h} \tag{10}$$

we obtain the tangential shear stress on the wall

$$\tau_1(i\omega)=-h\left(-\frac{\partial P}{\partial x}\right)\frac{1}{i\alpha_0^2}\left(\frac{i^{3/2}\alpha_0\eta(i\omega)\sin\left(i^{3/2}\alpha_0\eta(i\omega)\right)}{\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}\right) \tag{11}$$

Now we integrate both sides of formula (9) concerning variable y ranging from $-h$ to h , and obtain formulas for the fluid flow rate

$$Q_1=2h\left[\frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1-\frac{\sin\left(i^{3/2}\alpha_0\eta(i\omega)\right)}{\left(i^{3/2}\alpha_0\eta(i\omega)\right)\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}\right)\right] \tag{12}$$

Considering formula (12) that $Q_1 = 2h < u_1 >$, we find the longitudinal velocity averaged over the channel section

$$< u_1(i\omega) >= \frac{1}{\rho i\omega}\left(-\frac{\partial p_1(x)}{\partial x}\right)\left(1-\frac{\sin\left(i^{3/2}\alpha_0\eta(i\omega)\right)}{\left(i^{3/2}\alpha_0\eta(i\omega)\right)\cos\left(i^{3/2}\alpha_0\eta(i\omega)\right)}\right) \tag{13}$$

Here $\rho i\omega$ can be written in the following form

$$\rho i\omega = i\frac{\omega}{\nu}h^2 \cdot \frac{\eta_0}{h^2} = i\alpha_0^2\frac{\eta_0}{h^2}$$

Then formula (13), concerning (11), takes the following form:

$$< u_1(i\omega) >= -\frac{h}{3\eta_0}\tau_1 \cdot \frac{3i^{\frac{3}{2}}\alpha_0\eta(i\omega)\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)-\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{\left(i^{\frac{3}{2}}\alpha_0\right)^2\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)} \tag{14}$$

Using the formula (14), we determine the transfer function $W_{\tau,u}(i\omega)$ for the shear stress on the walls as

$$W_{\tau_1,u_1}(i\omega) = \frac{h}{3\eta_0}\frac{\tau_1(i\omega)}{u_1(i\omega)} \tag{15}$$

Taking into account (15) from equation (14), we obtain

$$W_{\tau_1,u_1}(i\omega) = \frac{h}{3\eta_0}\frac{\tau_1(i\omega)}{< u_1(i\omega) >} = -\frac{\left(i^{\frac{3}{2}}\alpha_0\right)^2\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)}{3(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\cos\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right)-\sin\left(i^{\frac{3}{2}}\alpha_0\eta(i\omega)\right))} = \chi + \beta i \tag{16}$$

The transfer function (16) is sometimes called the amplitude-phase frequency characteristic (APFC). This function makes it possible to estimate the hydraulic resistance for a given law of change in the longitudinal velocity averaged over the channel section since its real part allows us to determine the active hydrodynamic resistance, and the imaginary part is the reactive resistance or inductance of the oscillatory flow.

3 Results and Discussion

The hydrodynamic resistance under oscillatory flow in Newtonian and viscoelastic flows is determined by the ratio of the pressure gradient to the average velocity, sometimes referred to as the "impedance" of the flow. The ratio of the pressure gradient to the average velocity is found from the following formula (13)

$$Z=\frac{\left(-\dfrac{\partial p}{\partial x}\right)}{R_0<u_1(i\omega)>}=\left[\frac{1}{i\alpha_0^2}\left(1-\frac{\sin\left(M_1-i\overline{M}_1\right)}{\left(M_1-i\overline{M}_1\right)\cos\left(M_1-i\overline{M}_1\right)}\right)\right]^{-1}=R_n^0+iL_0 \qquad (17)$$

Here $R_0=\dfrac{\eta}{h^2}$ is the hydrodynamic resistance of the Newtonian fluid at a steady flow.

Separating the real and imaginary parts of formula (17), we determine the total hydrodynamic resistance $\overline{R}$ and inductance $\overline{L}$:

$$\overline{R}=\frac{\alpha_0^2\left(A_1^2+B_1^2\right)}{\left(A_2^2+B_2^2\right)}B_2,\quad \frac{\overline{L}}{\alpha_0^2}=\frac{\left(A_1^2+B_1^2\right)}{A_2^2+B_2^2}A_2,$$

where

$$A_1=\overline{A}\overline{M}_1+\overline{B}M_1,\quad B_1=\overline{A}M_1-\overline{B}\overline{M}_1,$$

$$A_2=\left(A_1^2+B_1^2\right)-A_1C-B_1D,\quad B_2=\left(B_1C-A_1D\right)$$

$$C=\sin M_1 ch\overline{M}_1,\quad D=-\cos M_1 sh\overline{M}_1.$$

$$\overline{A}=\sin M_1 sh\overline{M}_1,\quad \overline{B}=\cos M_1 ch\overline{M}_1,\quad M_1=\frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_1,\ \overline{M}_1=\frac{\alpha_0}{\sqrt{2}}\overline{\overline{G}}_2,$$

$$\overline{\overline{G}}_1=\overline{\overline{G}}_1+\overline{\overline{G}}_2,\quad \overline{\overline{G}}_2=\overline{G}_1-\overline{G}_2,\quad \sqrt{\frac{1}{\eta^*(i\omega)}}=\sqrt{G_1+G_2i}=\overline{G}_1+\overline{G}_2i,$$

$$\overline{G_1}=\sqrt{\sqrt{G_1^2+G_2^2}}\cos\frac{\varphi+2n\pi}{2},\quad \overline{G}_2=\sqrt{\sqrt{G_1^2+G_2^2}}\sin\frac{\varphi+2n\pi}{2},\ n=0,1;$$

$$\varphi=arctg\frac{G_2}{G_1},\qquad \frac{1}{\eta^*(i\omega)}=\frac{1+De^2X_1\alpha_0^4+iDe\alpha_0^2(1-X_1)}{1+De^2X_1^2\alpha_0^4}=G_1+G_2i$$

$$G_1=\frac{1+De^2X_1\alpha_0^4}{1+De^2X_1^2\alpha_0^4},\qquad G_2=\frac{De\alpha_0^2(1-X_1)}{1+De^2X_1^2\alpha_0^4}$$

$$\eta(i\omega)=\left(\frac{\eta_s}{\eta}+\frac{\eta_p}{\eta}\frac{1}{1+iDe\alpha_0^2}\right)=\left(X_1+Z_1\frac{1}{1+iDe\alpha_0^2}\right)=$$

$$\frac{1+iDeX_1\alpha_0^2}{1+iDe\alpha_0^2},\ \frac{\eta_s}{\eta}+\frac{\eta_p}{\eta}=1\ X_1=\frac{\eta_s}{\eta},\ Z_1=\frac{\eta_p}{\eta},X_1+Z_1=1,$$

$$De=\frac{\lambda\eta}{\rho h^2},\quad \alpha_0^2=\frac{\omega}{\nu}h^2.$$

The results of studies (17) for the Newtonian fluid are given in [7, 9, 10]. Figure 1 shows the dependence of the hydrodynamic resistance on the dimensionless oscillation frequency α_0 when the elastic number is $De=0.05$ and for different concentrations of the Newtonian fluid in the solution.

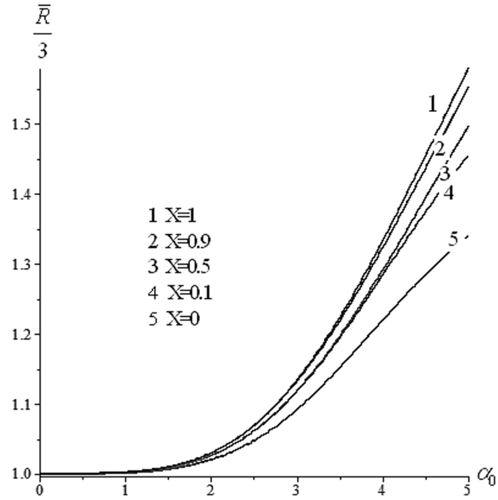

Fig.1. Dependence of hydrodynamic resistance on dimensionless frequency of oscillations α_0 for different concentrations of Newtonian fluid, for $De = 0.05$.

The graph in Figure 1 shows for $X = 1$ the change in the total hydrodynamic resistance of the Newtonian fluid in an oscillatory flow; it coincides with the results of other researchers [7, 9]. It can be seen from this graph that with an increase in the dimensionless oscillation frequency α_0, the total hydrodynamic resistance of the Newtonian fluid increases monotonically. Curves 2-5 in Figure 1 characterize the change in hydrodynamic resistance in an oscillatory flow of an elastic-viscous fluid with a low elastic Deborah number, with the addition of Newtonian fluid. Indeed, curves 2-5 differ little from curve 1, so, in this case, instead of the hydrodynamic resistance of an elastic-viscous fluid, one can take the hydrodynamic resistance of a Newtonian fluid.

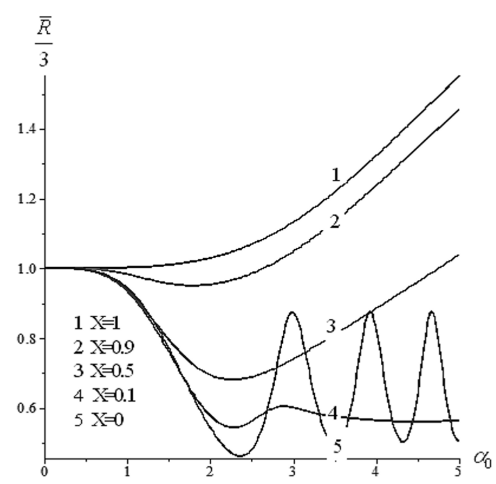

Fig. 2. Dependence of hydrodynamic resistance on dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid, for $De = 0.5$.

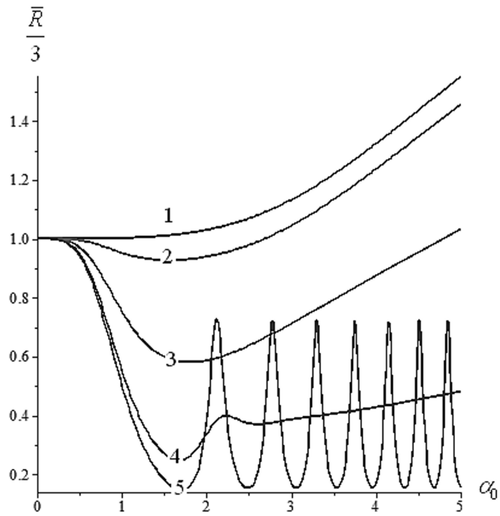

Fig. 3. Dependence of hydrodynamic resistance on dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid, for $De = 1$.

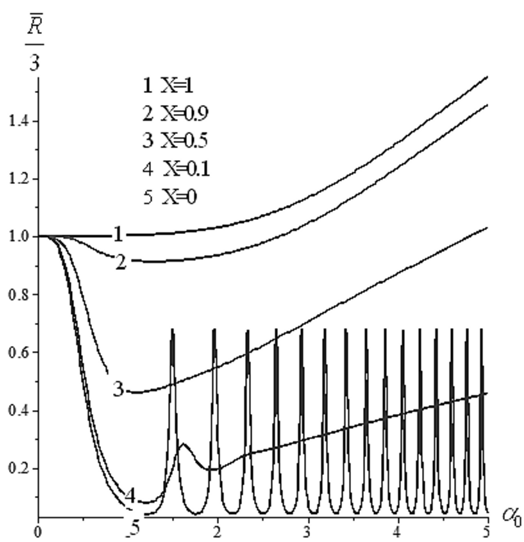

Fig. 4. Dependence of hydrodynamic resistance on dimensionless oscillation frequency α_0 at different concentrations of Newtonian fluid, for $De = 2$.

However, with an increase in the elastic Deborah number, there is a significant difference between the hydrodynamic resistances of an elastic-viscous fluid compared to the Newtonian fluid. This difference is shown in Figures 2.2- 2.4 for increasing values of elastic Deborah number. Starting from the elastic Deborah number $De = 0.5$, there is a decrease in hydrodynamic resistance depending on the concentration of the Newtonian fluid (curves 3-5 in Figure 2.3). The solution has viscoelastic properties when there is no Newtonian fluid in it. In such cases, in an oscillatory flow, the hydrodynamic resistance of elastic-viscous fluid changes in an oscillatory manner depending on the dimensionless oscillation frequency α_0, and it increases with an increase in the elastic Deborah number (curves 5 in Figures 2-4).

The content of the Newtonian fluid in the solution smooths out the oscillatory mode of the change in the hydrodynamic resistance (curves 3.4 in Figures 2-4). In the general case, at an oscillatory flow of a viscoelastic fluid, the hydrodynamic resistance decreases in the intermediate value $1 < \alpha_0 < 3$ of the dimensionless oscillation frequency to a maximum. Then it increases with an increase in the frequency. The effect obtained makes it possible to evaluate the hydrodynamic resistance for a given law of the change in the longitudinal velocity averaged over the channel section for the flow of a viscoelastic fluid in an unsteady flow; it makes it possible to determine the dissipation of the mechanical energy of the medium, which is important in the regulation of hydraulic and pneumatic systems.

4 Conclusions

The problems of an oscillatory flow of a viscoelastic fluid in a flat channel for a given harmonic oscillation of the fluid flow rate are solved based on the generalized Maxwell model. The transfer function of the amplitude-phase frequency characteristics is determined. Using this function, the dependence of the hydrodynamic resistance on the dimensionless oscillation frequency is studied for various values of the elastic Deborah number and the concentration of the Newtonian fluid. It is shown that in an oscillatory flow of a viscoelastic fluid, the hydrodynamic resistance decreases depending on the Deborah number. With an increase in this number, the decrease becomes more pronounced than before. This effect allows us to evaluate the hydrodynamic resistance for a given law; the change in the longitudinal velocity averaged over the channel section and for the motion of a viscoelastic fluid in an unsteady flow allows us to determine the dissipation of the mechanical energy of the medium, which is important in the regulation of hydraulic and pneumatic systems.

References

1. Jones J.R., Walters T.S. (1967). Rheol. Acta. Part 1. Vol. 6, 1967. 240-245
2. Khabakhpasheva E., Popov V., Kekalov A. and Mikhailova E.(1989). J. Non-Newtonian Fluid Mech. Vol.33(3), 1989. 289-304
3. Casanellas L., Ortin J. (2011). J. Non-Newtonian Fluid Mechanics. 166. 2011. 1315-1326
4. Casanellas L., Ortin J. (2012) Rheol . Acta., 51, 2012. 545-557
5. Hassan A.Abu-El and El-Maghawru E.M. (2013) Rheology-New concepts. Applications and Methods Ed by Durairaj R. Published by In Tech. 6, 2013. 91-106
6. AkilovZh.A., Dzhabbarov M.S. and Khuzhayorov B.Kh. (2021) Fluid Dynamics Research, Vol. 56, 2021. 189-199
7. Ding Z., Jian Y.,Tan W. (2019). J. Fluid. Mech., 863, 2019. 1062-1090
8. Ding Z., Jian Y. (2021) J. Fluid. Mech., 2021. doi:10.1017/j fm. 2021. 380 A20.1-31.
9. Ünsal B., Ray S., Durst F., Ertunç Ö. (2005). Fluid Dynamics Research. Vol. 37, 2005. 317-333
10. Marx U., Wallis H., Hoffmann S., Linder G., Harland R., Sonntag F., Klotzbach U., Sakharov D., Tonevitskiy A., Lonster R. (2012), ATLA Vol. 40, 2012. 235-257
11. Inman W., Domanskiy K., Serdy J., Ovens B., Trimper D., Griffith L.G. (2007), J.

Micromech. Microeng. Vol. 17, 2007. 891-899

12. Valueva E.P., Purdin M.S. (2015) 9, 2015. 24-33
13. Tsangaris S., Vlachakis N.W. (2003), ZAMP. Vol. 54, 2003. 1094-1100
14. Tsangaris S., Vlachakis N.W. (2007), Appl. Math. Modelling. Vol. 31, 2007. 1899-1906.
15. Popov D.N. Nonstationary hydromechanical processes.(1982), M.: Mashinostroenie
16. Astarita J., Marrucci J. Fundamentals of hydromechanics of non-Newtonian fluids. (1978), M.: Mir
17. Loitsyansky L.G. Mechanics of liquid and gas. (2003), M.: Drofa
18. Kolesnichenko V.I., Sharifulin A.N. Introduction to the mechanics of an incompressible fluid. (2019), Perm, Ed. Perm National research political university
19. Navruzov K., Khakberdiev Zh.B. Dynamics of non-Newtonian fluids.(2000), Tashkent, Fan
20. Schlichting G. Theory of the boundary layer.(1974), M.: Nauka
21. Momoniat, E. A point source solution for unidirectional flow of a viscoelastic fluid. Phys. Lett. A 372(22), 4041–4044 (2008).
22. Ali, F., Khan, I. & Shafie, S. Closed form solutions for unsteady free convection flow of a second grade fluid over an oscillating vertical plate. PLoS ONE 9(2), e85099 (2014).
23. Ali, F. & Sheikh, N. A. Introductory Chapter: Fluid Flow Problems (IntechOpen, 2018).
24. Ali, F., Sheikh, N. A., Saqib, M. & Khan, I. Unsteady MHD flow of second-grade fluid over an oscillating vertical plate with iso- thermal temperature in a porous medium with heat and mass transfer by using the Laplace transform technique. J. Porous Media 20(8), 671 (2017).