

# Upheaval buckling of underground pipelines of complex configuration located in liquefied soils

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**Abstract.** The problem of loss of stability of underground pipelines with a middle part in the form of a straight or  $\Pi$ -shaped part located in liquefied soils is considered in the article. The pipeline lifting process occurs under the action of buoyancy force in the liquefied soil zone and then under the action of the longitudinal compressive force that appears due to the temperature of the transported product and its pressure. The problem is solved by the finite element method. The results are presented as a table and graphs of changes in the values of transverse displacements along the coordinate depending on the pipeline geometric characteristics and the soil rheological properties. It is established that the  $\Pi$ -shaped part is a damper against pipeline buckling.

## 1 Introduction

Underground pipeline systems for various purposes work under high operational loads. This is especially true for pipelines intended to transport gaseous substances, liquefied masses, solid fuels, and other solid substances in the form of a mixture. Due to their large extent, such systems are subject to seismic effects.

Sufficient material has accumulated on damage and destruction of underground constructions due to powerful earthquakes. There have been repeated historical events of lifting of buried constructions during strong earthquakes in San Francisco (1906), California (1952), Niigata (1964), Miyagi (1972), Kum Dag (1983), Loma Prieta (1989), Kushiro-Oki (1993), Northridge (1994), Kobe (1995), Chi-Chi (1999), Kocaeli (1999), Tokachi-Oki (2003), Niigataken-Chuetsu (2004), Chuetsu Offshore Earthquake (2007), Darfield (2010), Christchurch (2011) and Turkey (2023) [1–9].

If the territory of liquefied ground is large, then the questions of buckling of pipelines come to the fore. In many events, pipelines are laid at shallow depths; therefore, the limit pipes motion is approximately 1 meter [10].

Events in history are valuable for supervising the capacity of construction on various occasions. Nevertheless, if the network of buried pipelines operates properly during and after possible earthquakes and surrounding soil liquefaction has not occurred near the

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ground surface. It is impossible to find out deformations of buried pipes, leading to their detriment and destruction.

So, real observations have several limitations: it is impossible to discover subsurface deformation, and it is impossible to get data about various arguments that influence the processes of detriment and collapse of constructions. Though, experimental modeling overcomes that restriction. Realty, it may require several days, months, and sometimes several years to establish the effect of an argument on pipes response. But this limitation allows overcoming numerical modeling, which allows studying a huge number of various events.

Experimental studies were held on the shaking table on the buckling of underground constructions [11]. The formation of empty space was revealed throughout the entire lifting process directly under construction. In [12], centrifuge experiments were carried out to analyze the influence of parameters on the rising displacement of underground construction during soil liquefaction. The experiments showed that the rise begins after the liquefaction starts and continues as long as the vibrations continue. An attempt was made to explain the mechanism of lifting pipes in liquefied soils [13]. The ground deformation within the rise of the pipeline under seismic impact was learned in full-scale model tests in [14]. The rise of construction was investigated via FEM (the finite element method) in [15]. In [16], the coupled finite element stress-flow procedure was improved to explore the rise of the underground pipes in the liquefied surrounding soil. Several problems in the study of the stability of underground pipelines with an initial deflection were solved in [17–19]. An extensive review of real observations, computational methods, and numerical simulations linked to the stability of underground pipelines in liquefied soils was presented in [20]. The papers investigate the seismodynamics of spatial systems of underground pipelines based on records of real earthquakes [21, 22].

Compensators in the form of geometrically complicated parts as a measure of protection against pipeline buckling were not previously taken into account. That's why the influence of a  $\Pi$ -shaped damper on the stability of the buried pipeline laid in liquefied soil is considered in this article.

Thus, the underground lifeline pipelines receive a lot of destruction, and a part of the network is turned off; it seriously threatens the social life of the population and leads to large economic losses. The main factors affecting the buckling of buried pipelines are geological and hydrogeological conditions, efficiency and technological loads and impacts, the configuration of the pipe, pipe material characteristics, service life, etc. Therefore, a preliminary assessment of the danger from static and dynamic loads and the development of appropriate protective measures is important to reduce pipeline damage. So it is necessary to ensure the stability of underground structures. Due to this, conducting specific scientific research to improve numerical methods for solving problems of stability of buried complicated pipelines located in liquefied soils is actual and necessary.

## **2 Objects and methods of research**

The problem of buckling a buried pipeline of finite length in a static formulation is considered. It is assumed that an earthquake brings the soil into a liquefied state in a certain area where the pipeline is buried, and the process of the uplifting pipeline occurs after the passage of seismic waves. The loss of pipeline stability is the state when it uplifts above the ground surface in the liquefied area of the soil. It is assumed that one part of the pipeline is in the liquefied soil, and the other two parts of the pipeline are in the natural soil. The pipeline is subjected along the axis to the action of a longitudinal compressive force, which occurs due to the temperature and pressure of the transported product. The interaction of the pipeline with the surrounding soil is described by the model given in [23–25].

The problem is non-linear, so the finite element method is used to solve this problem. The pipeline is divided into linear finite elements. It is known that a set of linear parts can approximate any curve.

The linear finite element is considered in the local coordinate system. The origin of the local coordinate system  $O\xi\eta\zeta$  is located at the center of the first section of the linear finite element of the pipe [26]. Axis  $O\xi$  is directed along the pipeline longitudinal axis, and axes  $O\eta$  and  $O\zeta$  are directed along the main central axes of the pipeline cross-section.

The linear section of the pipeline is modeled as a bar element, then the differential equilibrium equation of this element in an elastic medium has the form

$$\begin{aligned} -EF \frac{\partial^2 u}{\partial \xi^2} + K_\xi u &= 0 \\ -GJ_\xi \frac{\partial^2 \varphi}{\partial \xi^2} + K_\varphi \varphi &= 0 \\ EJ_\zeta \frac{\partial^4 w_\eta}{\partial \xi^4} + (P + kGF) \frac{\partial^2 w_\eta}{\partial \xi^2} + K_\eta w_\eta &= 0 \\ EJ_\eta \frac{\partial^4 (w_\zeta - w_0)}{\partial \xi^4} + (P + kGF) \frac{\partial^2 w_\zeta}{\partial \xi^2} + K_\zeta w_\zeta &= q_\zeta \end{aligned} \quad (1)$$

where  $EF$ ,  $EJ_\eta$ ,  $EJ_\zeta$ ,  $GJ_\xi$  are the longitudinal, bending in direction  $\eta$ ,  $\zeta$  and torsional stiffness of the bar per unit length;  $u$ ,  $w_\eta$ ,  $w_\zeta$  are the displacements of points of the median longitudinal axis of the bar along the axes  $\xi$ ,  $\eta$ ,  $\zeta$ ;  $w_0$  is the initial deflection;  $\varphi$  is the angle of rotation of the cross-section of the bar around the axis  $\xi$ ;  $P = EF \left( \frac{\partial u}{\partial \xi} - \varepsilon_0 - \alpha T \right)$  is the

longitudinal force;  $\varepsilon_0$ ,  $T$ ,  $\alpha$  are the preliminary longitudinal deformation, the temperature of the transported product in the pipeline and the coefficient of linear expansion of the pipe material;  $K_\xi$ ,  $K_\eta$ ,  $K_\zeta$  are the soil stiffness per unit length in the direction of axes  $\xi$ ,  $\eta$ ,  $\zeta$ ;  $K_\varphi$  is torsion resistance per unit of soil stiffness;  $q_\xi$ ,  $q_\eta$ ,  $q_\zeta$ ,  $m$  are the intensity of distributed external loads;  $G$  is the shear modulus;  $F$  is the pipeline cross-sectional area;  $k$  is the efficiency coefficient of shear area.

The temperature and pressure of the gas entering the pipeline should be considered when calculating the gas pipeline for stability. As noted above, the equivalent axial compression force  $P$  is appeared due to changes in temperature and high pressure of the gas; the following formula determines Force  $P$  [27]:

$$P = \left( \alpha E \Delta t + \frac{\mu p D}{2\delta} \right)$$

where  $\alpha$  is the coefficient of linear expansion of metal;  $E$  is the modulus of elasticity of the material;  $\Delta t$  is the temperature difference;  $\mu$  is Poisson's ratio;  $p$  is the internal pressure;  $D$  is the pipeline outside diameter;  $\delta$  is the pipe wall thickness.

The coefficients  $K_\xi$ ,  $K_\eta$ ,  $K_\zeta$  and  $K_\varphi$  are equal to zero in the zone of soil liquefaction. The uplifting buoyancy force acts in this zone, and it is determined by the formula

$$q_{axz} = \pi \cdot \frac{D^2}{4} \cdot g \cdot \rho_l$$

where  $\rho_l$  is the density of the liquefied soil. Then the resulting force, consisting of the own weight of a unit length of the pipeline and buoyancy force, has the form

$$q_{ax} = \pi \cdot \frac{D^2}{4} \cdot g \cdot \rho_l - \pi \cdot \left( \frac{D^2}{4} - \left( \frac{D}{2} - \delta \right)^2 \right) \cdot g \cdot \rho \quad (2)$$

where  $\rho$  is the density of the pipeline material.

The boundary conditions for one finite element are given as the values of six displacements and six rotations at its ends

$$\begin{aligned} u = u_0; w_\eta = w_{\eta 0}, w_\zeta = w_{\zeta 0}, \varphi = \varphi_0, \varphi_\eta = \varphi_{\eta 0}, \varphi_\zeta = \varphi_{\zeta 0} & \quad \text{when } \xi = 0; \\ u = u_l; w_\eta = w_{\eta l}, w_\zeta = w_{\zeta l}, \varphi = \varphi_l, \varphi_\eta = \varphi_{\eta l}, \varphi_\zeta = \varphi_{\zeta l} & \quad \text{when } \xi = \ell; \end{aligned} \quad (3)$$

Stiffness and interaction matrices are constructed to obtain the pipeline stability equation, and then they are summed over all finite elements to obtain a non-linear system of algebraic equations.

The non-linear problem of pipeline stability is solved in two stages. In the first stage, the problem of uplifting the pipeline by buoyancy force is considered under the following boundary conditions at the ends of the pipeline in the global Cartesian coordinate system; axis  $OX$  coincides with the axis of the main part of the pipeline, the origin of coordinates is located on its left end

$$\begin{aligned} u = 0; w_y = 0, w_z = 0, \varphi = 0, \varphi_y = 0, \varphi_z = 0 & \quad \text{when } x = 0; \\ u = 0; w_y = 0, w_z = 0, \varphi = 0, \varphi_y = 0, \varphi_z = 0 & \quad \text{when } x = \ell; \end{aligned}$$

The resulting solution is denoted by  $\{U_1\}$ .

At the second stage, a correction is introduced according to the values of  $\{U_1\}$  for the values of the coordinates of the nodal points of the finite elements, and the problem is solved under the action of the axial force acting on the ends of the pipeline

$$\begin{aligned} F_x = F_{pr}; w_y = 0, w_z = 0, \varphi = 0, \varphi_y = 0, \varphi_z = 0 & \quad \text{when } x = 0; \\ F_x = -F_{pr}; w_y = 0, w_z = 0, \varphi = 0, \varphi_y = 0, \varphi_z = 0 & \quad \text{when } x = \ell; \end{aligned}$$

The vector  $\{F\}$  contains elements with given external, temperature forces and forces caused by the internal pressure of the transported gas.

The Cholesky method for profile storage of stiffness matrices and interaction of the spatial system of the buried pipeline is used to solve the system of algebraic equations [28].

### 3 Results and discussion

The pipeline with a length of 100 m is considered. The middle part of the pipeline is located in the liquefied soil. The length of the pipeline, located in the liquefied soil, is chosen in three variants: 40 m and 60 m. Steel pipes with external diameters of 1.02 m, 0.53 m, and 0.219 m were used in the calculations. The remaining characteristics are as follows:  $E=2 \cdot 10^5$  MPa;  $G=7.69 \cdot 10^4$  MPa;  $\rho=7.8 \cdot 10^3$  kg/m<sup>3</sup>;  $\rho_{soil}=2 \cdot 10^3$  kg/m<sup>3</sup>;  $l=100$  m;  $\mu_{tube}=0.3$ ;  $D_H=1.02$  m;  $\delta=0.008$  m;  $EF=5.09 \cdot 10^9$  N;  $EJ_\eta = EJ_\zeta = 6.51 \cdot 10^8$  N·m<sup>2</sup>;  $GJ_\xi = 1.96 \cdot 10^9$  N;  $K_\zeta = 3.2 \cdot 10^7$  N/m<sup>2</sup>;  $K_\eta = K_\zeta = 8.33 \cdot 10^7$  N/m<sup>2</sup>;  $D_H=0.53$  m;  $\delta=0.007$  m;  $EF=2.3 \cdot 10^9$  N;  $EJ_\eta =$

$EJ_{\zeta}=7.87 \cdot 10^7 \text{ N} \cdot \text{m}^2$ ;  $GJ_{\xi}=8.85 \cdot 10^8 \text{ N}$ ;  $K_{\zeta}=1.67 \cdot 10^7 \text{ N/m}^2$ ;  $K_{\eta}=K_{\zeta}=4.33 \cdot 10^7 \text{ N/m}^2$ ;  
 $D_H=0.219 \text{ m}$ ;  $\delta=0.003 \text{ m}$ ;  $EF=4.07 \cdot 10^8 \text{ N}$ ;  $EJ_{\eta}=EJ_{\zeta}=2.37 \cdot 10^6 \text{ N} \cdot \text{m}^2$ ;  $GJ_{\xi}=1.57 \cdot 10^8 \text{ N}$ ;  
 $K_{\zeta}=6.88 \cdot 10^6 \text{ N/m}^2$ ;  $K_{\eta}=K_{\zeta}=1.79 \cdot 10^7 \text{ N/m}^2$ .

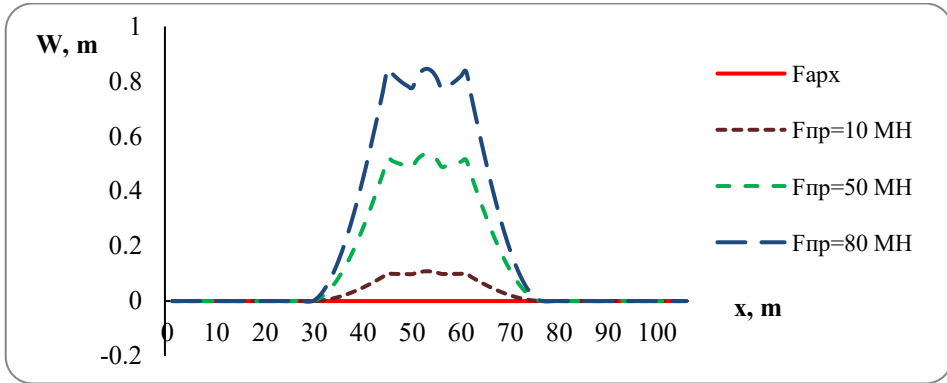
The results of calculations with the given data are discussed. The results of solving the problem are presented in the form of graphs. Table 1 and Fig. 1 – 3 show the processes of changing the uplifting straight pipeline and a pipeline with a  $\Pi$ -shaped part (dimensions of a  $\Pi$ -shaped configuration  $h=l=3$  and 5 m, where  $h$  and  $l$  are the height and length, respectively) at various values of the acting Archimedes power in the zone of liquefied soil and longitudinal compressive force. The graphs show significant changes in the process of uplifting the pipeline located in the sand with a liquefied zone. The process of buckling of the pipe is observed in this zone.

Under buoyancy force, the middle part of a straight pipeline located in liquefied soil is bent, taking the form of a half-wave of a sinusoid. For a visual comparison, the results are shown in the form of Table 1 only for the values of maximum transverse displacements of underground pipelines. After the action of buoyancy force, the axial compression force acts on the pipeline from both sides at a distance of 10 m from the liquefied part. As can be seen from the analysis of Table 1 that with an increase in the liquefied part, the axial force raises the pipeline less because the curvature of the pipe decreases due to its deformation.

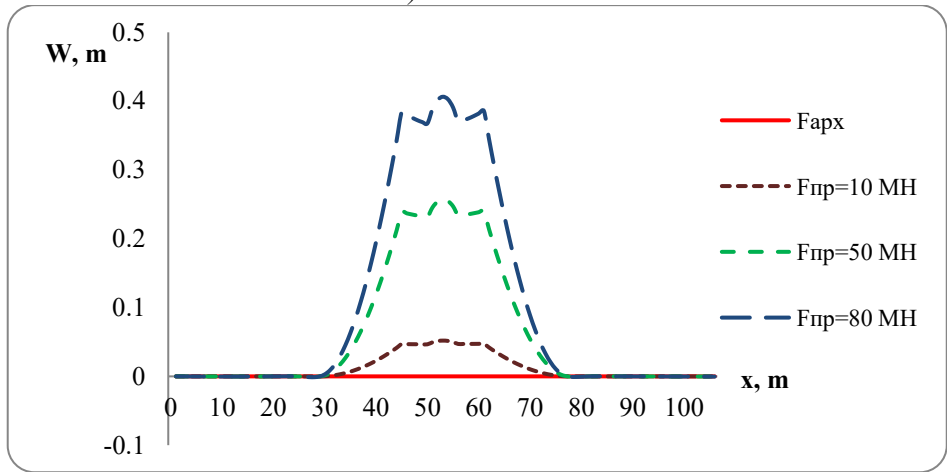
**Table 1.** The values of the maximum transverse displacements of the straight gas pipeline

Pipe diameter (m)	Liquefied area (m)	Lifting the pipe under the action of buoyancy force (m)	Lifting the pipe under the action of the force of 10 MN (m)	Lifting the pipe under the action of the force of 50 MN (m)	Lifting the pipe under the action of the force of 80 MN (m)
$D_H=0.53$	40	$3.21 \cdot 10^{-8}$	0.321	1.61	2.58
	60	$2.84 \cdot 10^{-8}$	0.284	1.43	2.31
$D_H=1.02$	60	$1.43 \cdot 10^{-8}$	0.143	0.723	1.16
	80	$1.31 \cdot 10^{-8}$	0.131	0.661	1.06

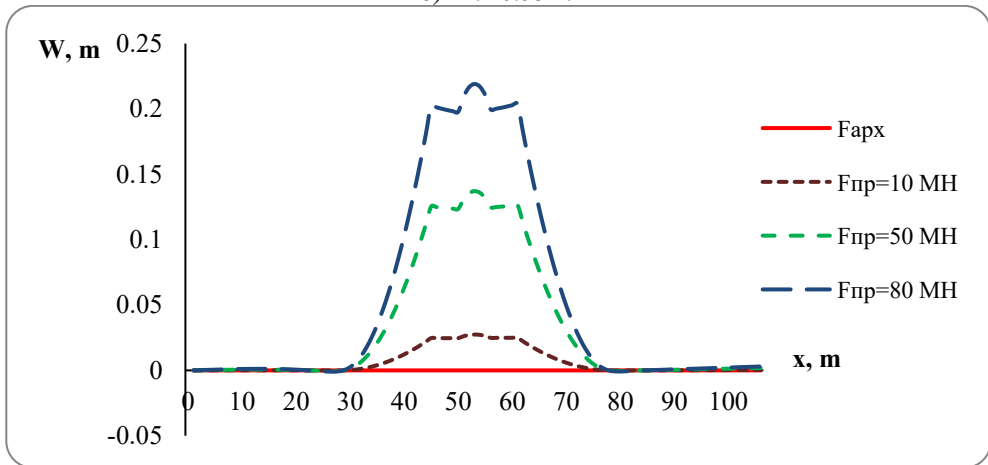
Fig. 1 illustrates the changes in the values of vertical transverse displacements along the axis of the underground pipeline when the middle part of the pipeline, equal to 40 m, is located in liquefied soil. The larger the diameter of the underground pipeline, the greater the value of buoyancy force; it affects the pipe geometry, i.e., its initial deflection. On the other hand, the larger the pipeline diameter, the greater its flexural rigidity. It has been found that the larger the diameter, the smaller the uplifting displacement (see Fig. 1). I.e., pipes with a larger diameter are more stable. For pipelines with a small diameter, with a liquefied area of 60 and 80 m, instability is revealed.



a)  $D_H=0.219$  m



b)  $D_H=0.53$  m



c)  $D_H=1.02$  m

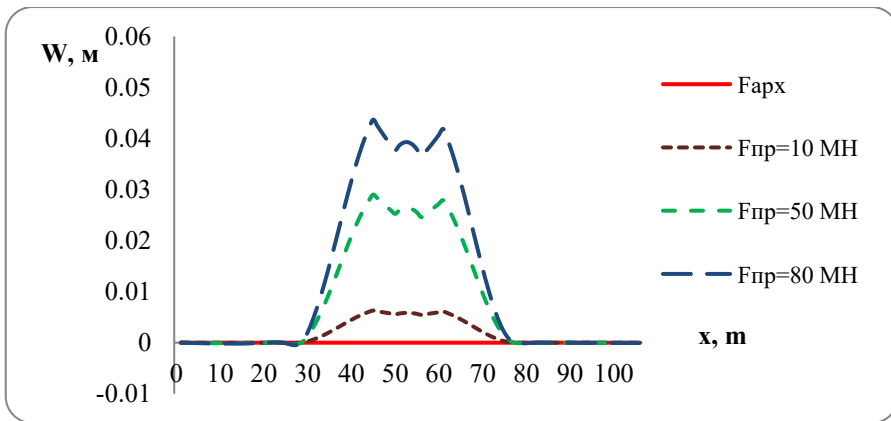
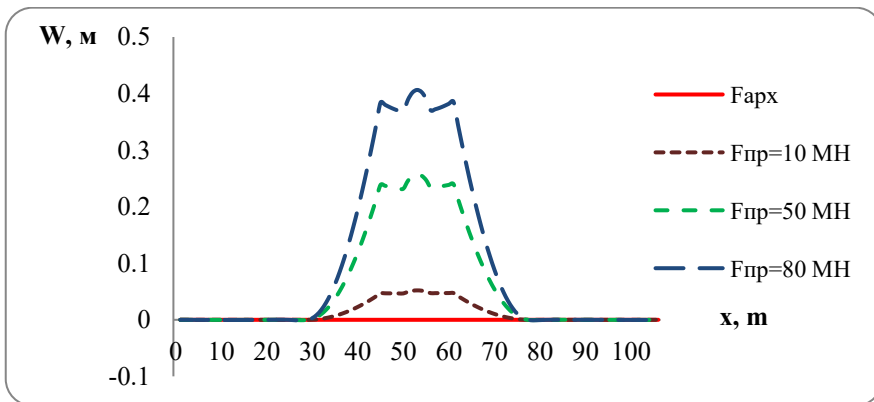
**Fig. 1.** Buckling of the buried gas pipeline with a  $\Pi$ -shaped part along axis  $Z$  with a configuration equal to 5 m:  $F_{apx}=579.3$  H;  $F_{np}=(10$  MH, 50 MH, 80 MH); liquefied area 40 m.

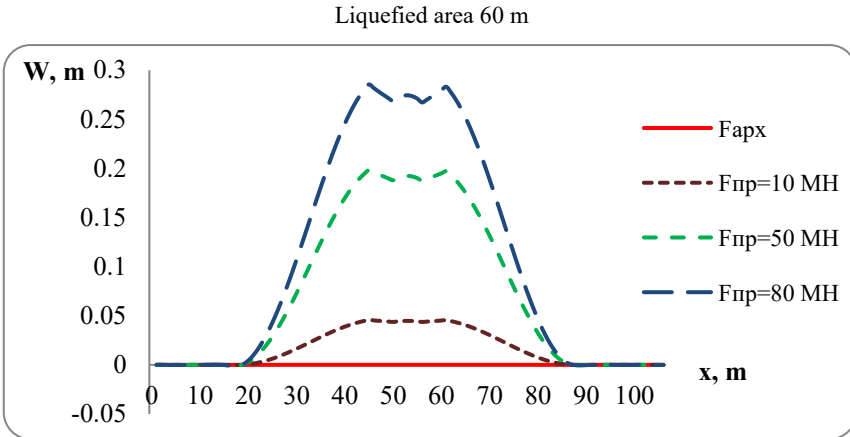
And with increased pressure and temperature of the transported product in the main gas pipelines, the value of the axial compression force increases accordingly. Fig. 1 – 3 illustrate that the greater the value of the longitudinal compressive force, the greater the maximum value of the uplifting displacement of the underground pipeline.

The following results are obtained to evaluate the effect of liquefied soil on pipeline stability. The graphs show that the length of the liquefied area significantly impacts the uplift of underground pipelines. It was found that the larger the range of the liquefied area (40 and 60 m), the greater the value of the uplifting displacement (see Fig. 1 – 3).

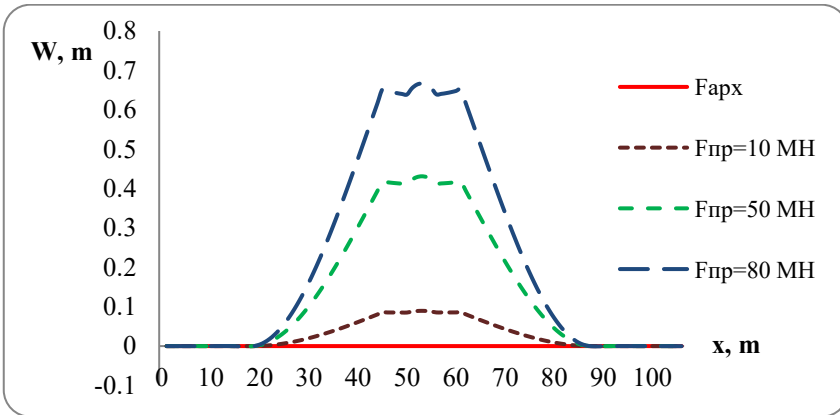
A complex process of distribution of values of transverse displacements along the axis of the underground pipeline is observed on graphs with a  $\Pi$ -shaped part in the liquefaction zone (see Fig. 1 – 3). Underground pipelines with a geometrically complex  $\Pi$ -shaped part in the liquefied areas have lower values of uplifting displacements than straight pipelines in the same areas due to the mutual influence of bending displacements and longitudinal force (see Fig. 1 – 3). Thus, the  $\Pi$ -shaped part is a damper in the process of buckling the pipeline.

Liquefied area 40 m

a)  $\Pi$ -shaped part of the pipeline along axis Y with a configuration equal to 5 mb)  $\Pi$ -shaped part of the pipeline along axis Z with a configuration equal to 5 m

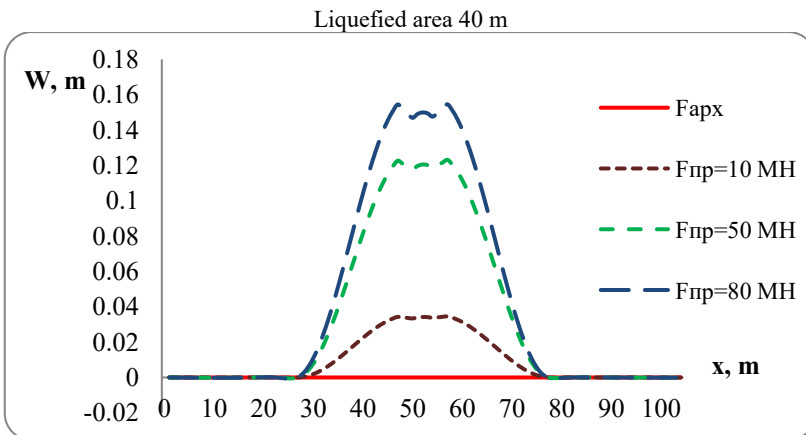


c)  $\Pi$ -shaped part of the pipeline along axis Y with a configuration equal to 5 m



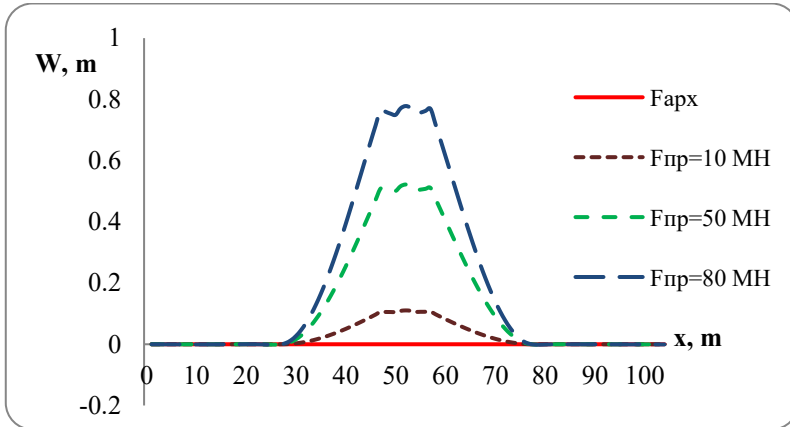
d)  $\Pi$ -shaped part of the pipeline along axis Z with a configuration equal to 5 m

**Fig. 2.** Buckling of the gas pipeline located in the soil:  $D_H=0.53$  m;  $F_{apx}=14036$  N;  $F_{np}=(10$  MN, 50 MN, 80 MN)

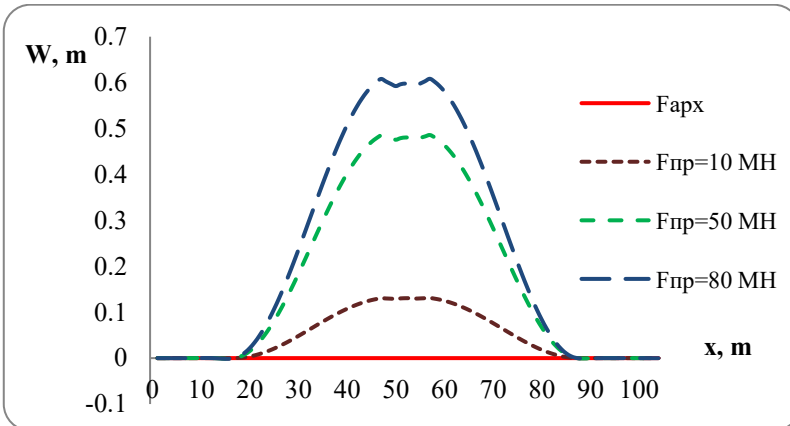


a)  $\Pi$ -shaped part of the pipeline along axis Y with a configuration equal to 3 m

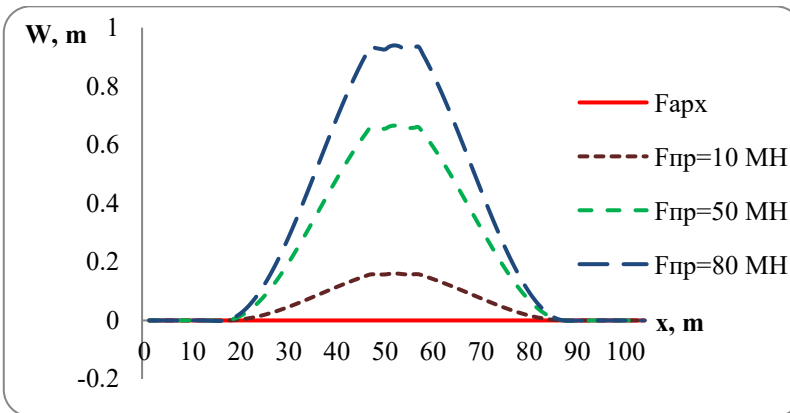




b)  $\Pi$ -shaped part of the pipeline along axis Z with a configuration equal to 3 m  
Liquefied area 60 m



c)  $\Pi$ -shaped part of the pipeline along axis Y with a configuration equal to 3 m



d)  $\Pi$ -shaped part of the pipeline along axis Z with a configuration equal to 3 m

**Fig. 3.** Buckling of the gas pipeline located in the soil:  $D_H=0.53$  m;  $F_{apx}=14036$  N;  $F_{np}=(10$  MH, 50 MN, 80 MN)

As seen from Fig. 1–3, a significant reduction in the uplifting of underground pipelines can be achieved by using dampers in different positions relative to the axis of the pipeline. From a comparison of graphs with a geometrically complex part along the Y and Z axes (see Fig. 2 and 3), it was found that underground pipelines with a  $\Pi$ -shaped damper along the Y axis have lower lift values, as it should be because the curvilinear configuration along the Z axis contributes to the increase in the uplifting of the pipe. For example, a pipeline with a  $\Pi$ -shaped damper along the Z axis has dimensions of 5 m: 1 m is located in the soil, and 4 m is above the ground surface, accelerating the buckling process of the pipe due to the lack of soil resistance. In the considered examples, the difference between the results of solutions with compensators along the Y and Z axes is 10 ... 17%. That's why the  $\Pi$ -shaped damper along the Y-axis is the best damper against pipeline buckling.

The presence of compensators with a complex configuration in underground pipelines plays an important role, as they help to reduce the uplifting of the pipeline from under the ground. As seen from a comparison of Fig. 2 and Fig. 3, the geometric dimensions of the compensator significantly affect the values of vertical displacement: the larger the geometric dimensions of the  $\Pi$ -shaped damper (the dimensions of the damper  $h=l=3$  and 5 m, where  $h$  and  $l$  are the height and length of the damper, respectively), the more stable the pipeline.

## 4 Conclusion

The problem of the stability of underground pipelines with a middle part in the form of a straight and  $\Pi$ -shaped part located in liquefied soils is considered. It has been established that with increasing pressure and temperature of the transported product in the main gas pipelines, the maximum value of the uplifting displacement of the pipe becomes greater. It was also found that the maximum value of the uplifting displacement of the pipe depends on the diameter of the underground pipeline and the range of the liquefied soil: the larger the diameter of the pipe, the lower the value of the lifting displacement, and the larger the range of the liquefied section, the greater the value of the lifting displacement. Pipes with larger diameters are more stable.

There is a significant change in the vertical, lateral displacements in the liquefied areas, which can lead to buckling in these areas. A significant reduction in the vertical transverse displacements of underground pipelines can be achieved when designing pipelines with a complex construction in the form of a  $\Pi$ -shaped part in the zone of liquefied soil. The presence of a  $\Pi$ -shaped part reduces the uplifting of the pipe, i.e., it is a damper. Thus, as a countermeasure against buckling of the underground pipeline in the area of liquefied soils, it is possible to offer curvilinear parts of  $\Pi$ -shaped pipelines.

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