Modeling of cargo flow processes in the logistics of the transport system

Jonibek Usmonov*, *Tulkindjon* Djuraev, *Nodira* Malikova, *and Avaz* Kuvnakov

Tashkent University of Information Technologies Named After Muhammad Al-Khwarizmi, Tashkent, Uzbekistan

Abstract. The management of cargo flows and transport activities is one of the most significant challenges currently. This article aims to discuss optimizing material flows in both transport and cargo management. It also develops mathematical software and material flow structures to facilitate management and production functions in supply enterprises. The article solves regulatory and forecasting issues using material flows and information models to optimize vehicle operational forms.

1 Introduction

Production (service) activities of all sectors of the economy of the Republic of Uzbekistan are carried out due to the movement of flow processes: material, financial, information and service .

Therefore, the movement of material flows, that is, finished products, raw materials, fuel resources, construction materials, agricultural products, industrial waste, etc. By space, it is necessary to navigate through the shortest distance, at the lowest cost [1].

Since the industries producing material goods are also branched, the nomenclature of manufactured and cultivated products is correspondingly numerous, the issue of optimizing the movement of cargo flows between enterprises and organizations is extremely relevant from a scientific and practical point of view[2, 3].

In general, the process of transporting raw materials for the construction of highways to factories for their processing consists of two stages [4]:

- transportation of raw materials of construction materials from quarries to processing plants (workshops) or warehouses;
- transportation of semi-finished raw materials from processing plants or warehouses-to factories for the production of finished products or to consumers.

Optimization of the flows of raw materials or semi-finished products required for transportation allows you to determine a plan that ensures a minimum of transportation work performed [5].

*Corresponding author: max_2011@inbox.ru

2 Methods and model

The formulation of the problem of optimizing cargo flows is formally expressed in the form of a linear programming transport model and formulated as follows [6–8].

Given the Sets of numbers of quarries that send raw materials $\{1,2,\ldots,I,\ldots,m\}$ and a set of numbers of plants that accept raw materials for processing $J=\{1,2,\ldots,\mathrm{j},\ldots\mathrm{n},\}$. The distances between all quarries or warehouses with the number $\boldsymbol{i}$ and factories with the number $\boldsymbol{j}$ are known from the L_{ij} matrix $||L_{ij}||\ I_{ij}$. In addition, the raw material shipment volume for each sending digital quarry or warehouse is a_i, and the raw material processing requirements for the digital prefabricated building materials plant J are b_i values. We formulate a mathematical model of the problem of optimizing raw material flows [3, 9] .

It is necessary to determine such positive values X_{ij} of the flow of construction raw materials between each i and j, that is

$$X_{ij}\geq 0,\qquad i\epsilon I,\qquad j\epsilon J \tag{1}$$

In this case, the flow of raw materials transported from each i quarry or warehouse to all $j\epsilon J$ plants $\sum_j X_{ij}$ does not exceed its shipping capacity a_i, i.e.

$$\sum_{j\epsilon J} X_{ij}\ \geq a_i \qquad i\in I \tag{2}$$

The flow of raw materials transported to each plant for the production of finished products, $\sum_j X_{ij}$ means that its ability to process raw materials cannot exceed a_i, i.e.

$$\sum_i X_{ij}\ \leq b_i,\quad j\in J \tag{3}$$

The volume of transport work performed by the sender and recipient of the interceptor flows of transportation of raw materials should be the smallest

$$\sum_i \sum_j X_{ij}\, L_{ij}\,.$$

$$\sum_{i\epsilon I}\sum_{j\epsilon J} X_{ij}L_{ij}\ \rightarrow min \tag{4}$$

In the above model of the transport problem, when the limiting conditions are unequal or have the form of equality $\sum_i\ a_i=\sum b_j$, such models are called closed models.

To determine the quantitative solutions of problems like the above, it will be necessary to transform them into an open model, that is, a model in which the limiting conditions consist only of equations (2.1). Let's consider an extended model of the transport problem and its matrix form [10]. For example, a set of numbers of quarries or warehouses that send raw materials

$$I=\{1,2,\ldots I,\ldots,m\}$$

And the numbers of factories producing finished products= $\{1,2,\ldots\mathrm{j},\ldots,\mathrm{n}\}$. Sender of raw materials the volume of shipment of each quarry i is equal to a_i, and the volume of shipment for all quarries $i\epsilon I$ is equal to $a_1\,,a_2\ \ldots\ a_i\ ,\ldots\ ,\ a_m$, while the volume of consumption of raw materials by all plants for the production of finished products is indicated in the form $b_1\ ,b_2\ ,\ldots\ ,\ b_j\ \ \ldots,b_n$. For each address of the sender i and the recipient j, we mathematically express the condition that the volume of the transported flow of raw materials cannot exceed its capacity for sending and receiving raw materials.

For example, it is necessary to draw up possible options for the formation of raw material flows for the recipient $j=1$: $j=1$-cargo flow sent to the recipient from $i=1$-sender is X_{11} in tons, $i=2$ -cargo flow from X_{21}, etc. from $\mathrm{i}=\mathrm{m}$ -sender, and X_{ml}

represents values in tons. The sum of the flows $X_{11} + X_{21} + \cdots + X_{ml}$, i..e. all $i = l, 2, \ldots, m$ -from sender $j - 1 -$ or 2 - or n - the sum of the raw material flows directed to the receiver must be equal to the consumption volume of the load of this receiver, i.e.

$$\left.\begin{array}{c} X_{11} + X_{21} + \ldots + X_{m1} = b_1 ; \\ X_{12} + X_{22} + \ldots + X_{m2} = b_2 ; \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ X_{1m} + X_{2m} + \ldots + X_{mn} = b_n ; \end{array}\right] \quad (5)$$

Now let's formulate the requirements for the sum of volumes of raw material flows coming from each sender's address. Known in the general case, i is the sum of cargo flows sent from a digital $j = 1, 2, \ldots j, \ldots, n$ to all consumers of digital explosives $X_{11} + X_{12} + \ldots + X_{1n}$ should be equal to the possibility of sending the goods from this sender, i.e.[11–13].

$$\left.\begin{array}{c} X_{11} + X_{21} + \ldots + X_{m1} = a_1 ; \\ X_{12} + X_{22} + \ldots + X_{m2} = a_2 ; \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ X_{1m} + X_{2m} + \ldots + X_{mn} = a_n ; \end{array}\right] \quad (6)$$

At the same time, the sum of X_{ij} transport works formed during the movement of the above-mentioned X_{ij}, L_{ij} flows should be the smallest, i.e.

$$\begin{array}{c} +L_{11} + X_{21} + L_{21} + \cdots + X_{m1} + L_{m1} + X_{12} + L_{12} + X_{22} + L_{22} + \cdots + X_{m2} + L_{m2} + \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ +X_{1m} + L_{1m} + X_{2m} + L_{2m} + \cdots + X_{mn} + L_{mn} + X_{11} + L_{11} + X_{12} + L_{12} + \cdots + X_{m1} L_{m1} + \\ X_{21} L_{21} + X_{22} L_{22} + \cdots + X_{m2} L_{m2} + X_{m1} L_{m1} + X_{m2} L_{m2} + \cdots + X_{mn} L_{mn} \rightarrow min \end{array} \quad (7)$$

Another condition of the extended model of the transport problem is that its volatility cannot be negative. Because the load current with a negative value does not physically exist and does not make sense. This condition is expressed under the condition that the values of the variables for all $I=\{1, 2, \ldots, I, \ldots, t\}$ and $J=\{l, 2, \ldots, j, \ldots, n\}$ are non-negative, that is

$$X_{ij} \geq 0, \quad i \epsilon I, \quad j \epsilon J \quad (8)$$

Thus, the open model of the transport problem is expressed as follows. It is necessary to determine the value of X_{ij} of cargo flows directed from the sender to the consumer, at which the total cost of performing the entire transport work, measured in ton-kilometers P_{ij} or the cost of performing one unit of transport work S_{ij}, is equal to the lowest value of N_{ij}.

$$P_{um} = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} \cdot l_{ij} \rightarrow min \quad (9)$$

$$\text{or } S_{um} = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} \cdot S_{ij} \rightarrow min \quad (10)$$

the following conditions must be met:

$$\sum_{i=1}^{m} X_{ij} = b_j, \; j \in \{1, 2, \ldots, m\}; \quad (11)$$

$$\sum_{j=1}^{n} X_{ij} = a_j, \; i \in \{1, 2, \ldots, n\}; \quad (12)$$

$$X_{ij} \geq 0, \; i \in \{\overline{1 - n}\}, \; j \in \{\overline{1 - m}\}, \quad (13)$$

To determine the numerical solution of the linear programming transport problem (LPTP), you must have $\sum_i a_i = \sum_j \hat{a}_j$. For the initial data of the problem, this condition may be impossible, i.e. 1) $\sum_i a_i > \sum_j a_j$ or, conversely, 2) $\sum_i a_i < \sum_j b_j$ In such cases, the initial data of the question is artificially output to $\sum_i a_i = \sum_j b_j$. For case 1, in which the above condition is met $\sum_i a_i = \sum_j b^1{}_j$, And for case 2 $\sum_i a^1{}_i = \sum_j b_{j_j}$ the values are taken and have a value $\sum_i a_i$ va $\sum_j b^1{}_j$ or $\sum_i a^1{}_i$ va $\sum_j b_{j_j}$ solved for values [11-15].

The feature of TPLP is that the problem statement and model can also be expressed in the form of a matrix for a given initial information system (Table 2.1).

The sum of X_{ij} in each row is equal to a_i, that is, $\sum_i x_{ij} = a_i$, and the sum of X_{ij} in each column is equal to b_j, that is, $\sum_j X_{ij} = b_j$. The sum of a_i for shippers is $\sum_i a_i$ and the sum of a_j for receivers is equal to $\sum_i b_j$.

$$\sum_i a_i = \sum_i b_j \tag{14}$$

At the next stage, it is possible to formulate the transport issue necessary for the optimization of raw material flows.

One of the methods used in most cases of solving the transport problem is the integration of the transport plan. The essence of the method is as follows: we look for the cell of the transport table with the lowest transport tariff [16, 17] .

Fulfillment of the condition (5) or (11) in the model of the above transport problem, the fulfillment of the requirement of consumption of raw materials of FPMP by all recipients, shows the fulfillment of the transportation volume that should be sent from all shippers according to the condition (6) or (12). In general, the algorithm for solving the transportation problem can be as follows [18, 19]:

- drawing up a schedule of transport work;
- investigation of the issue in private;
- drawing up a basic plan;
- check for changes to the basic plan;
- calculation of the potential for the transportation plan;
- checking the basic plan for optimality;
- redistribution of the transportation plan;
- if the optimal solution is found, go to item 9, if not, return to item 5;
-making a graph of waste transportation by calculating the total volume of cargo transportation.

Table 1. Representation of TPLP in the form of a matrix

j/i	1	2	…	J	…	n	a_i
1	$l_{11}(S_{11})$ $X_{11}=?$	$l_{12}(S_{12})$ $X_{12}=?$		$l_{1j}(S_{1j})$ $X_{1j}=?$		$l_{1n}(S_{1n})$ $X_{1n}=?$	a_1
	$l_{21}(S_{21})$ $X_{21}=?$	$l_{22}(S_{22})$ $X_{22}=?$		$L_{2j1}(S_{2j})$ $X_{2j}=?$		$L_{2n}(S_{2n})$ $X_{2n}=?$	a_2
…							…
i	$l_{i1}(S_{i1})$ $X_{i1}=?$	$l_{i2}(S_{i2})$ $X_{i2}=?$		$l_{ij}(S_{ij})$ $X_{ij}=?$		$l_{in}(S_{in})$ $X_{in}=?$	a_i
…							…
m	$l_{m1}(S_{m1})$ $X_{m1}=?$	$l_{m2}(S_{m2})$ $X_{m1}=?$		$l_{mj}(S_{mj})$ $X_{mj}=?$		$l_{mn}(S_{mn})$ $X_{mn}=?$	a_m
b_j	b_1	b_2	…	b_j	…	b_n	$\sum a_i = \sum b_j$

3 Results and discussions

The price matrix for delivering one unit of product from the shipping address to the receiving address is as follows (Table 1). We will check the necessary and sufficient condition for solving the problem.

$$\sum a = 1000 + 300 + 100 + 400 + 40 = 1840$$

$$\sum b = 600 + 250 + 450 + 240 + 300 = 1840$$

The balance condition is satisfied. Stocks are equal to demand. So, the model of the transport problem is closed.
We will enter the necessary data into the distribution table (Table 2).

Table 2. 1st stage. Looking for a first base plan.

	1	2	3	4	5	Stocks
1	875 [60]	450[250]	350[450]	380[240]	720	1000
2	646[300]	620	586	570	630	300
3	752,5	545	420	743	450[100]	100
4	714[200]	625	620	543	560[200]	400
5	980[40]	712	560	670	770	40
Demand	600	250	450	240	300	

1. Using the least cost method, we determine the first base plan of the transportation problem.
2. We determine the number of occupied cells of the table, they are 9, but they should be $m + n - 1 = 9$.

The amount of the objective function for this base plan is:

$$F(x) = 8 \cdot 560 + 8 \cdot 240 + 8 \cdot 200 + 5 \cdot 300 + 7 \cdot 100 + 6 \cdot 250 + 9 \cdot 50 + 12 \cdot 100 + 5 \cdot 40 = 13550$$

2nd Stage. Improve the base plan. We check the optimality of the basic plan. Based on the condition $u_1 = 0$ we determine the primary potential $u_i,\ v_j$ based on the occupied cells of the table, where $u_i + v_j = c_{ij}$.

$$u_1 + v_1 = 875; 0 + v_1 = 875;\ v_1 = 875$$

$$u_2 + v_1 = 646;\ 875 + u_2 = 646;\ \mathrm{u}_2 = -229$$

$$u_4 + v_1 = 714;\ 875 + u_4 = 714;\ u_4 = -161$$

$$u_4 + v_5 = 560;\ -161 + v_5 = 560;\ v_5 = 721$$

$$u_3 + v_5 = 450;\ 721 + u_3 = 450;\ u_3 = -271$$

$$u_5 + v_1 = 980;\ u_5 + 875 = 980;\ u_5 = 105$$

$$u_1 + v_2 = 450;\ 0 + v_2 = 450;\ v_2 = 450$$

$$u_1 + v_3 = 350;\ 0 + v_3 = 350;\ v_3 = 350$$

$$u_1 + v_4 = 380;\ 0 + v_4 = 380;\ v_4 = 380$$

The basic plan is not optimal because there is a value of empty cells, for which $u_i + v_j > c_{ij}$

$$(1;5): 0 + 721 > 720;\ D15 = 0 + 721 - 720 = 1$$

$$(5;5): 105 + 721 > 770;\ D55 = 105 + 721 - 770 = 56$$

$$max(1{,}56) = 56$$

We select the maximum value of the empty cell (5.5):770. To do this, we put the "+" sign in the perspective cell (5,5), we put the alternating "-", "+", "-" signs on the other edges of the polygon. The cycle is given in the table below (5.5> 5.1> 4.1> 4.5).

From the x_{ij} loads in the minus cells, we choose the smallest one, i.e $\gamma = \min(5{,}1) = 40$

We add 40 to the amount of loads in the plus cells, and subtract 40 from the amount of loads in the minus cells. As a result, we will have a new base plan. (Tables 3, 4, 5)

Table 3.

№	1	2	3	4	5	Stocks
1	875 [60]	450[250]	350[450]	380[240]	720	1000
2	646 [300]	620	586	570	630	300
3	752,5	545	420	743	450[100]	100
4	714[200][+]	625	620	543	560[100] [-]	400
5	980[40][-]	712	560	670	770[+]	40
Demand	600	250	450	240	300	

Table 4.

№	1	2	3	4	5	Stocks
1	875[60]	450[250]	350[450]	380[240]	720	1000
2	646[300]	620	586	570	630	300
3	752.5	545	420	743	450[100]	100
4	714[240]	625	620	543	560[160]	400
5	980	712	560	670	770[40]	40
Demand	600	250	450	240	300	

Table 5.

	v 1 =8	v2=5	v3=8	v4=8	v5=8
i 1=0	875 [60]	450[250]	350[450]	380[240]	720
i2=-3	646[300]	620	586	570	630
u3=-l	752,5	545	420	743	450[100]
u4=l	714[240]	625	620	543	560[160]
u5=—3	980	712	560	670	770[40]

We check the optimality of the basic plan. Based on the occupied cells of the table, we find the initial potentials u_i, v_j, according to the condition $u_1 = 0$, they have $u_i + v_j = c_{ij}$

$$u_1 + v_1 = 875; 0 + v_1 = 875;\ v_1 = 875$$

$$u_2 + v_1 = 646;\ 875 + u_2 = 646;\ \mathrm{u}_2 = -229$$

$$u_4 + v_1 = 714;\ 875 + u_4 = 714;\ u_4 = -161$$

$$u_4 + v_5 = 560;\ -161 + v_5 = 560;\ v_5 = 721$$

$$u_3 + v_5 = 450;\ 721 + u_3 = 450;\ u_3 = -271$$

$$u_5 + v_1 = 980;\ u_5 + 875 = 980;\ u_5 = 105$$

$$u_1 + v_2 = 450;\ 0 + v_2 = 450;\ v_2 = 450$$

$$u_1 + v_3 = 350;\ 0 + v_3 = 350;\ v_3 = 350$$

$$u_1 + v_4 = 380;\ 0 + v_4 = 380;\ v_4 = 380$$

The base plan is not optimal because there are values of empty cells for which $u_i + v_j > c_{ij}$.

$$(1;5): 0 + 721 > 720;\ D15 = 0 + 721 - 720 = 1$$

We select the maximum value of the empty cell (1.5):720. To do this, we put the "+" sign in the perspective cell (5,5), we put the alternating "-", "+", "-" signs on the other edges of the polygon.

Load cycle: 944260 (Table. 6)

Table 6.

0	1	2	3	4	5	Stocks
1	875[60][-]	450(250]	350[450]	380[240]	720[+]	1000
2	646(300]	620	586	570	630	300
3	752,5	545	420	743	450 (100]	100
4	714[240][+]	625	620	543	560 (160][-]	400
5	980	712	560	670	770[40]	40
Demand	600	250	450	240	300	

The cycle is given in the table below (1-5, 1-1, 4-1, 4-5). From the xij loads in the minus cells, we choose the smallest one, i.e. u = min (1, 1) = 60. We add 60 to the load size in the plus cells, and subtract 60 from the load size in the minus cells. As a result, we will have a new base plan. (Table 7) We check the optimality of the basic plan. According to busy cells of the table u_i, v_j we find primary potentials, according to the condition $u_1 = 0$ where $u_i + v_j = c_{ij}$.

$$u_1 + v_2 = 450;\ 0 + v_2 = 450;\ v_2 = 450$$

$$u_1 + v_3 = 350;\ 0 + v_3 = 350;\ v_3 = 350$$

$$u_1 + v_4 = 380;\ 0 + v_4 = 380;\ v_4 = 380$$

$$u_1 + v_5 = 720;\ 0 + v_5 = 720;\ v_5 = 720$$

$$u_3 + v_5 = 450;\ 720 + u_3 = 450;\ u_3 = -270$$

$$u_4 + v_5 = 560;\ 720 + u_4 = 560;\ v_4 = -160$$

$$u_4 + v_1 = 714;\ -160 + v_1 = 714;\ v_1 = 874$$

$$u_2 + v_1 = 646;\ 874 + u_2 = 646;\ \mathrm{u}_2 = -228$$

$$u_5 + v_5 = 770;\ 720 + u_5 = 770;\ u_5 = 50$$

The base plan is optimal because all values of empty cells satisfy the condition $u_i + v_j \le c_{ij}$.

Table 7. Base Plan (Optimal Plan)

	v1=8	v2=5	v3=8	v4=8	v5=8
u1=0	875	450[250]	350[450]	380[240]	720[60]
u2=-3	646[300]	620	586	570	630
u3=-1	752ю5	545	420	743	450[100]
u4=1	714[300]	625	620	543	560[100]
u5=-3	980	712	560	670	770[40]

$$F(x) = 450 \cdot 250 + 350 \cdot 450 + 380 \cdot 240 + 720 \cdot 60 + 646 \cdot 300 + 450 \cdot 100 + 714 \cdot 300 + 560 \cdot 100 + 770 \cdot 40 = 944200 \text{ t.km}$$

We analyze the optimal plan. In the optimal plan, the cargo turnover is 2300 tkm less than the first option.

We make a graph that explains the essence and result of the problem and is provided for in the algorithm [20, 21]: (Fig. 1).

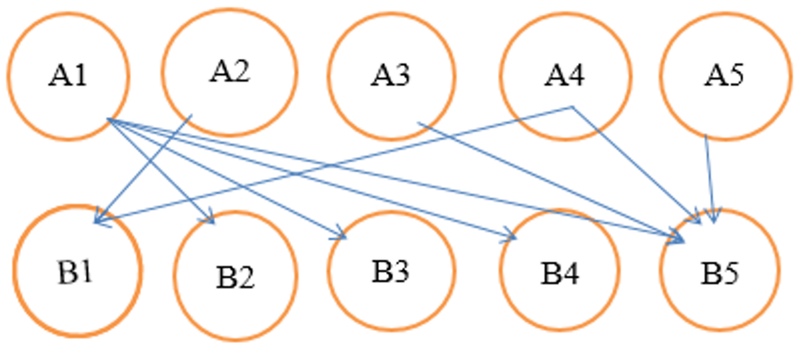

Fig. 1. Graph of cargo transportation

From the 1st warehouse, it is necessary to send goods to the 2nd (250), 3rd (450), 4th (240) and 5th customer
All goods from the 2nd warehouse should be sent to the 1st customer. All goods from the 3rd warehouse should be sent to the 5th customer.
It is necessary to ship from the 4th warehouse to the 1st customer (300) and to the 5th customer (100).
All goods from the 5th warehouse must be sent to the 5th customer.

4 Conclusions

As a result of the conducted research, the following can be noted:

1) the problem of optimization of cargo flows is formally expressed in the form of a transport model of linear programming and allows to determine a plan that provides a minimum of transport work.

2) the essence of solving the given problem is that we define some basic plan and check it for optimality. If the plan is optimal, then the solution has been found. If it is not optimal, the plan is improved using the method of potentials until the optimal plan is found. Cargo turnover is 944,200 ton km. The average cargo flow is 80 tons.

The average freight distance is 513.15 km.

PHP programming language was chosen for its development in order to make it possible to place the program on the Internet and make its use public.

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