Mathematical model of braking process of car

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Abstract. A mathematical model of braking a car using differential equations has been compiled. The calculation scheme of the plane-parallel motion of the car during braking is given. When braking a car, three forces are considered in the contact of each wheel with the road: the braking force, the slip resistance force, and the normal reaction of the road. A diagram of the components of the braking force acting in the plane contact of the wheel with the road when sliding. A scheme has been drawn up to determine the normal reactions on the wheels of a car. To assess the preservation of the car's controllability, a function of the driver's control action is introduced into the mathematical model, imitating the algorithm of his action. The angular speed of the steering wheel is limited by the driver's capabilities, and in emergencies is 250 ... 350 deg / s. If the heading angle decreased with the introduction of the driver's control action function, then the skid was considered controlled.

1 Introduction

One of the most important problems associated with the motorization of society is the problem of ensuring road safety. Among the important areas for reducing the number and severity of the consequences of road traffic accidents (RTA) is the improvement in the operation of vehicles of the technical -condition of their brake control. It accounts for about 50% of accidents related to the faulty state of vehicles. At the same time, the established requirements for vehicle stability during braking are not clear enough and, therefore, do not -adequately contribute to ensuring road safety. This determines the urgency of improving the regulatory framework regulating the braking properties of a car in operating conditions and methods for their control. This work aims to develop a method for standardizing and monitoring the braking properties of passenger cars in operation.

2 Objects and methods of research

The research methodology included the collection of statistical material on the output parameters of the brake system of taxi cars in operation, theoretical analysis of the effect of changes in the technical state of brake systems on the stability of the vehicle braking process using mathematical modeling, and experimental verification of the results obtained. The object of the study is middle-class cars.

During emergency braking, under the influence of uneven braking forces, the system and the

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center of mass move along a curved path with a turn. (Fig.1) LATERAL force acting in the wheel's contact with the road is determined by the non-linear theory of slip of the elastic wheel [1]. The use of this theory makes it possible to more accurately reproduce the operating conditions of an elastic wheel in braking mode, with In this case, the main factors affecting the magnitude of the lateral force are normal load, slip angle, tangential force, and coefficient of adhesion of the wheel to the road.

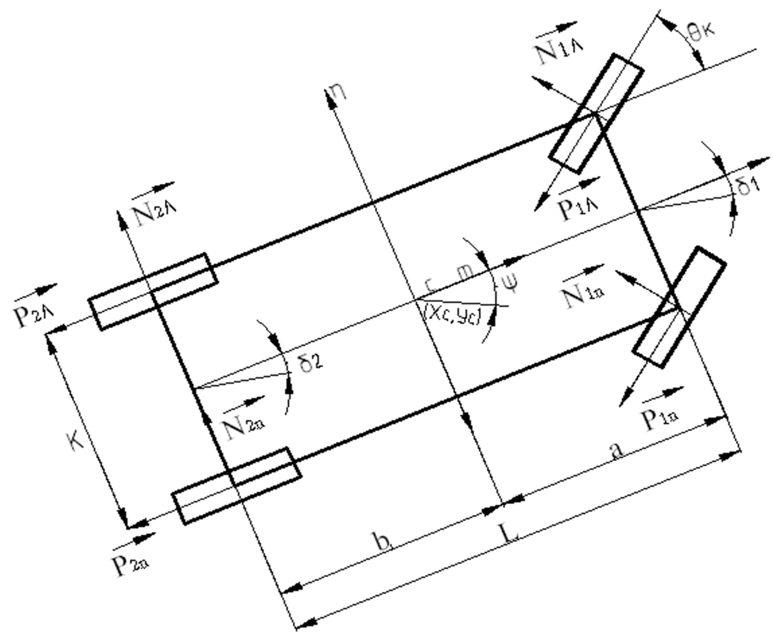

Fig. 1. Calculation scheme of plane-parallel motion car when braking

When deriving equations of motion of the system, the following assumptions are made [3]:

- the vehicle is an unsprung solid body,
- having a plane of symmetry;
- forces of rolling resistance and air resistance are absent;
- the moments of inertia of the rotating wheels are not taken into account;
- the car makes a plane-parallel movement;
- centrifugal force is distributed between the axles in inverse proportion to their distances from the center of mass of the car;
- the contact of the wheel with the road is considered a point contact [16-18].

Consider the car's movement concerning the inertial coordinate system *Oxy Z. The O Z* axis is perpendicular to the plane of motion and directed upwards. A moving system is connected to the vehicle coordinates $\xi\eta\chi$ with the origin at its center of mass. Axis χ directed perpendicular to the plane of motion, upward. Let us introduce the following notation:

X_o, Y_o are coordinates of the center of mass of the car; ψ is angle of the car's turn relative to the axis OX; m is mass of the car; I is the moment of inertia of the car about the axis χ; a, b are distance from the front and rear axles of the vehicle to the center of gravity; L is car base; H is height of the center of mass; θ_{κ} is kinematic angle of rotation of the steered wheels; $\theta_{вв}$ is the angle of rotation of the steered wheels due to the control action of the driver; $\theta = \theta_{к} + \theta_{dd}$ is the total angle of rotation of the steered wheels; ξ_i, η_i are projections of the speed of the center i - wheels on the moving coordinate axes; g is acceleration of gravity; $N_{1l(n)}$ is side slip resistance force i - wheels; $R_{1l(p)}$ is braking force on i - wheel; To is track of the car; $Z_{1l(n)}$ is normal reactions on the wheels of the car; φ is coefficient of adhesion of the wheel to the road.

Following the design scheme, the kinetic energy of the car is:

$$T = \frac{I \cdot \psi_p^2 + M(x^2 + y^2)}{2} \quad (1)$$

Using Lagrange's finite increment method, the differential equations of the car's motion will be written in the form:

$$\begin{aligned} m\ddot{x} &= (P_{1л} + P_{1n}) \cdot \cos(\theta + \psi_p) + (N_{1л} + N_{1n}) \cdot \sin(\theta + \psi_p) + \\ &+ (P_{2л} + P_{2n}) \cdot \cos\psi_p + (N_{2л} + N_{2n}) \cdot \sin\psi_p \end{aligned} \quad (2)$$

$$\begin{aligned} m\ddot{y} &= (N_{1л} + N_{1n}) \cdot \cos(\theta + \psi_p) - (P_{1л} + P_{1n}) \cdot \sin(\theta + \psi_p) + \\ &+ (N_{2л} + N_{2n}) \cdot \cos\psi_p - (P_{2л} + P_{2n}) \cdot \sin\psi_p \end{aligned} \quad (3)$$

$$\begin{aligned} J\ddot{\psi}_p &= \left[(P_{1л} - P_{1n}) \cdot \cos\theta + (N_{1л} - N_{1n}) \cdot \sin\theta + P_{2л} - P_{2n}\right] \cdot \kappa + \\ &+ \left[(P_{1л} + P_{1n}) \cdot \cos\theta + (N_{1л} + N_{1n}) \cdot \sin\theta\right] \cdot a - (N_{2л} + N_{2n}) \cdot b \end{aligned} \quad (4)$$

When solving the above system of equations, an approximate method of stepwise integration is used, according to which the quantities included in the right parts of the equations are substituted in the form of constants determined at the previous integration step.

When braking a car, three forces are considered in the contact of each wheel with the road: the braking force, the slip resistance force, and the normal reaction of the road. The braking force on the i-wheel, lying in the plane of the road, is applied at the point of contact and is directed in the opposite direction of its rolling [2, 20]. The values of the braking forces are determined by the following expressions:

$$P_i = P_{T\max i} \cdot K_{oi}^{рег} (1 - e^{-\alpha_i (t - t_c)}) \cdot F_{oi} \quad (5)$$

where : $P_{T\max i}$ is maximum braking force on the i -wheel at regulatory effort on the governing body; α_i is coefficient of intensity of growth of the drive efforts; t_{ci} is in the delay time of the braking force on the i - volume wheel; F_{oi} is step function that takes into account the delay brake drive; $K_{oi}^{рег}$ is coefficient that considers the presence of a brake force regulator in the drive and depends on its characteristics.

The slip resistance force lying in the plane of the road and directed perpendicular to the plane of the wheel is determined from the equation [5]:

$$N_i = q_{zi} \cdot q_{Ti} \cdot q_{\varphi i} \cdot K_{уоэо} \cdot \delta_i \quad (6)$$

Where: q_{zi} is coefficient taking into account the effect of the normal load on the wheel;

q_{Ti} is coefficient taking into account the influence of the braking force for withdrawal;

$q_{\varphi i}$ is coefficient that takes into account the influence of the grip of the wheel with the road on its withdrawal;

$K_{yoýi}$ is extreme value of the coefficient of resistance to lateral withdrawal;

δ_i is at the goal at water i - wheels.

- for front axle wheels: $\delta_{1л(n)} = arctg\left(\dfrac{\eta_i}{\xi_i}\right) - \theta$
- for rear axle wheels: $\delta_{2л(n)} = arctg\left(\dfrac{\eta_i}{\xi_i}\right)$,

where η_i, ξ_i are projections of the speed vector of the wheel center on the axis of the movable coordinate systems [6-9].

The slip angle is assumed to be positive if the lateral component of the wheel center velocity is directed away from the instantaneous center of velocities and negative in the opposite case.

When the braking force is equal to the limiting static friction force.

$$P_{Ti} = \sqrt{(z_i \cdot \varphi)^2 - N_i^2} \tag{7}$$

the forces acting in the wheel's contact with the road are calculated by the above formulas.

The condition for the transition from sliding to rolling is the condition:

$$P_{Ti} \prec \sqrt{(z_i \cdot \mu \cdot \varphi)^2 - N_i^2} \tag{8}$$

The normal reactions acting in the wheel's contact with the road, with a uniform movement of the car, are determined by its mass, the degree of loading, and the method of its distribution. When the car brakes, a redistribution of the normal reactions on the wheels occurs due to the centrifugal force and the forces acting in the plane of the supporting surface (Fig. 3), their magnitude can be determined from the expressions:

- for front axle wheels:

$$z_{1ё(ï)} = \frac{K \cdot m \cdot (g \cdot b + h \cdot \xi + \psi_{ð} \cdot \xi) \mp m \cdot \xi \cdot \psi_{ð} \cdot b}{L \cdot K} \tag{9}$$

- for rear axle wheels :

$$z_{2ё(ï)} = \frac{K \cdot m \cdot (g \cdot a + h \cdot \xi + \psi_{ð} \cdot \xi) \pm m \cdot \xi \cdot \psi_{ð} \cdot a}{L \cdot K} \tag{10}$$

Sign " + " **V** formulas corresponds to internal, relative **to** the instantaneous center of speed wheels, and "- " respectively external.

To assess the preservation of the car's controllability at the end of the considered time interval, a function of the driver's control action is introduced into the mathematical model, imitating the following algorithm of its action. When a lateral shift occurs, accompanied by an increase in the angular turn rate, the driver begins to turn the steering wheel. At the same time, he seeks to align the point located on the continuation of the longitudinal axis of the car with the center line of the corridor of movement so that the velocity vector of this point (called the "guiding point" in the theory of controllability and stability of the car coincides with the axis of the corridor [13-15**]**.

The above function was used for verification calculations of those options in which the value of the heading angle at time t_{rsh} exceeded 10°. If, with the introduction of the driver's control action function, the heading angle decreased, then the skid was considered controlled.

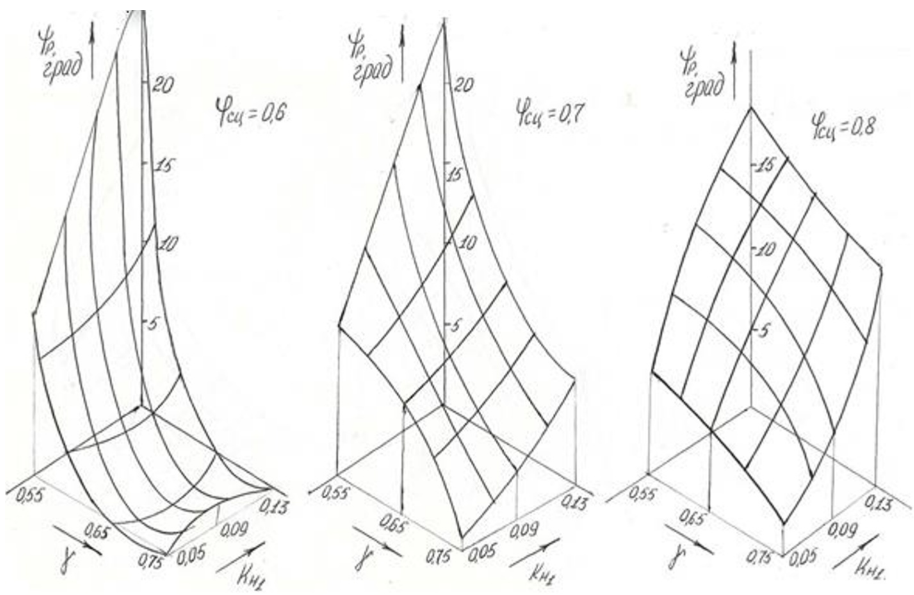

Fig. 2. The dependence of the angle of the car's turn during the total braking time on the magnitude of the uneven braking forces on the wheels of the front axle and the specific braking force. Curb weight. $V = 40 km/h$, without RTS

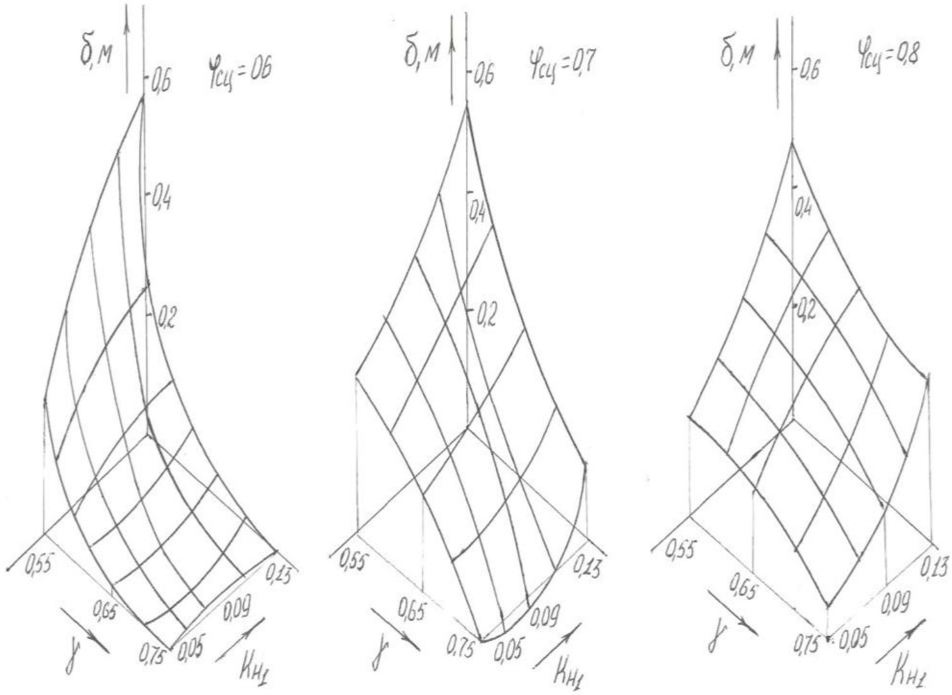

Fig. 3. The dependence of the lateral deviation of the car during the total braking time on the magnitude of the uneven braking forces on the wheels of the front axle and the specific braking force. Curb weight. $V = 40\ km/h$, without RTS

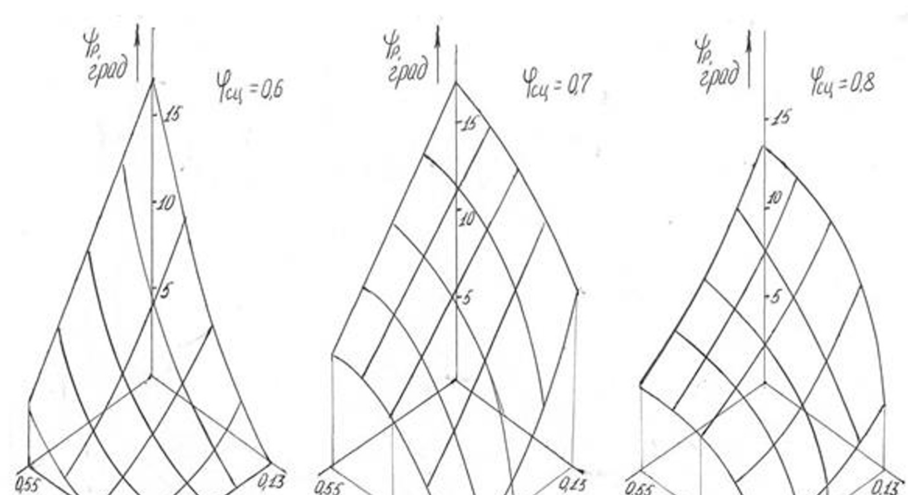

Fig.4. The dependence of the car's turn angle for the total braking time on the magnitude of the uneven braking forces on the wheels of the front axle and specific braking force.
Full mass. $V = 40 km/h$.

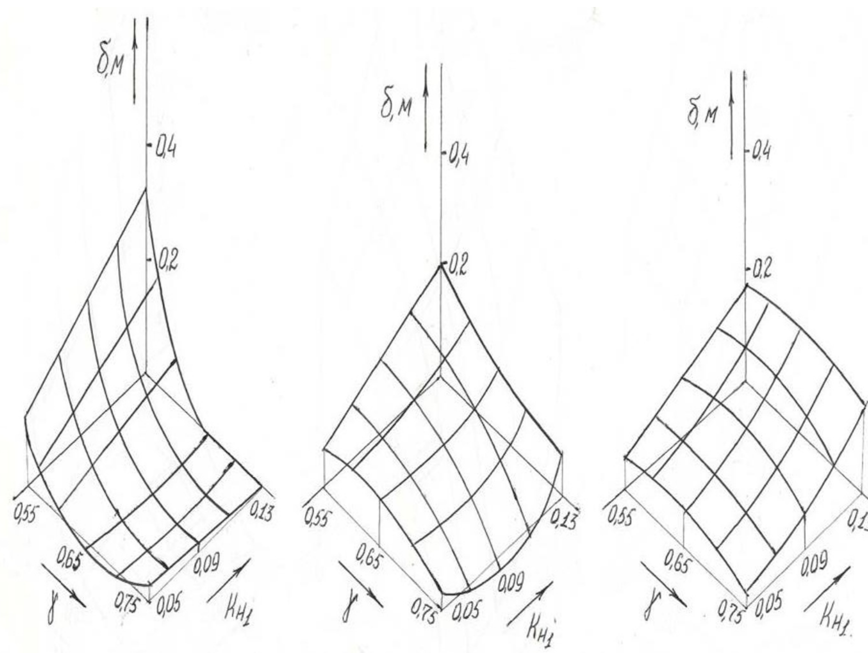

Fig.5. Dependence of the lateral deflection of the car during the total braking time on the magnitude of the uneven braking forces on the wheels of the front axle and the specific braking force. Full mass. V =40km/h.

3 Results and Discussion

When analyzing the behavior of the car during the reaction time of the driver, the angle of the car's turn was taken as the estimated indicator for the options in which the rear axle wheels are blocked and in the absence of their blocking (curb weight with RTS), lateral or linear deviation. This is since in the first case, the greatest danger is the car getting into an uncontrolled skid and, in the second case, going beyond the safety corridor. On fig. Figures 2...5 show the graphs of the turning angles and lateral deviation of the simulated car when braking from a speed of 40 km/h; the largest turning angles correspond to the - equipped state of the car. It can be seen from the graphs that with an increase in the friction coefficient and uneven braking forces on the front axle, as well as with their magnitudes, the turning angle increases . When braking with ($\gamma \geq 0.64$) and $\varphi_{st} = 0.6$, there is a zone of minimum turning angles; due to a decrease in the disturbing factor due to the blocking of the front wheels, with an increase in φ_{st}, this zone disappears.

In the presence of RTS in the rear axle brake circuit, the slip angles during the driver's reaction time are insignificant, and the lateral deviation depends on the combination of the magnitude of the braking forces of the front axle, their unevenness, and the adhesion coefficient, the maximum lateral deviations correspond to the minimum braking forces on the front axle wheels, the minimum value of φ_{cs} and the maximum value of $K_{н1}$. However, even with $K_{n1} = 0.13$ and $\gamma = 0.55$, the car is within the safety corridor.

4 Conclusion

Based on the results of theoretical studies, the following conclusions can be formulated:

1. When testing vehicles under the conditions prescribed by GOST I 25478-91, it should be borne in mind that the heading angle and the magnitude of the lateral deviation significantly depend not only on the magnitude of the axial unevenness of the braking forces but also on the distribution of the braking forces along the axles and the coefficient of adhesion of the wheels to the road when changing it within limits corresponding to the type of coating specified in GOST.

A decrease in the effectiveness of the front axle brakes and an increase in the coefficient of adhesion of the wheels to the road with the same axial uneven braking forces lead to an increase in lateral deviation and heading angle; however, this is true for the case of emergency braking, accompanied by blocking of the rear wheels. If the brake force regulator is in good condition and there is no blocking and leveling of the rear wheels, then the deviations of the car from the set course are minimal.

2. The most dangerous combination of parameters of the brake system in terms of loss of stability during braking is the reduction in the efficiency of the front axle brakes and the presence of uneven braking forces on it. Moreover, emergency braking of a loaded car is more dangerous than an equipped one, which is explained by the magnitude and duration of the turning moment due to the uneven braking forces of the front axle are greater than that of an equipped car.

3. The unevenness of the braking forces of the car's rear axle affects the stability during braking much less than the front axle. If it is present, the car goes beyond the braking efficiency limits faster than the standard for stability.

4. During emergency braking, the vehicle remains stable only if the wheels of both axles are blocked or if the wheels of the rear axle are not blocked, regardless of whether the front wheels are blocked.

5. The analysis of options showed that, concerning passenger cars, compliance with the standard γ=0.64 and axial unevenness of braking forces on both axles K n =0.09 does not guarantee vehicle stability during emergency braking, which indicates the existing shortcomings in the accepted system of standardization of brake performance. These shortcomings result from the actual implementation of the braking forces and their distribution along the axes are not taken into account. A simple tightening of standards to bring them closer to the values of the corresponding new vehicles in operating conditions is inefficient since it significantly increases the cost of maintaining the technical condition of the rolling stock at the required level. In addition, tightening, for example, the coefficient K_{H1} to a value of 0.05, which is less than allowed for new cars (K_{H1} = 0.08) in some cases, does not guarantee the car's stability. Thus, a new method for normalizing vehicle stability indicators is required, which takes into account the dynamics of the braking process under operating conditions.

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