# Transverse vibrations of underground pipeline with pinched ends

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**Abstract.** The study of transverse oscillations of extended underground structures from the action of constant harmonic forces is of practical interest. Transverse vibrations of an underground cylindrical structure of finite length under the action of a system of concentrated forces that change according to a harmonic law are considered. The environment surrounding the structure is assumed to be elastic, while the interaction of the "environment-structure" system is studied. When considering the problem, the Fourier method and the results of solving the problem of vibrations of an unbounded cylindrical structure in an elastic medium from the action of concentrated harmonic forces were used. The problem of transverse vibrations of a cylindrical rod of finite length interacting with an elastic homogeneous medium is solved by the method of compensating loads. In this case, the structure is considered a source of waves radiated into space and satisfying the conditions of radiation to infinity. The resulting solution is approximate and reliable within the length of the rod. The analysis of the influence of soil conditions and geometric dimensions on the amplitude of oscillations of an underground structure was carried out according to the graphs of the amplitude-frequency characteristic of the object.

## 1 Introduction

Most extended structures (transport and hydraulic structures, pipelines for various purposes, and others) have a cylindrical shape with a circular cross-section. Such structures can be underground and above ground. The calculation of ground structures in the form of beams and slabs on an elastic foundation is considered in [1]. Underground structures can be deeply buried and not deeply buried; the latter are considered structures with a burial depth equal to no more than the structure's diameter. Surface waves are dangerous for not deeply laid structures. In [2], the oscillation of an elastic half-space with a cylindrical cavity under the influence of Rayleigh (surface) waves was studied, and in [3], the possibility of damping the oscillations of a cylindrical structure not deeply embedded in the elastic half-space using a single-mass dynamic vibration damper was considered. The damping of vibrations of an underground cylindrical structure using a multi-mass dynamic vibration

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damper was considered [4], and the effectiveness of this approach was proved. The longitudinal vibrations of an infinite cylindrical rod from the action of harmonic forces are considered in [5], and the possibility of damping the longitudinal vibrations of a cylindrical rod from constantly acting harmonic forces with a stable frequency of action is studied in [6].

Massive structures are of particular interest in the dynamics of structures. Studies of the spatial stress-strain state of earthen dams are devoted to [7,8].

Underground cylindrical structures with an annular cross-section, as a rule, have a sufficiently high rigidity in cross-section and therefore are considered non-deformable, rigidly movable in the transverse direction. The rigid movement of a cylindrical disk in the section plane and the possibility of damping its vibrations using multi-mass dynamic vibration dampers were considered [6].

In [9], the dynamics of a physically nonlinear viscoelastic cylindrical shell with a concentrated mass was studied. Nonaxisymmetric vibrations of axisymmetric structures with attached masses and troughs, spatially stressed state, and dynamic characteristics of earth dams are considered in [10, 11].

The dynamic behavior of various structures has been studied in the works [12, 13, 14, 15]. At the same time, nonlinear oscillations of an orthotropic viscoelastic rectangular plate under periodic loads are considered [12], an assessment of the level of vibration waves at different distances [13], an assessment of the stress state and dynamic characteristics of a planar and spatial structure [14] forced axisymmetric oscillations of a viscoelastic cylindrical shell [15]. Interesting results were obtained when evaluating the dynamics of the dynamic damper's elastic plate and the liquid section [16]. Transverse vibrations of underground pipelines under different laws of pipe interaction with the surrounding soils and the dynamic interaction of the embedded cylindrical rod under axial harmonic forces are considered respectively by the authors of [17, 18].

The stress state of commonly used two-dimensional structures was investigated in [19, 20]. The bending of multilayer plates lying on an elastic half-space, taking into account tangential stresses, was studied in [19], and in [20], a load compensation method was proposed concerning the problems of equilibrium, vibration, and stability of plates and membranes.

## 2 Decision

To solve the problem of transverse vibrations of a cylindrical rod of finite length (Fig. 1a), we use the method of compensating loads [20]. The sought solution, in this case, will consist of the sum of two solutions - the main one and the compensating one: $W = W_{о} + W_{к}$. The main solution, having singularities within $2\,nL$, must satisfy the inhomogeneous equation

$$EJ\frac{d^4W_o}{dz^4} + m\frac{d^2W_o}{dt^2} + q = P(t) \tag{1}$$

Where

$$\mathrm{P(t)}=P_o \cdot e^{-i\omega t} \tag{2}$$

However, it does not satisfy the boundary conditions; therefore, it is necessary to find a solution to the problem shown in the diagram (Fig. 1b).

Using the results of the problem on vibrations of an unlimited pipeline [4], we write the main solution in the form

 https://doi.org/10.1051/e3sconf/202340103005

$$W_0 = \frac{1}{2}\sum_{m=1}^{n} C_m \alpha_m \{\frac{p^2 H_1(m_o\alpha)}{H_1(\alpha)}[H_0(\alpha)\frac{1}{\alpha}H_0(m_o\alpha)\frac{p^2 H_1(\alpha) + 2H_1'(\alpha)}{H_1(m_o\alpha)\{2(m_o\alpha)^{-2} - 1\}}] - $$
$$-H_0(m_o\alpha) + \frac{2}{m_o\alpha}H_1(m_o\alpha)\} sin\alpha_m z = \frac{1}{2}\sum_{m=1}^{n} C_m \Delta_1 \, sin\alpha_m z \quad (3)$$

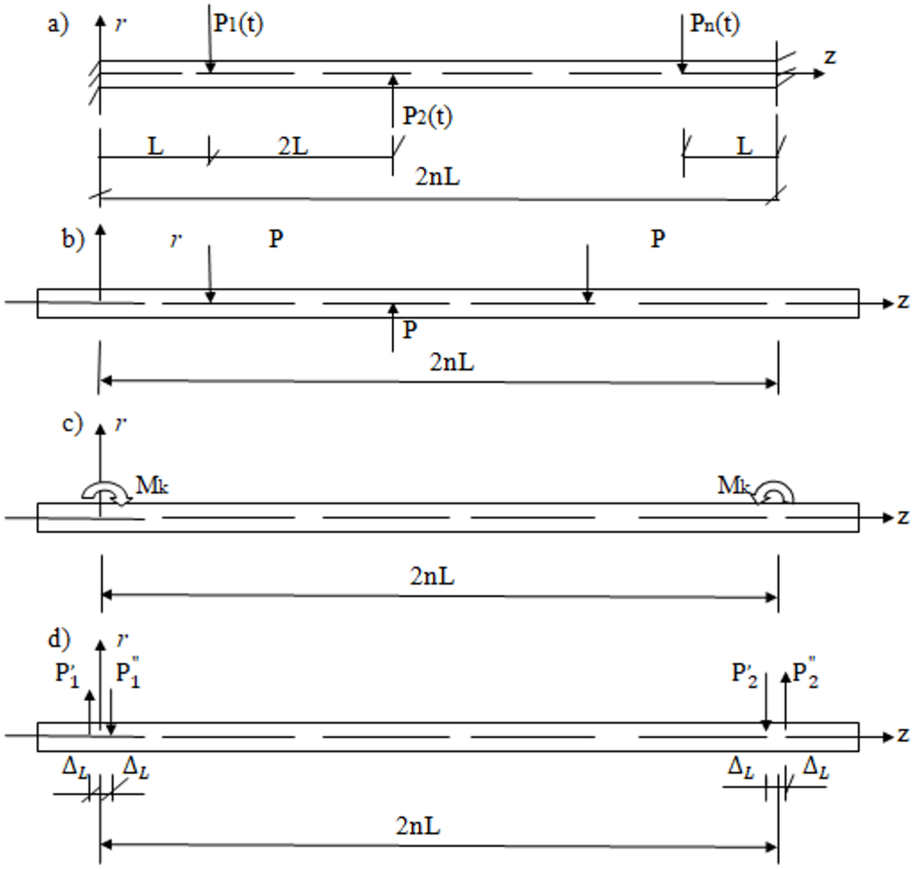


**Fig. 1.** Design schemes: a is pipelines of finite length with pinched ends; b is definitions of main solution; c, d are definitions of compensating solution

The next step is to determine the compensating solution. The final solution $W = W_0 + W_к$ must satisfy the boundary conditions of our problem, that is, at z = 0 and z = 2nL

$$\frac{dW}{dz} = 0 \quad (4)$$

therefore, to obtain a compensating solution, we use the conditions for the dissatisfaction with the main solution $W_0$ about the boundary conditions. According to Fig. 2, at the points z = 0 and z = 2nL, the angles of rotation appear

$$\frac{dW_0}{dz} = \frac{1}{2}\sum_{m=1}^{n} C_m \Delta_1 \, sin\alpha_m z \quad (5)$$

which are not equal to zero. To satisfy the boundary conditions of our problem, it is necessary to create the angles of rotation (at z = 0 and z = 2nL) opposite in sign and equal in magnitude using compensating loads. The compensating load will be the moments applied at the boundaries of the final beam.

Figure 1c will serve as a design scheme for determining the compensating solution. But first, it is necessary to determine the directions and magnitude of the moments $M_к$ based on the conditions formulated above.

To determine the compensating solution, the design scheme (Fig. 1c) will be slightly changed, passing from moments to concentrated forces (Fig. 1d) [4].

For each of these forces, we will compose a solution separately. First of all, consider the effect of the force $P'_1$, on an infinite beam.

We express the displacements of points in the medium in terms of elastic potentials, which have the form

$$\varphi = A \cdot H_1^{(1)}(k_1^* r) \cdot sin\alpha_1(z - \Delta_L)cos\theta, \quad \alpha_1 = \frac{\pi}{2L}$$
$$\Psi_1 = B \cdot H_1^{(1)}(k_2^* r) \cdot cos\alpha_1(z - \Delta_L)sin\theta \tag{6}$$
$$\Psi_2 = C \cdot H_1^{(1)}(k_2^* r) \cdot cos\alpha_1(z - \Delta_L)cos\theta.$$

The conditions that were used to solve the problem of vibrations of an infinite pipeline are preserved when considering vibrations of a pipeline of finite length [4]. Omitting intermediate calculations identical in [4] and taking into account the relations

$$A = C\frac{(\alpha_1^2 - k_1^2)H_1(k_2 r_0)}{\alpha_1 \cdot H_1(k_1 r_0)}$$
$$B = -C\frac{[\frac{(\alpha_1^2 - k_1^2)H_1(k_1 r_0)}{\alpha_1^2} + 2H'_1(k_2 r_0)]H_1(k_2 r_0)\alpha_1}{[\frac{H_0(k_2 r_0)}{k_2 r_0} + H''_1(k_2 r_0)]H_1(k_1 r_0)k_1\alpha_1} \tag{7}$$

we give the final form of the equilibrium equation for the case of the action of the force $P'_1$,

$$EJ\frac{1}{2}C \cdot \alpha_1^4 \Delta'_1 sin\alpha_1(z - \Delta_L) - \omega^2\rho_0 F_0 \frac{1}{2} C\Delta'_1 sin\alpha_1(z - \Delta_L) +$$
$$+\mu C\alpha_1 \left[\frac{\lambda}{\mu}\Delta'_2 - \Delta'_3 \alpha_1^2 r_0^2\right] sin\alpha_1(z - \Delta_L) = P'_1 \tag{8}$$

($\Delta'_1, \Delta'_2, \Delta'_3$ are the coefficients defined in [4], but instead of the parameter $\alpha_m = (2m - 1)\pi/2L$, you need to substitute α = π / 2L).
From (8), we obtain

$$C = \frac{P'_1/[2\alpha_1 L\omega^2\rho F_0]}{\{\left[\frac{T}{\alpha^2+\pi^2\eta^2} - \Omega\right]\cdot\frac{\Delta'_1}{2} + \frac{2}{m^2}\frac{[(m_0^2-2)\Delta'_1 - \Delta'_3\pi^2\eta^2]}{(4\alpha^2+\pi^2\eta^2)}\}} \tag{9}$$

The equation of vibrations of an infinite beam from the action of the force $P'_1$ has the form

$$W'_1 = N_1 P'_1 sin\alpha_1(z - \Delta_L)e^{-i\omega t} \tag{10}$$

where

$$N_1 = \frac{\Delta'_1}{4L\omega^2\rho F\left\{\left[\frac{T}{\alpha^2+\pi^2\eta^2} - \Omega\right]\cdot\frac{\Delta'_1}{2} + \frac{1}{m_0^2}\frac{[(m_0^2-2)\Delta'_2 - \Delta'_3\pi^2\eta^2]}{(\alpha^2+\pi^2\eta^2)}\right\}} \tag{11}$$

Similarly, you can get a solution under the action of the forces $P''_1$, $P'_2$, $P''_1$. Without repeating intermediate calculations, we present the final form of the solution:

by force $P_1^{"}$

$$W_1^{"} = N_1 P_1^{"} sin\alpha_n (z + \Delta_L) e^{-i\omega t} \quad (12)$$

by force $P_2^{'}$

$$\text{W}= N_2 P_2^{'} sin\alpha_n [z + (nL - \Delta_L)] e^{-i\omega t}, \quad (13)$$

Where

$$N_2 = \Delta_1^{"}/4L\omega^2 \rho F_0 \{\Big[T \cdot \frac{\Omega(2n-1)^4}{[\alpha^2+(2n-1)^2\pi^2\eta^2]} \cdot \frac{\Delta_1^{"}}{2} + \frac{[(m_0^2-2)\Delta_2^{"}-\Delta_2^{"}(2n-1)^2\pi^2\eta^2]}{m_0^2[\alpha^2+(2n-1)^2\pi^2\eta^2]}\Big\}, \quad (14)$$

by force $P_2^{"}$

$$W_2^{"} = N_2 P_2^{"} sin\alpha_n [z + (nL + \Delta_L) e^{-i\omega t} \quad (15)$$

According to the principle of addition of influences taking into account $P_1^{'} = P_1^{"}, P_2^{'} = P_2^{"}$

$$W^* = \left(W_1^{'} - W_1^{"}\right) + \left(W_2^{"} - W_2^{'}\right) = -2[N_1 P_1 \Delta_L \sin(\alpha_1 \Delta_L) \cdot \frac{\cos(\alpha_1 z)}{\alpha_1 \Delta_L} +$$

$$+N_2 P_2 \Delta_L \sin(\alpha_n \Delta_L) \cdot \frac{\cos[\alpha_n(z+nL)]}{\alpha_n \Delta_L}]\alpha \cdot e^{-i\omega t}. \quad (16)$$

Because $P_1\Delta_L = M_{k_1}$ and $P_2\Delta_L = M_{k_2}$,

$$W^* = -2[N_1 \alpha_1 M_{k_1} \sin(\alpha_1 \Delta_L) \cdot \frac{\cos(\alpha_1 z)}{\alpha_1 \Delta_L} +$$

$$+N_2 \alpha_n M_{k_2} \sin(\alpha_n \Delta_L) \cdot \frac{\cos[\alpha_n(z+nL)]}{\alpha_n \Delta_L}] e^{-i\omega t} \quad (17)$$

$$\lim_{\Delta_L \to 0} W^* = W_k,\ \lim_{x \to 0} \frac{sinx}{x} = 1,$$

$$W_k = N_1 M_{k_1} \frac{\pi}{2L} \cos\left(\frac{\pi}{2L} z\right) + \frac{(2n-1)}{2L} \pi N_2 M_{k_2} \cos[\frac{(2n-1)}{2L} \pi (z + nL)]\} e^{-i\omega t}. \quad (18)$$

Unknown moments $M_{k_1}$ and $M_{k_2}$ will be determined from the condition

$$\frac{dW_0}{dz} + \frac{dW_k}{dz} = 0, \quad (19)$$

From (19) at z = 0, we determine the value of the moment $M_{k_2}$

$$M_{k_2} = -[(2L)^2 \sum_{m=1}^{n} C_m \alpha_m \Delta_1]/\{(2n-1)^2 \pi^2 N_2 \cdot 4\sin[(2n-1)n\pi/2]\}, \quad (20)$$

and for z = 2nL

$$M_{k_1} = \left\{\cos\left[\frac{(2n-1)n\pi}{2}\right] \sum_{m=1}^{n} C_m \alpha_m \Delta_1\}/[2N_1 \left(\frac{\pi}{2L}\right)^2 sin\left(\frac{n\pi}{2}\right)\right] -$$

$$-[\sum_{m=1}^{n} C_m \alpha_m \Delta_1 \cos(\alpha_m 2nL)]/\left[N_1 \left(\frac{\pi}{2L}\right)^2 sin\left(\frac{n\pi}{2}\right)\right]. \quad (21)$$

Substituting (20), (21) into (18) and summing with (3), we obtain a solution to the problem

$$W=\frac{1}{2}\sum_{m=1}^{n} C_m\Delta_1 sin\alpha_m z-\{\cos[\frac{(2n-1)n\pi}{2}]\sum_{m=1}^{n} C_m\alpha_m\Delta_1\}\cos\left(\frac{\pi z}{2L}\right)/[\frac{\pi}{2L}\sin(\frac{n\pi}{2})]+$$
$$\sum_{m=1}^{n} C_m\alpha_m\Delta_1\cos(\alpha_m nL)]\cdot\cos\left(\frac{\pi z}{2L}\right)/[\frac{\pi}{L}sin\,(\frac{n\pi}{2})]+$$
$$+L\cos[\frac{(2n-1)\pi}{2L}(z+nL)\sum_{m=1}^{n} C_m\alpha_m\Delta_1]/\{(2n-1)\pi sin\left[\frac{(2n-1)n\pi}{2}\right]\}. \qquad (22)$$

## 3 Results

The expression for the transverse displacements of a rod of finite length (22) includes several quantities (η, $R_v$, Ω), which represent the physical and mechanical properties of the system under consideration in the form of a "medium-structure". To analyze the influence of these constants on the amplitude-frequency characteristics (AFC) of the system, the expression of the transverse displacements of the structure (22) will be presented as a function of the dimensionless frequency parameter α and in the range 0≤α≤5, we will carry out calculations for the AFC.

The arguments of the Hankel function, depending on the values of α, $R_v$, η, can take imaginary values. In this case, the Hankel function $H_n^1(iz)$ goes over the Macdonald function, and the following dependence is used

$$K_\vartheta(z)=\frac{\pi i}{2}e^{\frac{\vartheta\pi i}{2}}H_\vartheta^1\left(ze^{\frac{\pi i}{2}}\right) \qquad (21)$$

## 4 Discussion

Calculations were made according to the obtained equation of pipeline vibrations (22). The graphs of the amplitude-frequency characteristic (AFC) of the pipeline with pinched ends are shown in Fig. 2. If we compare the graphs of the frequency response of an infinite pipeline [4] and a pipeline of finite length with pinched ends (Fig. 1a), we can see that the values of the resonant frequencies are the same for both cases. For a pipeline with pinched ends, the amplitude of displacements at resonance is three times less than for a free pipeline (Fig. 2).

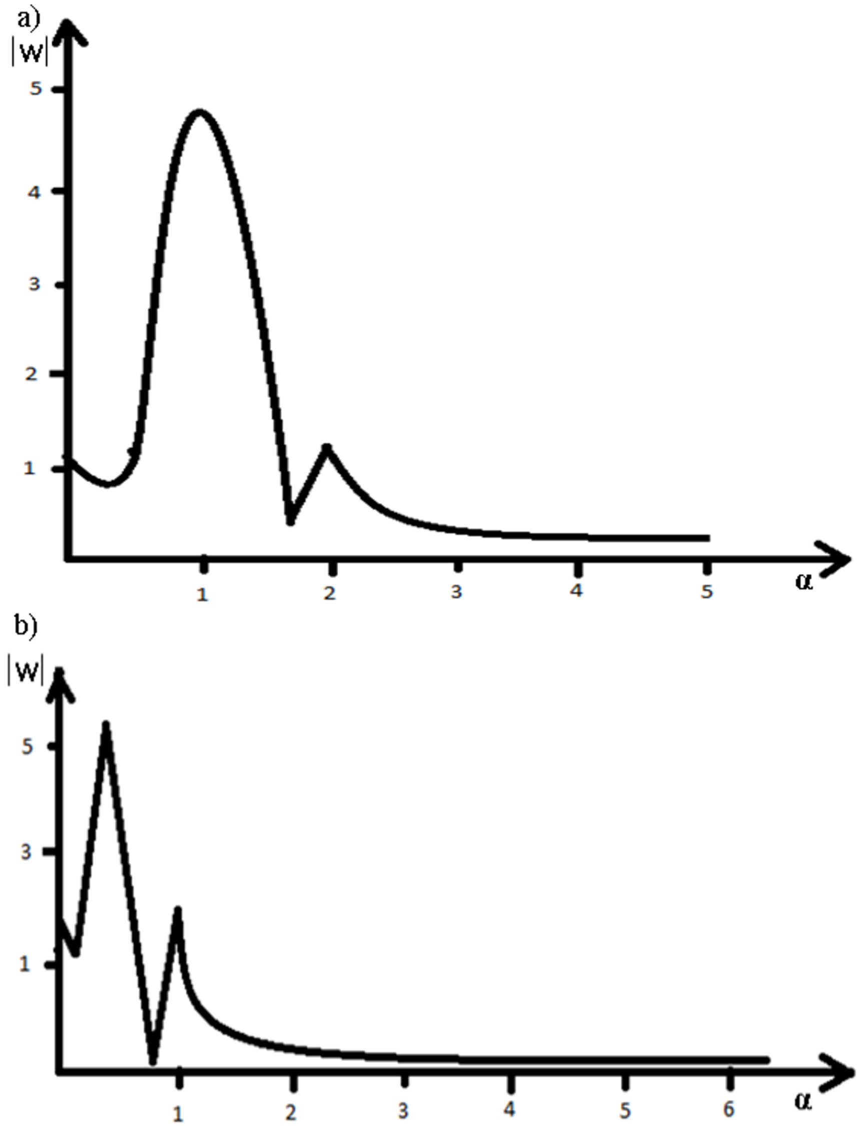


**Fig. 2.** AFC of structure at Ω=5, η=0,01: a - Rv=1; b - Rv=0,5.

## 5 Conclusions

The above studies of the lateral vibrations of an extended pipeline showed that the most unfavorable state for the construction is the first (main) mode of vibration. In this case, the first resonant zone has a rather wide range in the frequency domain, and the resonant amplitude is quite high compared to the peaks in other regions. The oscillation amplitudes outside the resonance zone are relatively small and, as a rule, are less dangerous for the structure. Changing the characteristics of the environment greatly affects the nature of the vibrations of the structure. A decrease in the value of Rv, which expresses the rigidity of the environment, leads to a shift of the resonance frequencies to the low frequency zone and leads to a narrowing of the resonance region (Fig. 2b). According to the frequency response graphs for an infinite structure and a structure of finite length with clamped ends (Fig. 1a), one can notice the proximity of the values of the resonant frequencies. In this case, the amplitude of displacements of a structure with pinched ends in a resonant state is three times less than for an endless structure.

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